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A Novel Decomposition and Coordination Approach for Large Day-Ahead Unit Commitment with Combined Cycle Units

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***Abstract*—Day-Ahead Unit Commitment (UC) is an important problem faced by Independent System Operators (ISOs). Midcontinent ISO as the largest ISO in US, solves a complicated UC problem involving over 45,000 buses and 1,400 generation resources. With the increasing number of combined cycle units (CCs) represented by configuration-based modeling, solving the problem becomes more challenging. The state-of-the-practice branch-and-cut method suffers from poor performance when there are a large number of CCs. The goal of this paper is to solve such large UC problems with near-optimal solutions within time limits. In this paper, our recently developed Surrogate Lagrangian Relaxation, which overcomes major difficulties of Lagrangian Relaxation by not requiring dual optimal costs, is significantly enhanced through adding quadratic penalties on constraint violations to accelerate convergence. Quadratic penalty terms are linearized through a novel use of absolute value functions. Therefore, resource-level subproblems can be formulated and solved by branch-and-cut. Complicated constraints within a CC unit are thus handled within a subproblem. Subproblem solutions are then effectively coordinated. Computational improvements on key aspects are also incorporated to fine tune the algorithm. As demonstrated by MISO cases, the method provides near-optimal solutions within a time limit, and significantly outperforms branch-and-cut.**

***Index Terms*--Branch-and-cut, combined cycle unit, mixed integer linear programming, linearization, Surrogate Augmented Lagrangian Relaxation, unit commitment**

I. Introduction

DAY-AHEAD Unit Commitment (UC) is an important problem faced by Independent System Operators (ISOs). The UC problem is formulated as a Mixed Integer Linear Programming (MILP) problem, and has specified solving time limits and solution quality requirements. Midcontinent Independent System Operator (MISO), as the largest ISO in US, manages a network with over 45,000 buses and 1,400 internal generation resources. It also has a large number of (over 10,000) virtual trading undertaken by participants [1] in day-ahead market. Considering the extended network size and the increasing number of generation resources, solving the UC problem becomes challenging.

Combined Cycle units (CCs), as in its name, combine Combustion Turbines (CTs) and Steam Turbines (ST) into one unit. Since high-temperature gas from CTs is not released into the atmosphere but is used by STs to generate extra power, a CC unit produces electricity at high efficiency and with low CO_2 emissions. There is thus an upward trend of installing CC units worldwide [2]. However, CC units bring significant challenges to UC problems in view of their complicated operation characteristics. A CC unit can operate in different modes or configurations, each with a particular set of commitment states of CTs and STs. Switching commitment states among CTs and STs should follow pre-defined configurations and allowable transition paths. For example, a ST cannot generate electricity if there is not enough heat from CTs. In view of the complicated transitions among configurations, a CC unit is commonly modeled as an aggregated unit [3], which does not utilize possible transitions. Some ISOs, such as MISO and CAISO, are looking into configuration-based modeling to save energy costs and incentivize participants to offer true marginal costs [4]-[7].

The current state-of-the-practice to solve large UC problems is using commercial MILP solvers, which are Branch-and-Cut (B&C) based and combined with heuristics. The B&C method attempts to obtain a convex hull of the entire problem by using valid cuts that gradually cut off areas outside the convex hull. The optimal solution can then be obtained at one of vertices by solving a linear programming problem. The B&C method explores problem linearity, but has no "local" concept. With the aggregated CC modeling, B&C generally performs well. However, when there are many CCs with configuration-based modeling, configuration transition constraints within a CC unit complicate the entire solution process. Significant computational efforts are required since cuts generated by B&C thus may not be tight and heuristics may not be effective. For a particular MISO case with 1,092 conventional units, 80 CCs (as stress tests), and about 15,000 virtuals looking ahead

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36 hours, the method could not obtain a feasible solution with MIP gap less than 3% in 3,600s. The goal of this paper is to solve such large and difficult problems for near-optimal solutions within time limits using MILP solvers.

To solve such a large and difficult problem, a decomposition and coordination approach is preferred. Lagrangian Relaxation (LR) was traditionally used to solve UC to exploit problem separability [8]. However, standard LR suffers from major difficulties. Our recently developed Surrogate Lagrangian Relaxation (SLR) overcomes difficulties of LR by not requiring the optimal dual value and not fully optimizing the relaxed problem [9]. SLR is further combined with B&C to simultaneously exploit separability and linearity [10]. However, when there are a large number of CC units, levels of constraint violations may not be reduced sufficiently fast within a time limit.

In this paper, SLR is significantly enhanced by adding quadratic penalties on constraint violations to accelerate the convergence for large and difficult UC problems. Surrogate Augmented Lagrangian Relaxation (SALR) is thus developed with subproblem formulations and effective coordination. Due to the introduction of quadratic penalties, the augmented relaxed problem is nonlinear, and existing Mixed Integer Quadratic Programming (MIQP) solvers are not efficient to solve large-scale problems. To linearize quadratic penalties, a novel use of absolute value functions is established by exploiting the fact that an absolute value function and a quadratic function have the same minimum. This linearization approach is much more accurate than other traditional linearization methods. The relaxed problem is then decomposed into individual resource subproblems solved by B&C. Subproblem solutions are then coordinated based on updating multipliers after solving one or multiple subproblems subject to surrogate optimality condition. For such a large problem, every aspect should be carefully handled to achieve an overall good performance. Several enhancements on key aspects including grouping resources in subproblem formulation, filtering out inactive transmission constraints, and developing effective heuristics to search feasible solutions are provided. Our work is timely and critical to solve large UC problems with increasing number of configuration-based CC units, and can be extended to other complicated large-scale MILP problems in power system and beyond.

The rest of the paper is organized as follows. Section II reviews the CC modeling and solution methodologies for UC problems with CC units. In Section III, the UC formulation with configuration-based CC modeling is summarized. In Section IV, the solution methodology including linearization scheme, subproblem formulation, subproblem solution coordination, and convergence proof is presented. In Section V, computational improvements on key aspects are introduced to enhance the overall performance. In Section VI, numerical testing results of one simple example and two MISO datasets are presented to demonstrate the efficiency and robustness of our method.

II. Literature Review

Subsection II-A reviews the modelling of CCs. Subsection II-B reviews solution methodologies for UC with CCs.

A. *Combined cycle unit modeling*

A CC unit typically consists of one to four CTs and one or two STs, and may operate in different modes/configurations based on combinations of commitment states of CTs and STs. Considering the widely used linear solvers to solve UC problems, CC units are typically modeled within MILP framework in three different representations. The simplest one is the aggregated modeling, which represents a CC unit as a conventional unit, ignoring all possible transitions among different modes.

A second CC representation is the component-based modeling, which represents CTs and STs as individual components with its own unit parameters, such as ramping rate limits, minimum up/down time limits, and startup/shutdown costs. Transitions within a CC unit are specified based on specific characteristics of the unit [11]. This model describes operating constraints for each component but requires modeling coupling steam constraints and may incur significant computational complexity especially when the number of CCs is large.

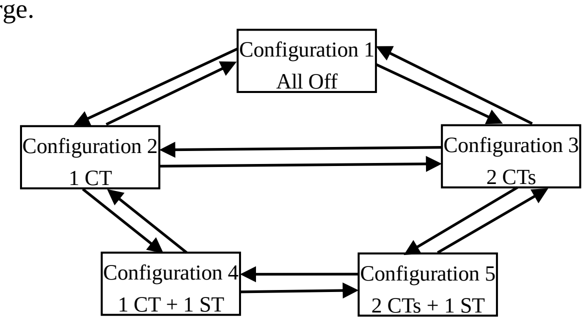

Fig. 1. Allowable transitions in a CC unit with 2 CTs and 1 ST.

