

# A Dynamic Regrouping Based Dynamic Programming Approach for Unit Commitment of the Transmission-constrained Multi-site Combined Heat and Power System

Aiying Rong and Peter B. Luh, *Fellow, IEEE*

**Abstract**— Combined heat and power (CHP) systems offer additional advantage and flexibility for addressing power grid balance resulting from large-scale introduction of intermittent renewable energy sources (RES) in contrast to power-only systems. The dependence between heat and power production in the CHP plant can be utilized to adjust power production level to accommodate more RES. Furthermore, electricity can be transformed into heat by electric heater and heat pump to avoid starting up heat led CHP plants when RES production is abundant. This paper focuses on solving efficiently unit commitment of the interconnected multi-site CHP system without considering RES. A relaxed ON/OFF state based dynamic programming (DP) applying sequential commitment scheme in conjunction with dynamic regrouping (MDRDP-RSC) is used to coordinate heat and power production in each site (region) as well as power transmission across sites. Computational experiments for real-life daily scheduling demonstrate that our method generates solutions much more quickly than a standard high-performance optimizer (CPLEX) with comparable solution quality, and lays foundation for the future handling of uncertainties of intermittent RES.

**Index Terms**—Dynamic programming, dynamic regrouping, relaxed states, multi-site combined heat and power system, transmission-constrained generation unit commitment.

## NOMENCLATURE

### Indices

- $i, k$  Index for the site.
- $(i, k)$  Index for the arc (line connection) in the power transmission network.
- $j$  Index for the extreme point.
- $j_u^{OFF}$  Index for the extreme point representing OFF-state of plant  $u$ ,  $j_u^{OFF} \in J_u$ .
- $p, q$  Prefixes or superscript/subscripts for power and heat in the system.

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A. Rong is with the Department of Mechanical Engineering, Aalto University, Aalto, 00076, Finland (email: aiying.rong@aalto.fi).

P. B. Luh is with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269-4257 USA (e-mail: peter.luh@uconn.edu).

- $u$  Index for the plant.
- $t$  Index of a point in time or a period. The period  $t$  from points  $t-1$  to  $t$ . The length of period is one hour in the current study.

### Index sets

- $F$  Set of indices for generation and demand sites (nodes).
- $A$  Set of indices for arcs  $A = \{(i, k) : i, k \in F, i \neq k\}$ .
- $J$  Set of indices for extreme points of the operating regions for all plants.
- $J_i$  Set of indices for extreme points of the operating regions for plants at site  $i$ .
- $J_u$  Set of indices for extreme points of the operating region of plant  $u$ ,
- $U$  Set of indices for all plants.
- $U_i$  Set of indices for plants at site  $i \in F$ .

### Parameters

- $c_{i,k,t}$  Transmission cost in €/MWh on arc  $(i, k)$ .
- $g_{i,k}$  Capacity of arc  $(i, k)$  in MWh.
- $(c_{j,t}, p_{j,t}, q_{j,t})$  (cost, power, heat) of extreme point  $j \in J_u$  at plant  $u$  in period  $t$ .
- $c_{i,p\pm,t}$  Power surplus/slack penalty cost in €/MWh at site  $i$  in period  $t$ .
- $c_{i,q\pm,t}$  Heat surplus/slack penalty cost in €/MWh at site  $i$  in period  $t$ .
- $c_u^{cold}$  Cold start-up cost in €/MWh of plant  $u$ .
- $c_u^{hot}$  Hot start-up cost in €/MWh of plant  $u$ .
- $s_u(w_{u,t-1}, y_{u,t}, y_{u,t-1})$  Start-up and shut-down cost in €/MWh of plant  $u \in U$  in period  $t$ , varying between 0 and  $c_u^{cold}$ .
- $P_{i,t}$  Power demand in MWh at site  $i$  in period  $t$ .
- $Q_{i,t}$  Heat demand in MWh at site  $i$  in period.
- $Y_{u,OFF}$  Set of indices of periods for plant  $u \in U$  at forced OFF-states.
- $Y_{u,ON}$  Set of indices of periods for plant  $u \in U$  at forced ON-states.
- $CT_u$  Cold start-up periods of plant  $u \in U$ .
- $DT_u$  Minimum down periods of plant  $u \in U$ .
- $UT_u$  Minimum up periods of plant  $u \in U$ .
- $T$  Number of periods over the planning horizon.