Another CC representation is the configuration-based modeling, which captures CC unit operational characteristics by using multiple configurations with possible transitions. Figure 1 shows allowable transitions among five configurations within a CC unit containing two CTs and one ST [12]. In this modeling [5], [10], [12] each configuration was modeled as a conventional unit and had its own operating characteristics. Constraints such as generation limits and ramping rates of a CC configuration were formulated following those of a conventional unit. There are several ways to model transitions. In [10], startup/shutdown variables and big "M" operations were used to indicate a transition among configurations. However, it led to major computational efforts. In [5] and [12], transition variables were used such that transition constraints can be directly captured in linear forms.

B. *Methodologies for UC with CC units*

The B&C-based MILP solvers are widely used to explore problem linearity with the above CC presentations [5], [12]. The B&C method attempts to obtain a convex hull of the problem by using valid cuts that gradually cut off areas outside the convex hull. The optimal solution can then be obtained at one of vertices by solving a linear programming problem. However, strongest possible cuts that define facets of the convex hull are problem-dependent and may not be easily obtained. In the absence of these facet-defining cuts, branching operations and heuristics are required. MILP solvers typically work well for UC problems with the aggregated CC modeling. However, when there are many CC units with configuration-based modeling, transitions among commitment states significantly complicate the problem convex hull. Cuts generated may not be effective, and time-consuming branching operation or heuristics are typically required.

Lagrangian Relaxation (LR) with subgradient methods to update multipliers was traditionally used to solve UC problems without CC units by exploiting separability [8]. After relaxing system-coupling constraints and decomposing the relaxed problem into subproblems, subproblem solutions are coordinated based on subgradient directions, which are obtained after solving all subproblems with given multipliers. However, standard LR requires the knowledge of the optimal dual value or its estimates.

Major difficulties of LR has been overcome by our recently developed surrogate Lagrangian relaxation (SLR) [9], in which surrogate subgradient directions are obtained after solving one or a few subproblems at a time subject to the simple surrogate optimality condition to ensure surrogate subgradient directions form acute angles with the direction toward optimal multipliers. Computational effort and multiplier zigzagging are much reduced. More importantly, convergence to the optimal multipliers has been proved based on contraction mapping and does not require the knowledge of the optimal dual value. Reference [9] is mainly on the theory with no large and difficult MILP problems tested.

The SLR has been further synergistically combined with B&C to simultaneously exploit separability and linearity. The SLR+B&C had been applied to a particular UC example with 300 conventional and 40 CC units – the latter modeled by using logical expressions [10]. Transmission capacity constraints are ignored, and each unit has a single-block cost function. Computational time was significantly reduced compared with that of B&C. However, for MISO's cases with many transmission constraints, conventional and CC units with multiple-block cost functions, and virtual variables, levels of constraint violations are not reduced sufficiently fast to obtain near-optimal solutions within a limited solving time.

Augmented Lagrangian Relaxation (ALR) has a fast convergence by penalizing violations of coupling constraints [13]. Due to the introduction of quadratic terms, the relaxed problem becomes nonlinear and non-separable. The presence of quadratic terms can cause significant computational difficulties. Performance of existing MIQP solvers are often dramatically worse than that of linear cases. To address this issue, traditional way of linearization based on Taylor series expansion [14] was widely used. However, the approach is not effective since solution values tend to jump from one vertex to another as slopes of linear functions change. Adding proximal terms can alleviate this issue, and the proximal terms can be linearized following the way provided in PySP [15] by a simple interpolation. Our conference paper [16] summarized our working progress on linearization of ALR. It presented our investigation on maintaining the fast convergence of ALR by rewriting the entire Lagrangian function as summation of square functions with respect to each variable. However, due to the mathematical difficulties in completing squares and proof of preserving minima for practical problems as well as the implementation complexity, the method proposed in [16] is not applicable for large and practical UC problems.

III. PROBLEM FORMULATION

Subsection III-A summarizes the UC problem formulation with constraints of configuration-based CC modeling descried in subsection III-B.

A. Unit commitment formulation

The goal is to commit and dispatch conventional units, CC units, as well as dispatchable variables including virtuals, dispatchable demands, and dispatchable transactions to minimize energy supply and reserve costs while satisfying individual resource-level constraints, power balance, reserve requirements, and transmission capacity constraints for all looking ahead hours. The formulation presented in this section is based on MISO UC with configuration-based CC modeling [1], [12]. For CC units, each configuration is treated as a conventional unit with its own operating constraints. Transitions among configurations are developed by using transition variables and constraints. For presentation brevity, linear production costs are used and reserve is not considered. However, these features can be easily incorporated following [17], [18], and are considered in Examples 1-3 in Section VI.

For each conventional unit i at time t, decision variables include binary on/off status $x_{i,t}$, binary startup decision $u_{i,t}$, binary shutdown decision $y_{i,t}$, and continuous generation level $p_{i,t}$. For each configuration f of a CC unit j, decision variables include binary on/off status $x_{j,f,t}$, binary transition variable $v_{j,ff't}$ indicating a transition is made from configuration f to configuration f' following the allowable path, and continuous generation level $p_{j,f,t}$. Virtual, dispatchable demand, and dispatchable transaction variables $p_{n,t}$ are continuous and integer-independent.

<u>Objective function</u>

The objective of UC is to minimize total operating costs of all resources including conventional units (no-load cost $C_{NL\,i,t}x_{i,t}$, energy cost $C_{i,t}p_{i,t}$, and startup cost $C_{SU\,i,t}u_{i,t}$), CC units (no-load and energy costs for each configuration f, and transition cost $C_{j,f'f,t}v_{j,f'f,t}$ for each allowable transition) and virtuals, dispatchable demands, and dispatchable transactions (energy cost $C_{n,t}p_{n,t}$):

$$\min_{\{x,u,y,v,p\}} \sum_i \sum_t \left(C_{NL\,i,t} x_{i,t} + C_{i,t} p_{i,t} + C_{SU\,i,t} u_{i,t} \right)$$
$$+ \sum_j \sum_{f \in F_j} \sum_t \left(C_{NL\,j,f,t} x_{j,f,t} + C_{j,f,t} p_{j,f,t} + \sum_{f' \in F_j^{from,f}} C_{j,f'f,t} v_{j,f'f,t} \right)$$

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$$+\sum_n \sum_t C_{n,t}\, p_{n,t}, \quad (1)$$

where F_j denotes a set of configurations within CC unit j, and $F_j^{from,f}$ denotes a set of configurations f' ($f' \neq f$) that have an allowable transition to configuration f within CC unit j. Similarly, $F_j^{f,to}$ denotes a set of configurations f' ($f' \neq f$) that have an allowable transition from configuration f .

Individual conventional unit constraints

Conventional unit-level constraints include startup/shutdown, minimum up/down time, generator limits, and ramping up/down constraints [19].

Individual CC unit constraints

Constraints for a CC unit are summarized in Subsection III-B.