*Decision variables*

- $w_{u,t}$  State variable of plant  $u \in U$  in period  $t$ , with given initial state  $w_{u,0}$ , negative values indicating number of OFF periods.
- $x_{j,t}$  Continuous variables indicating the operating level of each plant  $u$  regarding extreme points  $j \in J_u$  in period  $t$ .
- $x_{i,p\pm,t}$  Power surplus/slack in MWh at site  $i$  in period  $t$ .
- $x_{i,q\pm,t}$  Heat surplus/slack in MWh at site  $i$  in period  $t$ .
- $y_{u,t}$  Binary variable indicating ON/OFF states of plant  $u \in U$  in period  $t$ .
- $z_{i,k,t}$  Power flow in MWh on arc  $(i,k)$  in period  $t$ .

I. INTRODUCTION

ENVIRONMENTAL impact mitigation of energy supply and harnessing as well as the dependence on fossil fuels drive the development of energy technology utilizing natural resources efficiently and large-scale introduction of renewable energy sources (RES) like wind, solar, geothermal and biofuel into energy systems. Combined heat and power (CHP) can improve energy efficiency by recycling otherwise wasted heat in electricity production process. This means that heat and power production depend on each other in the CHP plant. When electricity and heat demand need to be satisfied simultaneously, CHP can provide more cost- and energy-efficient supply than traditional separate heat and power production. This results in fuel savings and emission reductions from 10% to 40% [1] for a fossil fuel-based CHP plant as well as efficient use of scarce biofuel for a renewable biofuel-based CHP plant.

Several countries in Europe, for example, Finland Denmark, and the Netherlands have taken lead in using CHP, which contributes to 30-50% of national electricity production though CHP currently only accounts for 10% of global electricity production [2]. It is anticipated that CHP will cover wide regions in coming decades [2] with strong support for utilizing CHP by European Union (EU) [3] and many other countries [4].

Similar to the traditional condensing power-only generation plant, CHP can also contribute to balancing the grid [5] when intermittent RES is penetrated into power and energy system significantly. In contrast to power-only systems, CHP systems have additional advantage and flexibility. First, coupling between heat and power production in the CHP plant can be exploited to affect power production level [6] to accommodate more RES. Second, electricity can be directly converted into heat by means of electric heater and heat pump to avoid or delay starting up heat led CHP plants [7] when RES production is plentiful.

Usually, CHP systems were approached from the angle of distributed energy systems (DES) for utilizing RES via interconnection of utility power networks at a lower level, e.g., within the city infrastructure [8]. This paper deals with the transmission-constrained multi-site CHP system at relatively high level utility network interconnections such as at municipal or national levels. The schematic system structure is shown in Fig.1. The multi-site CHP system was introduced in

[9] and viewed as the extension of multi-site power-only systems [10-12]. Each site (Fig.1 (a)) is treated as a regional energy system that includes CHP plants and other heat and power production plants to satisfy heat and power demands at heat and power nodes respectively.

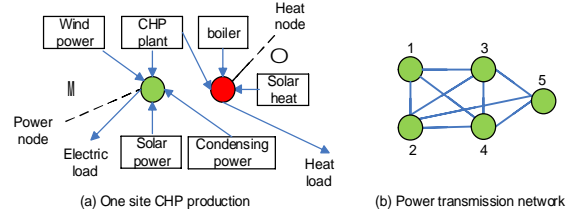


Fig. 1 A transmission-constrained multi-site CHP system from [22]

Similar to [12], the transmission constrained multi-site CHP system has potential to address electricity market situation when the share of power production from CHP on the market is high. Naturally, large-scale penetration of intermittent RES into CHP systems [13] has also created the critical needs for coordinating between the decentralized regional heat and power supply energy systems (described in numerical experiments) to utilize low cost RES production more efficiently by combining the flexibility of CHP and interconnections [5].