Individual virtual transaction, dispatchable demand, and dispatchable interchange transaction constraints

Virtual trading is undertaken by participants that do not necessarily have physical loads to serve or physical resources to offer. Participants submit bids, either loads or supplies, for the financial purchase or sale of energy in the day-ahead market. Together with dispatchable demands and dispatchable transactions, these are associated with continuous variable $p_{n,t}$ only. Unlike generation resources, there is no integer variables and no other constraints except the MW limit constraint:

$$p_{n,t}^{\min} \le p_{n,t} \le p_{n,t}^{\max}, \forall n,t, \quad (2)$$

where $p_{n,t}^{\min}$ and $p_{n,t}^{\max}$ are limits with either $p_{n,t}^{\min}$ = 0 or $p_{n,t}^{\max}$ = 0. For a virtual supply offer, $p_{n,t}^{\min}$ is 0, and $p_{n,t}^{\max}$ is a positive value. For a virtual demand bid, $p_{n,t}^{\max}$ is 0, and $p_{n,t}^{\min}$ is a negative value. Dispatch demand is always a negative value leading to $p_{n,t}^{\max}$= 0 and $p_{n,t}^{\min}$ as a negative limit. Dispatchable transactions could be either purchases or sales as similar to virtuals.

System-coupling power balance constraints

$$P_t = \sum_i p_{i,t} + \sum_j \sum_{f\in F_j} p_{j,f,t} + \sum_n p_{n,t}, \forall t, \quad (3)$$

where P is the sum of fixed demand bids at all nodes. At time t, P_t is equal to the total power generated by all generation resources and all other dispatchable variables.

System-coupling DC transmission capacity constraints

$$f_{l,t}^{\min} \le f_{l,t} \le f_{l,t}^{\max}, \ \forall l,t,$$

$$\text{where } f_{l,t} = \sum_i \alpha_{i,l}\, p_{i,t} + \sum_j \sum_{f\in F_j} \alpha_{j,f,l}\, p_{j,f,t} + \sum_n \alpha_{n,l}\, p_{n,t} + P_{F_{l,t}}. \quad (4)$$

Power flow $f_{l,t}$ of transmission line l at time t is modeled as a linear function of injections from all nodes weighted by generation shift factors α plus a fixed demand $P_{F_{l,t}}$, and restricted by power flow limits.

B. Constraints of CC units with configuration-based model

Within the configuration-based CC modeling, each configuration is treated as a conventional unit. Since $\sum_{f'\in F_j^{from,f}} v_{j,f'f,t}$ and $\sum_{f'\in F_j^{f,to}} v_{j,ff',t}$ can be viewed as startup and shutdown variable for configuration f, respectively, conventional unit-level constraints are applied to each configuration by using transition variables. The following are additional constraints defined to restrict configuration commitment states and transitions within a CC unit.

Configuration transition constraints

A CC unit can only have one configuration committed at t:

$$\sum_{f\in F_j} x_{j,f,t} = 1, \forall j,t. \quad (5)$$

Configuration startup/shutdown, minimum up/down time, and (5) guarantee $v_{j,f'f,t}$ = 1 when there is a transition from configuration f' to f, and $v_{j,f'f,t}$ = 0 otherwise [12].

Unique commitment constraints

These constraints have demonstrated strong ability in improving the computational performance [5] by defining that at most one transition is allowed at time t for a CC unit:

$$\sum_{f\in F_j} \sum_{f'\in F_j^{from,f}} v_{j,f'f,t} \le 1, \forall j,t. \quad (6)$$

IV. Solution Methodology

This section presents the solution methodology. Subsection IV-A presents Surrogate Augmented Lagrangian Relaxation with a novel linearization scheme. Subsection IV-B provides the convergence proof.

A. Surrogate Augmented Lagrangian Relaxation with absolute value function linearization

As reviewed in subsection II-B, our recently developed Surrogate Lagrangian Relaxation (SLR) overcomes major difficulties of LR. However, when there are many CC units, levels of violation of coupling constraints are still large when reaching solving time limits. This brings difficulty in searching feasible solutions. In addition, lower bounds may not provide a sufficiently good measure of solutions quality.

Motivated by the fast convergence of Augmented Lagrangian Relaxation, a novel approach Surrogate Augmented Lagrangian Relaxation (SALR) is developed by incorporating the idea of using quadratic penalties into SLR. Subproblems can then be extracted by fixing variables in other subproblems at values obtained from the previous iteration. Subproblem solutions are coordinated through updating multipliers subject to surrogate optimality condition. In the end, a near-optimal solution is obtained using heuristics.

After relaxing system-coupling constraints and penalizing violations, the augmented relaxed problem of UC at iteration k becomes:

$$\min_{\{x,u,y,v,p,w\}} L\text{: with}$$

$$L \equiv \sum_i \sum_t \left(C_{NLi,t} x_{i,t}^k + C_{i,t}\, p_{i,t}^k + C_{SUi,t} u_{i,t}^k \right) + \sum_n \sum_t C_{n,t}\, p_{n,t}^k$$

$$+\sum_j \sum_{f\in F_j} \sum_t \left(C_{NL\,j,f,t} x_{j,f,t} + C_{j,f,t}\, p_{j,f,t} + \sum_{f'\in F_j^{from,f}} C_{j,f'f,t} v_{j,f'f,t} \right)$$

$$+\sum_t \lambda_t^k \left(P_t - \sum_i p_{i,t}^k - \sum_j \sum_{f\in F_j} p_{j,f,t}^k - \sum_n p_{n,t}^k \right)$$
$$+\sum_l \sum_t \mu_{l,t}^k \left(\sum_i \alpha_{i,l} p_{i,t}^k + \sum_j \sum_{f\in F_j} \alpha_{j,f,l} p_{j,f,t}^k + \sum_n \alpha_{n,l} p_{n,t}^k + P_{F_{l,t}} - f_{l,t}^{\max} + w_{l,t}^k \right)$$
$$+\frac{c^k}{2} \sum_t \left(P_t - \sum_i p_{i,t}^k - \sum_j \sum_{f\in F_j} p_{j,f,t}^k - \sum_n p_{n,t}^k \right)^2$$
$$+\frac{c^k}{2} \sum_l \sum_t \left(\sum_i \alpha_{i,l} p_{i,t}^k + \sum_j \sum_{f\in F_j} \alpha_{j,f,l} p_{j,f,t}^k + \sum_n \alpha_{n,l} p_{n,t}^k + P_{F_{l,t}} - f_{l,t}^{\max} + w_{l,t}^k \right)^2, \quad (7)$$

subject to individual resource-level constraints. In (7), λ^k and μ^k are Lagrangian multipliers (left-hand side of transmission constraints (4) are ignored in the presentation), c^k (>0) is a penalty coefficient, and $\{w_{l,t}\}$ are non-negative slack variables converting inequality constraints to equalities.

The augmented relaxed problem is nonlinear and non-separable because of the introduction of quadratic penalties. The presence of quadratic terms can cause significant computational difficulties. For large-scale problems, performance of existing Mixed Integer Quadratic Programing (MIQP) solvers is dramatically worse than that of linear cases. To maintain the linearity so as to exploit MILP solvers, our idea is to linearize quadratic terms by a novel use of absolute value functions. Conceptually, a quadratic function can be replaced by a V-shape absolute value function with the minimum preserved as shown in Figure 2 for a simple quadratic function $y = x^2$. Although not differentiable, absolute value functions have the advantage of being exactly linearizable through extra variables and constraints.

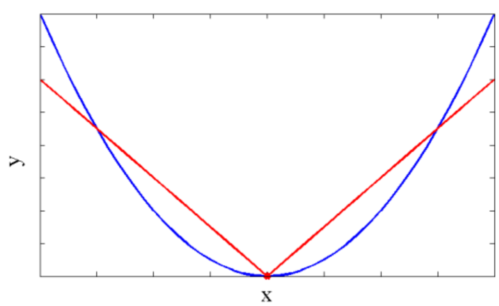

Fig. 2. Absolute value function linearization.