To authors' knowledge, there is no research related to scheduling the multi-period transmission-constrained multi-site CHP system except our attempt [14]. The reasons behind this as well as coordination challenges and ranking difficulties for CHP plants were discussed in [9].

This paper focuses on developing efficient algorithms for the deterministic unit commitment (UC) of the transmission-constrained multi-site CHP system ignoring intermittent RES. Efficient solution of the deterministic problem is critical for addressing the uncertainty of the intermittent RES in both stochastic programming approach [15] and discrete Markov process [16]. In the former approach, multiple scenarios with certain probability need handling and each scenario is regarded as a deterministic problem instance. In the latter approach, multiple states need considering and each state is viewed as a deterministic problem instance.

In the following, some general-purpose solution approaches are reviewed. There are roughly three categories of solution methods. The first category resorts to decomposition approaches including Lagrangian relaxation (LR) [12], [17], [18] and dynamic programming (DP) [14], [19]-[22]. The second category utilizes various heuristics such as sequential approach [23], merit-order-based method [10], system of system engineering method [24], and meta-heuristics [25]. The last category applies mixed integer linear programming (MILP) techniques [26]-[30]. For solving single-site UC of CHP system, LR [31], DP [32]-[36] and heuristic approaches [37], [38] have been used.

The contributions of the current study are summarized below.

First, time-oriented DP-based decomposition is applied to

handle coordination challenges in contrast to site-oriented LR-based decomposition [12], [17] commonly used for dealing with multi-site power-only systems. DP-based approach is more robust than LR-based approach, especially for dealing with linear programming (LP)-based models, as discussed in [35].

Second, complicated coordination challenges are handled on hourly basis by solving efficiently the hourly multi-site CHP system (the underlying economic dispatch (ED) sub-problem) in an integrated way according to hourly heat and power demand using a specialized efficient network power Simplex algorithm [9].

Third, solutions of the hourly sub-problems are coordinated according to recursive equations based on Bellman's principle of optimality [39] using a more efficient state representation scheme than that in [35].

Fourth, to overcome the curse of dimensionality for the pure DP algorithm, a relaxed ON/OFF state based DP (DP-RSC) algorithm resorting to sequential commitment for dealing with UC for the single-site CHP system [33], [35] has been extended to the multi-site context, called MDP-RSC [14]. Introducing relaxed states aims at reducing the dimension of DP algorithm. With plants at relaxed states, the corresponding ON/OFF states can temporarily be ignored as decision variables. To enhance further the solution quality of MDP-RSC, a modified dynamic regrouping scheme (Fig.3 of Section IV) is introduced in the current study and the resulting DP algorithm is called MDRDP-RSC2.

Finally, numerical experiments for daily scheduling with real-life data justify the effectiveness of MDRDP-RSC2.

The paper is structured as follows. Section 2 revisits the CHP plant model and the corresponding relaxed state, as well as the model for the power transmission network. Section 3 formulates the UC of the interconnected multi-site CHP system. Section 4 describes the DP-based solution approach. Section 5 reports computational experiments according to real-life instances.

## II. CHP PLANT MODEL, RELAXED STATES AND POWER NETWORK MODEL

### A. CHP plant model and relaxed state

Plant modeling techniques are similar to those discussed in [14] and [35]. The convex plant characteristic can be modeled based on linear programming (LP) techniques [40], [41] and the non-convex characteristic on MILP approaches [42], [43]. Plants are assumed to be convex for simplicity because the convexity assumption is widely accepted in practice as commented in both [9] and [40]. The convexity assumption means that the underlying ED of the UC can be solved more efficiently by general LP optimizers or a specialized LP optimizer [9]. Similarly, the ED should be solved using an MILP solver if there are non-convex plants.