Linearization

A simple MILP problem (8) is used to illustrate the linearization scheme.

$$\min_x f(x), \text{ s.t. } g(x) = 0, \text{ where } f \text{ and } g \text{ are linear.} \quad (8)$$

After relaxing the constraint, penalizing the violation, the augmented relaxed problem at iteration k becomes

$$\min_{x^k} f(x^k) + \lambda^k \cdot g(x^k) + 0.5c^k \cdot g(x^k)^2. \quad (9)$$

During the iterative process, replace $g(x^k)^2$ with an absolute value function:

$$\min_{x^k} f(x^k) + \lambda^k \cdot g(x^k) + 0.5c^k \cdot a^k \cdot |g(x^k)|, \quad (10)$$

where a^k is the slope of the absolute value function determined by the value of variable obtained from the previous iteration. At the beginning iterations, $a^k = \max(|g(x^{k-1})|, 1)$ is used in case $|g(x^{k-1})|$ becomes zero, and the constraint violation is not penalized. After a few iterations, the slope is fixed at a certain constant $\bar{a}$ to ensure the convergence.

The absolute value function is then linearized following a standard procedure [20] by introducing two non-negative variables (z_1, z_2) and one additional constraint. Equation (10) can be equivalently rewritten in a linear form as:

$$\min_{\{x^k, z_1^k, z_2^k\}} f(x^k) + \lambda^k \cdot g(x^k) + 0.5c^k \cdot a^k \cdot (z_1^k + z_2^k) \quad (11)$$

$$\text{s.t.} g(x^k) = z_1^k - z_2^k;\ z_1^k \ge 0;\ z_2^k \ge 0. \quad (12)$$

Following the above linearization scheme, the augmented relaxed problem at iteration k for UC can be linearized as:

$$\min_{\{x,u,y,v,p,w,z\}} L' : \text{with}$$
$$L' \equiv \sum_i \sum_t \left(C_{NL\,i,t} x_{i,t}^k + C_{i,t} p_{i,t}^k + C_{SU\,i,t} u_{i,t}^k \right) + \sum_n \sum_t C_{n,t} p_{n,t}^k$$
$$+\sum_j \sum_{f\in F_j} \sum_t \left(C_{NL\,j,f,t} x_{j,f,t} + C_{j,f,t} p_{j,f,t} + \sum_{f'\in F_j^{from,f}} C_{j,f'f,t} v_{j,f'f,t} \right)$$
$$+\sum_t \lambda_t^k \left(P_t - \sum_i p_{i,t}^k - \sum_j \sum_{f\in F_j} p_{j,f,t}^k - \sum_n p_{n,t}^k \right)$$
$$+\sum_l \sum_t \mu_{l,t}^k \left(\sum_i \alpha_{i,l} p_{i,t}^k + \sum_j \sum_{f\in F_j} \alpha_{j,f,l} p_{j,f,t}^k + \sum_n \alpha_{n,l} p_{n,t}^k + P_{F_{l,t}} - f_{l,t}^{\max} + w_{l,t}^k \right)$$
$$+\frac{c^k}{2} \sum_t a_t^k (z_{1,t}^k + z_{2,t}^k) + \frac{c^k}{2} \sum_l \sum_t b_{l,t}^k (z_{3,l,t}^k + z_{4,l,t}^k), \quad (13)$$

subject to individual resource-level constraints and additional constraints to linearize absolute value functions:

$$z_{1,t}^k - z_{2,t}^k = P_t - \sum_i p_{i,t}^k - \sum_j \sum_{f\in F_j} p_{j,f,t}^k - \sum_n p_{n,t}^k, \forall t, \quad (14)$$

$$z_{3,l,t}^k - z_{4,l,t}^k = \sum_i \alpha_{i,l} p_{i,t}^k + \sum_j \sum_{f\in F_j} \alpha_{j,f,l} p_{j,f,t}^k + \sum_n \alpha_{n,l} p_{n,t}^k + P_{F_{l,t}} - f_{l,t}^{\max} + w_{l,t}^k, \forall l,t, \quad (15)$$

$$z_{1,t}^k, z_{2,t}^k, z_{3,l,t}^k, z_{4,l,t}^k \ge 0, \forall l,t. \quad (16)$$

In (13), a_t^k and $b_{l,t}^k$ are slopes of absolute value functions defined as

$$a_t^k = \max\left(\left| P_t - \sum_i p_{i,t}^{k-1} - \sum_j \sum_{f\in F_j} p_{j,f,t}^{k-1} - \sum_n p_{n,t}^{k-1} \right|, 1 \right), \forall t, \quad (17)$$

$$b_{l,t}^k = \max\left(\left| \sum_i \alpha_{i,l} p_{i,t}^{k-1} + \sum_j \sum_{f\in F_j} \alpha_{j,f,l} p_{j,f,t}^{k-1} + \sum_n \alpha_{n,l} p_{n,t}^{k-1} + P_{F_{l,t}} - f_{l,t}^{\max} + w_{l,t}^{k-1} \right|, 1 \right), \forall l,t. \quad (18)$$

Decomposition

Eq. (13) is now linear, and variables coupled in the original quadratic terms are now coupled through (14) and (15). Subproblems can still be extracted by fixing decision variables

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of other resources at values obtained from the previous iteration. The augmented relaxed problem is thus "decomposed" into individual resource-level subproblems. A subproblem for CC unit *j* at iteration *k* is formulated as:

$$\min_{\{x,v,p,w,z\}} L'_{CC} : \text{with } L'_{CC} \equiv$$

$$\sum_{f\in F_j}\sum_t \left(C_{NL\,j,f,t} x^k_{j,f,t} + C_{j,f,t} p^k_{j,f,t} + \sum_{f'\in F_j^{from,f}} C_{j,f',f,t} v^k_{j,f',f,t} \right)$$

$$+\sum_t \lambda^k_t \left(-\sum_{f\in F_j} p^k_{j,f,t} \right) + \sum_l \sum_t \mu^k_{l,t} \left(\sum_{f\in F_j} \alpha_{j,f,l} p^k_{j,f,t} + w^k_{l,t} \right)$$

$$+\frac{c^k}{2} a^k \sum_t \left(z^k_{1,t} + z^k_{2,t} \right) + \frac{c^k}{2} b^k \sum_l \sum_t \left(z^k_{3,l,t} + z^k_{4,l,t} \right), \quad (19)$$

subject to individual CC unit constraints and constraints to linearize absolute value functions:

$$z^k_{1,t} - z^k_{2,t} = P_t - \sum_i p^{k-1}_{i,t} - \sum_{j'\neq j}\sum_{f\in F_j} p^{k-1}_{j,f,t} - \sum_n p^{k-1}_{n,t} - \sum_{f\in F_j} p^k_{j,f,t}, \forall t, \quad (20)$$

$$z^k_{3,l,t} - z^k_{4,l,t} = \sum_i \alpha_{i,l} p^{k-1}_{i,t} + \sum_{j'\neq j}\sum_{f\in F_j} \alpha_{j,f,l} p^{k-1}_{j',f,t} + \sum_n \alpha_{n,l} p^{k-1}_{n,t} +$$

$$\sum_{f\in F_j} \alpha_{j,f,l} p^k_{j,f,t} + P_{F_{l,t}} - f^{\max}_{l,t} + w^k_{l,t}, \forall l,t, \quad (21)$$

$$z^k_{1,t}, z^k_{2,t}, z^k_{3,l,t}, z^k_{4,l,t} \geq 0, \forall l,t. \quad (22)$$

Complicated transitions within a CC unit are now handled within a CC unit subproblem, and do not complicate the entire solving process. Note that for a subproblem, when solving from one iteration to the next, only coefficients (λ^k, μ^k, c^k, a^k, b^k) and values of variables from other subproblems are changed.

A conventional unit subproblem can be similarly formulated. Other dispatchable resources including virtuals, dispatchable demands, and dispatchable transactions are integer-independent and only with simple bounds. Due to the simplicity, the original quadratic forms of these individual resource subproblems can be taken to derive analytical solutions based on unconstrained minima and bounds with slack variables fixed. Ideally, individual resource-level subproblems can be solved iteratively. For MISO UC with a large number of conventional units (more than 1,000), CC units (about 50), and virtuals (more than 10,000), slow convergence would be expected when solving individual resource subproblem. In the Subsection V-A, a grouping strategy to create subproblems will be discussed to improve the convergence.

Coordination

To update multipliers, subproblem solutions are coordinated subject to the surrogate optimality condition to ensure that multiplier directions form acute angles with directions toward optimal multipliers:

$$\tilde{L}'_{c^k}(x^k, u^k, y^k, v^k, p^k, w^k, z^k, \lambda^k, \mu^k) <$$

$$\tilde{L}'_{c^k}(x^{k-1}, u^{k-1}, y^{k-1}, v^{k-1}, p^{k-1}, w^{k-1}, z^{k-1}, \lambda^k, \mu^k), \quad (23)$$

where $\tilde{L}'_{c^k}(x^k, u^k, y^k, v^k, p^k, w^k, z^k, \lambda^k, \mu^k)$ is the surrogate augmented dual value of (13). After satisfying (23), multipliers are updated using stepsizes s^k as:

$$\lambda^{k+1}_t = \lambda^k_t + s^k \cdot \tilde{h}^k_t, \quad \mu^{k+1}_{l,t} = \mu^1_{l,t} + s^k \cdot \tilde{e}^k_{l,t}, \quad (24)$$

where $\tilde{h}^k_t$ and $\tilde{e}^k_{l,t}$ are components of surrogate subgradients $\tilde{g}(p^k, w^k)$. The stepsizing formula developed in SLR [9, (14)-(15), (20)] is used:

$$s^{k+1} = \gamma^k \frac{s^k \left\| \tilde{g}(p^{k-1}, w^{k-1}) \right\|}{\left\| \tilde{g}(p^k, w^k) \right\|}, \quad (25)$$

where

$$\gamma^k = 1 - \frac{1}{Mk^d}, d = 1 - \frac{1}{k^r}, M \geq 1, 0 < r < 1 \quad (26)$$

Penalty coefficient is increased as:

$$c^{k+1} = \beta \cdot c^k, \ \beta > 1. \quad (27)$$

If surrogate optimality condition (23) is not satisfied after solving all subproblems, penalty coefficients will be reduced to enforce the satisfaction of the surrogate optimality condition as:

$$c^{k+1} = c^k / \beta, \ \beta > 1. \quad (28)$$

SALR with the absolute value function linearization is named SALRL for short. When synergistically combined with B&C to solve subproblems, SALRL+B&C is defined. The key steps of SALRL+B&C are summarized as follows:

Step 0 Initialize λ^0, μ^0, s^0, c^0, a^0, and b^0. Multipliers are initialized as 0.

Step 1 For given λ^k and μ^k, create a subproblem and solve it by B&C. If (23) is satisfied, go to next step. If the surrogate optimality condition is not satisfied, skip this iteration and solve the next subproblem. If the surrogate optimality condition is not satisfied after solving all subproblems, update penalty coefficients c^{k+1} per (28).

Step 2 Update γ^k and s^k per (25)-(26), update λ^{k+1} and μ^{k+1} per (24) and update c^{k+1} per (27).

Step 3 If a stopping criteria (a time limit is used for MISO UC problem) is satisfied, go to the next step. Otherwise, go back to Step 1.

Step 4 Search for feasible solutions.

Step 5 Calculate the duality gap by using the feasible solution and the dual value.

B. *Convergence proof*

This subsection presents the convergence proof of SALR and SALRL+B&C methods.

Proposition 1: The SALR method converges if surrogate optimality condition

$$\tilde{L}_{c^k}(x^k, u^k, y^k, v^k, p^k, w^k, \lambda^k, \mu^k) <$$

$$\tilde{L}_{c^k}(x^{k-1}, u^{k-1}, y^{k-1}, v^{k-1}, p^{k-1}, w^{k-1}, \lambda^k, \mu^k), \quad (29)$$

in which $\tilde{L}_{c^k}(x^k, u^k, y^k, v^k, p^k, w^k, \lambda^k, \mu^k)$ is the surrogate augmented dual value of the augmented relaxed problem, is satisfied after solving all subproblems once, and penalty coefficient approaches a finite constant.

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Proof: Within SALR, the augmented Lagrangian function (7) can be viewed as the Lagrangian function associated with Problem A:

$$\min_{\{x,u,y,v,p,w\}} f(x,u,y,v,p)+0.5c\cdot g(p,w)^2, \qquad (30)$$

s.t. all system-coupling and resource-level constraints.

This problem is equivalent to the original UC problem when c approaches a finite constant ensuring no change in the objective function. Moreover, Problem A satisfies all assumptions listed in SLR (linearity of constraints [9, p. 178] and boundedness of constraint norms [9, p. 176]; but linearity of the objective function is not required). Therefore, the surrogate Lagrangian relaxation framework can be used whereby multipliers and stepsizes are updated in the same way as within the SLR method subject to the surrogate optimality condition, and the convergence proof in Theorem 2.1 of SLR [9, pp. 180-186] can be applied to SALR. If the surrogate optimality condition is not satisfied after solving all subproblems, penalty coefficients will be reduced to enforce the satisfaction of the surrogate optimality condition and the updating of multipliers. Multipliers thus converge to λ^*, μ^* which maximize the dual function:

$$q_{c^k}(\lambda,\mu)\equiv \min_{\{x,u,y,v,p,w\}} L_{c^k}(x,u,y,v,p,w,\lambda,\mu), \qquad (31)$$

corresponding to the augmented relaxed problem.

Proposition 2: SALRL+B&C converges if the surrogate optimality condition (23) is satisfied after solving all subproblems once, penalty coefficient approaches a finite constant, and the slopes of absolute value functions are fixed at constant values.

Proof: Within SALRL+B&C, the linearized surrogate augmented Lagrangian function (13) can be viewed as the Lagrangian function associated with Problem B defined as follows:

$$\min_{\{x,u,y,v,p,w\}} f(x,u,y,v,p)+0.5c\cdot a\cdot\left|g(p,w)\right|, \qquad (32)$$

s.t. all system-coupling and resource-level constraints. The objective of Problem 2 is the objective of the original UC problem plus the absolute value function terms.

At the beginning of the iterative process, slopes $a=\max\left(\left|g(p^{k-1},w^{k-1})\right|,1\right)$ would accelerate the reduction of $g(p,w)$. After several iterations, a would be fixed at $\bar{a}$, and the penalty coefficient c approaches $\bar{c}$, the problem (32) would be equivalent to the original problem. Moreover, (32) satisfies all assumptions listed in SLR. Therefore, following Proposition 1, convergence of SALRL+B&C can be guaranteed.