Fig.2 shows the characteristic region of a convex plant at three ON/OFF states (ON, OFF, relaxed ON/OFF). Relaxed ON/OFF state is an artificial state to facilitate implementing sequential commitment scheme. At ON-state, the plant

operates in the region formed by the extreme points  $(c_j, p_j, q_j)$  ( $j=1, \dots, 5$ ), where  $c_j$  represents the operating cost for producing heat  $q_j$  and power  $p_j$ .  $c_j$  is the fuel cost for a fuel-based plant. Point  $(0, 0, 0)$  represents OFF-state. It means that the plant does not incur any cost when no energy is produced. At the relaxed ON/OFF state, the plant would run in the continuum defined by  $(0, 0, 0)$  and  $(c_j, p_j, q_j)$  ( $j=1, \dots, 5$ ) as indicated by the dash lines in Fig.2. If  $(0, 0, 0)$  happens to be in the true (ON-state) region, then the relaxed and true regions coincide.

The extreme points of the plant can be obtained by an empirical method [42] or an analytical method [44]. The analytical method recorded the feasible solutions (corner points of the feasible region) of a mathematical model that encoded the plant according to ordinary mass and energy balances using a Simplex based LP solver [44]. i.e., the extreme point formulation of the plant (1) aims at reformulating a general plant model in a special way. This reformulation facilitates developing specialized efficient algorithms.

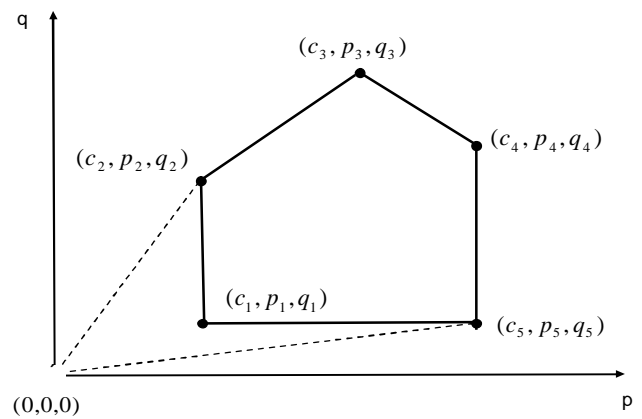


Fig.2 The characteristic operating region of a convex CHP plant in the power-heat plane at different ON/OFF states. Extreme points  $(c_j, p_j, q_j)$  ( $j=1, \dots, 5$ ) form the operating region at the ON-state and  $(0,0,0)$  indicates the OFF-state.  $p$ =power,  $q$ =heat,  $c$ =cost. ([14] and [35])

Due to convexity, power  $(P_{u,t})$  and heat  $(Q_{u,t})$  generation of plant  $u$ , as well as the corresponding operating costs  $C_{u,t} = C_{u,t}(P_{u,t}, Q_{u,t})$  can be formulated as a convex combination [45] of extreme points  $(c_{j,t}, p_{j,t}, q_{j,t})$ :

$$\begin{aligned}
 C_{u,t} &= \hat{\mathbf{a}}_{j \in J_u} c_{j,t} x_{j,t} \\
 P_{u,t} &= \hat{\mathbf{a}}_{j \in J_u} p_{j,t} x_{j,t} \\
 Q_{u,t} &= \hat{\mathbf{a}}_{j \in J_u} q_{j,t} x_{j,t} \\
 \hat{\mathbf{a}}_{j \in J_u} x_{j,t} &= 1 \\
 x_{j,t} &\geq 0, \quad j \in J_u,
 \end{aligned} \tag{1}$$

The above extreme point formulation can accommodate hourly operating (the number of points and coordinate values of the point) change with fixed  $|J_u|$ . It means that  $J_u$  includes all possible extreme points of plant  $u$ . The non-active points can easily be excluded by forcing corresponding  $x_{j,t}$  to be zero.

If  $J_u$  contains point  $(0, 0, 0)$  index  $j_u^{\text{OFF}}$ , then formulation (1)



and shut-down costs can be computed easily according to number of ON/OFF periods obtained from state transitions (12) with no need to encode as linear relations, similar to those in [35]. For MILP formulation of UC, non-linear relations (11) and (12) as well as start-up and shut-down costs would be transformed into linear relations with integer variables as described in [26-28] and [47].