V. Computational Improvements

This section presents improvements on key aspects of the algorithm. It includes a grouping strategy to improve the convergence in Subsection V-A, a method to filter out inactive transmission constraints in Subsection V-B, and an effective heuristic to search feasible solutions in Subsection V-C.

A. Grouping resources within the same type

As discussed in Subsection IV-A, a subproblem can be extracted by fixing values of coupled resources in other subproblems in (14)-(15). Ideally, individual resource-level subproblems can be formulated and then solved iteratively. However, for large MISO UC problem, there are more than 1,000 unit-wise subproblems, and over 10,000 virtual subproblems. The enormous number of subproblems and less information in each small subproblem bring difficulty in satisfying the surrogate optimality condition and result in slow convergence. Take a conventional unit as an example, if all other resources are fixed and the penalty coefficient is large, absolute value terms become dominant. Solution then tends to be "feasible" to satisfy the current constraint violation rather than "optimal" to the original primal problem. There may not exist solutions that can satisfy the surrogate optimality condition. The wasted efforts on solving these individual unit-wise subproblems make the algorithm inefficient or does not converge within a time limit. Moreover, due to the inefficiency in most commercial optimization packages, the total time of solving 1,000 subproblems is typically much longer than that of solving 10 subproblems in which each contains 100 units.

In view of a large amount of resources in MISO system and the difficulty of satisfying surrogate optimality condition in solving individual resource subproblems, the relaxed problem is decomposed into limited number of subproblems. In each subproblem, certain resources within the same type are solved together to improve the convergence and reduce the computational time. Conventional units are divided into ten subproblems, each containing about 120 units. Decision variables of these coupled units are now solved together within

$$z_{1,t}^k - z_{2,t}^k = P_t - \sum_{i=1}^{120} p_{i,t}^k - \sum_{i=121}^{I} p_{i,t}^{k-1} - \sum_j \sum_{f\in F_j} p_{j,f,t}^{k-1} - \sum_n p_{n,t}^{k-1},\ \forall t. \qquad (33)$$

Solving such a subproblem could have more chances to satisfy surrogate optimal condition because the solution $p_{i,t}^k$ for these units could likely move from the previous iteration $p_{i,t}^{k-1}$. Improvements on grouping resources within the same type will be demonstrated in Case 1 of Example 2 in Section VI.

In view of the complicated transitions within a CC unit, the number of CC units in one subproblem should be limited (10 is used for MISO UC cases) to avoid the potential increased complexity. Virtuals, dispatchable demands, and dispatchable transactions are only associated with continuous variables and simple MW limit constraints. In view of these features, all these variables are solved together in one subproblem.

B. Identification of inactive transmission constraints

Within SALRL+B&C, since all transmission constraints are relaxed and violations are penalized, a large number of such constraints would result in a large number of multipliers and absolute value functions. To overcome these difficulties, inactive transmission constraints are identified and removed following [21]. An analytical estimate of the worst-case power flow through each line is obtained. If it is within the transmission capacity, the corresponding transmission constraint is removed. This method is extended to CCs by only considering the worst-case configuration (all components "on" when the generation shift factor is positive, and all components "off" when the generation shift factor is negative). Virtuals, dispatchable demands, and dispatchable transactions are treated similarly to conventional units since they have MW

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limits. This process reduces solving time for subproblems and computational time of surrogate subgradients and multipliers.

C. Obtaining feasible solutions

Solutions to subproblems are typically feasible with respect to each subproblem, but these solutions may not satisfy relaxed constraints. To obtain a feasible solution to the original problem, heuristics are used to free certain discrete variables to solve a smaller MILP problem by B&C.

Considering the complexity associated with CC units, binary variables of CC units are fixed at values obtained from the iterative process. Level of violation for each transmission constraint at each time interval can be calculated as:

$$g_{l,t} = \sum_i \alpha_{i,l} p_{i,t} + \sum_j \sum_{f \in F_j} \alpha_{j,f,l} p_{j,f,t} + \sum_n \alpha_{n,l} p_{n,t} + P_{F_{l,t}} - f_{l,t}^{\max}. \quad (34)$$

For example, if $g_{l,t} > 0$, it indicates that extra power is generated. Commitment variables of online units with positive $\alpha_{i,l}$ and off-line units with negative $\alpha_{i,l}$ are set free and will be resolved to potentially reduce the power flow. Number of selected units and magnitude thresholds of $\alpha_{i,l}$ can be tuned based on testing cases. Additional out-of-money units are identified if their subproblem costs are positive based on MISO's heuristics [1] to ensure that sufficient units are resolved in the small MILP problem. Since levels of constraint violations are penalized, SALRL+B&C provides better commitment decisions, and fewer units are freed as compared with those from SLR+B&C as demonstrated in Section VI.

VI. Numerical Testing

The method has been implemented on an Intel Core i7-6700K 4.0GHz 16 GB server with AIMMS 4.2 and CPLEX 12.6. Three examples are presented. In Example 1, a 5-bus system is tested to demonstrate the efficiency of the configuration-based CC modeling and the convergence of SALRL+B&C. In examples 2 and 3, two different MISO datasets are tested to demonstrate near-optimal solutions are effectively obtained within a time limit.

Example 1: 5-Bus system

A 5-bus system with eight conventional units, one combined cycle unit, and 224 virtuals looking ahead 36 hours is tested. After relaxing system coupling constraints, the relaxed problems are decomposed into nine unit subproblems and one virtual subproblem. Iterative process stops when the number of iterations (solving one subproblem is defined as one iteration) reaches 200, feasible solution search is then started based on heuristics. The following two cases are studied to demonstrate the efficiency of configuration-based CC modeling and the convergence of our solution methodology.

Case1: This case compares the performance of configuration-based CC modeling and the aggregated CC modeling. In the former, the CC unit contains two CTs and one ST, and allows five configurations as shown in Figure 1. The feasible solution of configuration-based CC modeling is 0.54% less than that of the aggregated modeling. Comparisons of commitment states and generation levels are shown in Figure 3. As can be seen, in the aggregated modeling, the CC unit is committed in the last eight intervals with a high generation level. In the configuration-based modeling, the CC unit is committed four hours earlier and the generation level is reduced by switching to configurations with smaller minimum generation requirement. This is because in the aggregated modeling, high capacity, high startup/energy cost, and long minimum run time are used to ensure that the operating costs can be recovered. The configuration-based model introduces flexibility because different operational configurations can be selected and the energy costs and minimum up/down time are more accurate for each configuration than the simplified aggregated model.

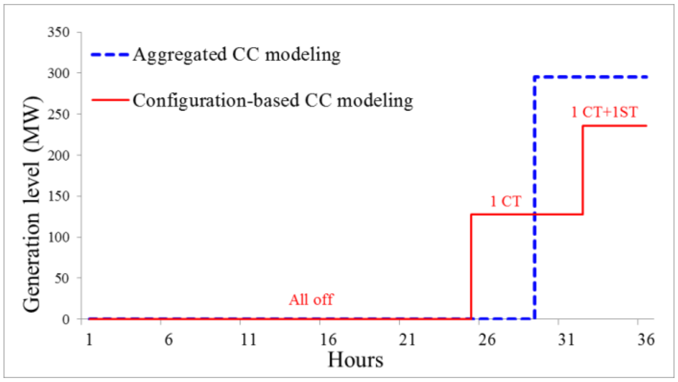

Fig. 3. Generation level and configuration transitions of the CC unit.

Table 1. Performance comparisons of SALRL+B&C, SALR+MIQP, SLR+B&C, and ALR+MIQP.

	SALRL + B&C	SALR + MIQP	SLR + B&C	ALR + MIQP
Feasible solution ($)	604,903	604,903	604,903	604,903
CPU time (s)	40	53	39	258

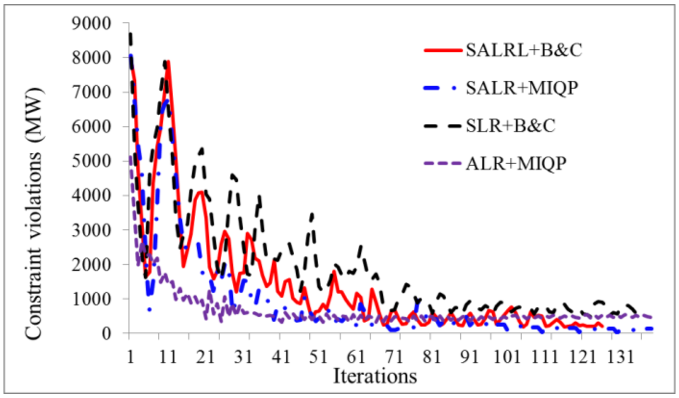

Fig. 4. Convergence of SALRL+B&C compared with SALR+MIQP, SLR+B&C, and ALR+MIQP.

Case 2: This case compares the performance of SALRL+B&C, SALR+MIQP (with MIQP solver to solve subproblems), SLR+B&C, and ALR+MIQP (with MIQP solver to solve the full augmented relaxed problem). Feasible solutions and computational time of these four approaches are summarized in Table 1. All approaches obtain the same optimal solution $604,903. Among them, ALR+MIQP requires the longest time since the problem is more complex than other decomposition approaches during each iteration. The computational time of SALRL+B&C is similar to that of SLR+B&C demonstrating the efficiency of our linearization approach. SALR+MIQP requires longer computational time than SALRL+B&C because of the complexity to solve a

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quadratic problem. For this simple example with only a few units, MIQP works with acceptable performance. When solving large-scale MIQP problems, the complexity would increase exponentially.

Levels of constraints violations in squared L2-norm are depicted in Figure 4 to illustrate the convergence of all approaches. The two SALR-based approaches accelerate the reduction of levels of constraint violations compared with SLR. Constraint violations of SALRL+B&C are slightly higher than that of SALR+MIQP because of the linear approximation. ALR approach reduces the constraint violation fast at the beginning because it fully optimizes the augmented relaxed problem. However, the level of constraint violations is still high at convergence due to the major issues as discussed in LR method.

Example 2

One MISO dataset containing 1,143 conventional units, 20 CC units converted from aggregated modeling to configuration-based modeling, and 14,955 virtuals with 36 looking ahead hours is tested. The case is also extended to 40 and 80 CC units with configuration-based modeling. All system-coupling constraints including power balance, transmission, and reserve are relaxed. Among them, power balance and transmission constraints are penalized with quadratic terms.

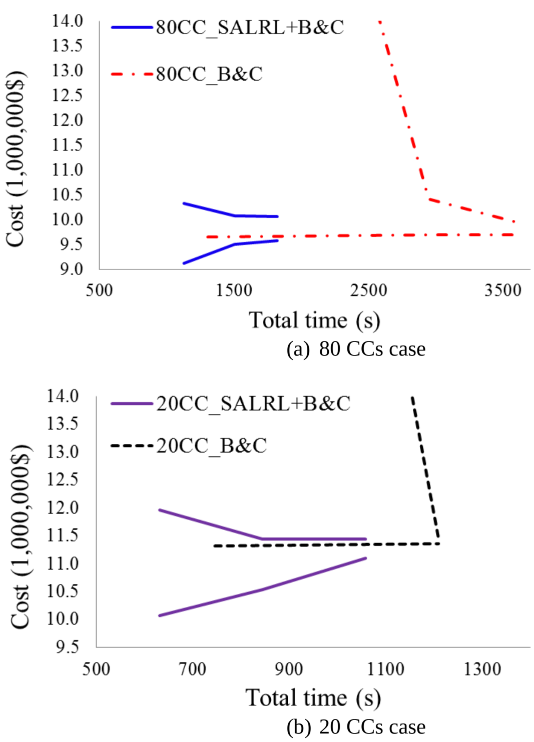

Fig. 5. Comparison of SALRL+B&C and B&C for (a) 80 CCs, and (b) 20 CCs in Example 2.

Performance of SALRL+B&C and comparison with B&C are depicted in Figure 5. For 80 CCs as a stress test, at the time limit 1,800s, B&C cannot reach a feasible solution with a reasonable MIP gap. It requires 3,600s to first obtain a good feasible solution. The SALRL+B&C method provides a near-optimal solution with an acceptable duality gap 5.5% within 1,800s. Along the solution process, it achieves feasible solution much faster than B&C. Although the feasible objective cost obtained from B&C at 3,600s is slightly less than that of SALRL+B&C at 1,800s, the 3,600s solving time is too long for MISO to accept. For less complicated 20 CC case, B&C obtains a good feasible solution with a very small MIP gap 0.8%, which is hard for SALRL+B&C to outperform. This demonstrates that SALRL+B&C is powerful to provide good-quality feasible solutions within limited time and significantly outperforms B&C for difficult cases.

The following cases 1-3 are conducted for 80 CCs to demonstrate the efficiency of our approach on grouping strategy, filtering out inactive transmission constraints, and searching for feasible solutions, respectively. In Case 4, our method is compared with B&C for 20, 40 and 80 CCs.

Case 1: This case shows the performance of grouping resources in subproblems. The nominal grouping strategy is defined as follows: 120 conventional units are grouped into one subproblem, 10 CCs are grouped into a CC unit subproblem, and all virtuals, dispatchable demands, and dispatchable transactions are grouped together into one subproblem. Therefore, totally 19 subproblems are created and solved iteratively. Comparisons of nominal grouping and solving individual unit-wise subproblems are presented in Table 2. Due to the large number of virtual variables, all virtuals and other dispatchable variables are solved together in both scenarios, and we only compare for generation units.

With the nominal grouping strategy, the initial model generation time and solving time for completing 19 subproblems once are 71s and 75s, respectively. The total time 164s also includes computational efforts on multiplier updating, data transfer, surrogate subgradient calculation, etc. When solving individual unit-wise subproblems, solving all 1,173 subproblems once requires more than 1,000s. The large computation time of solving individual unit-wise subproblems mainly results from the large overhead in AIMMS. Moreover, at the targeted 1,800s, levels of constraint violations with the nominal grouping strategy are small, and a near-optimal feasible solution is obtained. However, when solving individual unit-wise subproblems, since less information is used in each small subproblem, levels of constraint violations are not much reduced leading to a very slow convergence.

Table 2. Comparison of nominal grouping with solving individual unit-wise subproblem.

		Solving all subproblems once			At 1,800s
	# of sub-problems	Generation Time (s)	Solving Time (s)	Total Time (s)	MIP gap (%)
Nominal grouping	19	71	75	161	5.5
Individual unit-wise subproblem	1,173	312	392	1,042	>100

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Case 2: This case compares the performance of SALRL+B&C with and without filtering inactive transmission constraints (TCs). Numbers of original and remaining TCs after filtering are shown in Table 3. Performance of SALRL+B&C with and without filering out inactive TCs are compared in Table 4. This process reduces computational time and improves the feasible solution.

Case 3: Performance of heuristics to search feasible solutions is shown in Table 5. The number of commitment variables selected based on generation shift factor is reduced by 21.9% from SLR, and the number of out-of-money units identified by SALRL+B&C is reduced from 136 to 112 as compared with SLR. Feasible solution is also improved.

Case 4: This case shows the scalability of SALRL+B&C with 20, 40 and 80 CC units as depicted in Figure 6. SALRL+B&C obtains near-optimal solutions for all scenarios within the targeted time. For 20 CCs and 40 CCs, SALRL+B&C provides good-quality feasible costs within 1,200s. When the number of CC units increases, the total cost is reduced because of the energy efficiency of configuration-based modeling. Meanwhile, computational time increases because more CC subproblems need to be solved and the feasible solution search becomes complicated. SALRL+B&C thus requires a longer time to obtain a small duality gap. As it can be seen, B&C is powerful to provide a good lower bound while our method provides a high-quality feasible solution. To take advantages of the two methods, we may run SALRL+B&C and B&C in a parallel manner. Along the iterative process, the lower bound from B&C can be taken and compared with our feasible solution to calculate the MIP gap.

Table 3. Numbers of original and remaining TCs.

Original TCs		Remaining TCs		Filtered
Right side	Left side	Right side	Left side	48.8%
8,347	7,032	5,503	2,372	

Table 4. Comparison of filtering and not filtering inactive TCs.

	UB* ($)	LB** ($)	Gap (%)	Total time (s)
SALRL+B&C[1]	10,064,124	9,525,597	5.5	1,896
SALRL+B&C[2]	10,082,321	9,517,789	5.6	1,995

[1]: w/ filtering inactive TCs; [2]: w/o filtering inactive TCs.
*: upper bound, the feasible solution.
**: lower bound.

Table 5. Comparison of UB search between SALRL and SLR.

	# of binary freed[1]	# of units freed[2]	Feasible cost ($) w/ 1	Feasible cost ($) w/ 1 and 2
SALRL+B&C	1,683	112	10,075,603	10,064,124
SLR+B&C	2,155	136	10,124,712	10,084,577

[1]: binary variables freed based on violations of transmission constraints;
[2]: number of out-of-money units freed based on MISO heuristic.

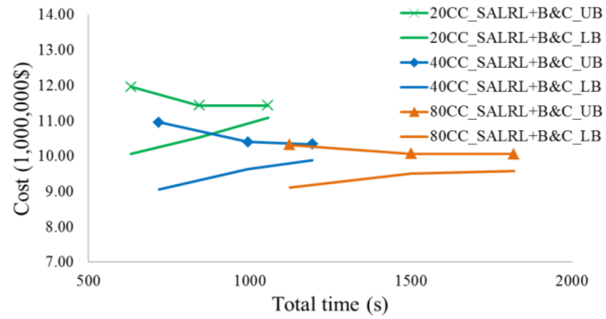
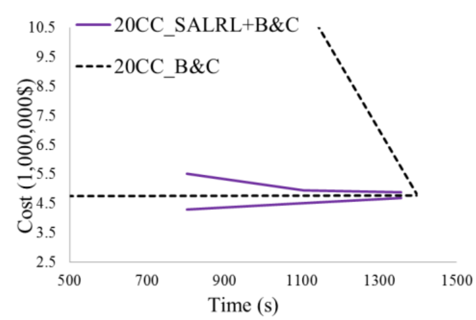


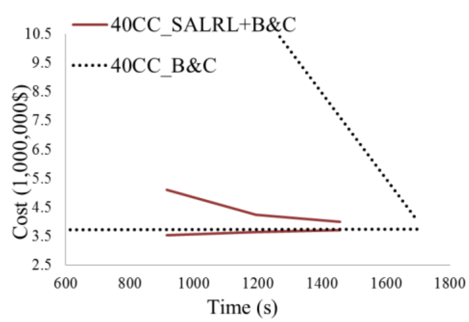
Fig. 6. SALRL+B&C for 20, 40 and 80 CC units in Example 2.

Example 3

To test the robustness of our method, another MISO case containing 1,129 conventional units and 15,843 virtuals with 36 looking ahead hours is tested. Performance of SALRL+B&C for 20, 40, and 80 CC units is compared with B&C, SLR+B&C as in Figure 7. With 20 and 40 CC units, both SALRL+B&C and B&C methods obtain near-optimal solution within 1,800s. However, when the number of CC units increases to 80, the complexity increases exponentially and hits the bottom of B&C method. Therefore, B&C cannot obtain a feasible solution with a small MIP gap within limited time. Even after 3,600s, it is still difficult for B&C to find a feasible solution with an acceptable MIP gap. For 80 CC cases, SALRL+B&C is powerful to obtain a near-optimal solution at targeted 1,800s by fully exploiting exponential reduction of complexity. Our SALRL+B&C method also outperforms SLR+B&C in terms of lower bound, feasible solution, and computational time as shown in Table 6. It converges fast and reduces the duality gap along the iterative process. These improvements could result in significant annual savings for ISOs considering that such a large UC problem involves millions of dollars each day.



(a) 20 CCs case



(b) 40 CCs case

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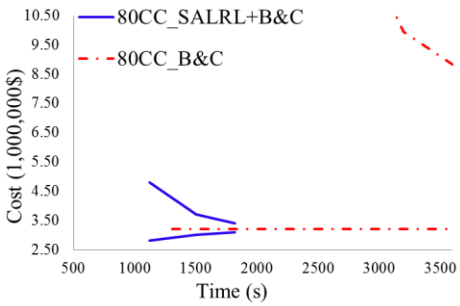

(c) 80 CCs case

Fig. 7. Comparison of SALRL+B&C and B&C for (a) 20 CCs, (b) 40 CCs and (c) 80 CCs in Example 3.

Table 6. Results of SALRL+B&C, SLR, and B&C for 80 CCs in Example 3.

	UB ($)	LB ($)	Gap (%)	Total time (s)
SALRL+B&C	3,308,783	3,172,256	4.3	1843
SLR+B&C	3,329,845	3,151,576	5.6	1859
B&C	14,947,775	3,204,067	>100	1902

VII. Conclusion

This paper targets to solve a large and difficult UC problem which contains over 1,000 units and 10,000 virtuals looking ahead 36 hours. With an increasing number of combined cycle units represented by configuration-based modeling, current state-of-the-practice B&C cannot solve the problem with targeted MIP gap or within a time limit. Decomposition and coordination is a must for such large-scale complicated problems. Our recently developed surrogate Lagrangian relaxation is thus significantly enhanced through adding quadratic penalties on constraint violations to fully exploit exponential reduction of complexity with fast convergence. Quadratic terms are innovatively linearized through a novel use of absolute value functions. Enhancements on certain key aspects are also incorporated to fine tune the algorithm and improve the overall performance. As demonstrated by MISO cases, the method provides near-optimal solutions within a time limit, and significantly outperforms B&C.

Our work is timely and critical to solve large UC problems, and can be extended to other complicated MILP problems in power systems and beyond. There is still room to improve the method, and we propose two directions as in future work: 1) Formulation tightening and 2) Distributed and asynchronous implementation. Tightening subproblem formulation is important since if constraints directly delineate a problem convex hull, the MILP problem can be directly solved by linear programming. Distributed and asynchronous implementation of SALRL+B&C will also be investigated thus that subproblems will be solved in a distributed way and multipliers will be updated asynchronously to improve the efficiency.

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