#### IV. SOLUTION APPROACHES

Dynamic programming (DP) is formulated based on multi-stage recursive process, where sub-problems in each stage are represented as states. The recursive equations are set up to describe the transition between states following the principle of optimality [39]. The principle of optimality guarantees that each state contains the optimal solution up to that state and provides enough information to determine the future state.

##### A. DP framework for the UC

The UC can be interpreted as a multi-stage problem when the period  $t$  is treated as a stage. The states can be represented based on either general integer state variables  $w_{u,t}$  [33],[35] or binary ON/OFF state variables [21]. The number of states for the former and the latter approaches are  $2^{|U|}$  and  $T \times 2^{|U|}$  respectively. Here the latter approach is adopted because it has advantage of reduced number of states as compared with the former one.

Given state  $(t,i)$  at stage  $t$  and initial state  $(0,k_0)$ , let  $R(t,i)$  represent the minimum cost to state  $(t,i)$ ,  $C_{ED}(t,i)$  the ED cost of state  $(t,i)$ ,  $SC(t-1,k:t,i)$  the start-up or shut-down costs from states  $(t-1,k)$  to  $(t,i)$ , then the recursive equations can be written as follows.

$$R(t,i) = \min\{R(t-1,k) + C_{ED}(t,i) + SC(t-1,k:t,i), k \in S_{t-1}\}, \hat{i} \in S_t, \\ t = 1, \dots, T, |S_t| \leq 2^{|U|}, \quad (17)$$

$$R(0,k_0) = 0, \quad (18)$$

where  $C_{ED}(t,i)$  can be computed based on the algorithm in [9], and  $SC(t-1,k:t,i)$  take the similar form as those in [27].

##### B. Ranking measures and MDP-RSC procedures

As  $|U|$  increases, the number of states for the pure DP increases exponentially as shown in (17). To overcome this disadvantage, the sequential commitment scheme in conjunction with relaxed states is adopted, where plants are dispatched sequentially based on the pre-ordered plant sequence. At one time the subset of at most  $|G|$  ( $|G| \leq |U|$ ) plants are committed simultaneously based on DP principle and the plants whose states are not determined are temporarily set at relaxed states (ignoring ON/OFF states). The process continues till the ON/OFF states of all plants are determined. This is MDP-RSC algorithm [14], i.e., multi-site version DP-RSC [33], [35] combining relaxed state with sequential commitment. The DP-RSC procedure was embedded in the dynamic regrouping based DP-RSC (DRDP-RSC) procedure [35] (refer to the inner loop of the algorithm).

The choice of  $|G|$  seeks a trade-off between the solution speed and solution accuracy. Usually the solution accuracy has a tendency to improve when  $|G|$  ( $|G| < |U|$ ) increases but not

monotonically. Based on numerical experiment for our test instances, it seems that the solution speed is sufficiently fast with acceptable solution accuracy when  $3 \leq |G| \leq 10$ .

The effectiveness of the sequential commitment scheme is associated with ranking measures. However, it is difficult to find a single measure to judge the relative efficiency of CHP plants in all situations. Considering the interdependence between heat and power in a CHP plant, it is better for the ranking measure to capture prices of heat and power production simultaneously. Numerical experiments showed that the relaxed-state problem (with all plants at relaxed states) can be used to obtain a reasonably good ranking measure.

The relaxed solutions can help to partition the plants into two categories: category one contains the plants producing heat and/or power over the planning horizon and category two the remaining plants. The plants in category one is heuristically more efficient than those in category two. The plants in category two are preceded by those in category one. Within the category the less efficient plants are placed first. This arrangement has tendency to shut down the less efficient plant because when ON/OFF combinations of the less efficient plants are determined the remaining plants including more efficient plants are at relaxed states, where the plants operate based on the relaxed characteristics as discussed in Section II A. Though the operations based on relaxed characteristics are different from those based on the true characteristics, the relative efficiency of the plants should not change much.

The plants in category one are ranked based on a non-increasing order of measures  $M_{R,u,j}$ , 
$$M_{R,u,j} = \frac{\sum_{i=1}^j \lambda_{q,i,t} Q_{u,t} + \lambda_{p,i,t} P_{u,t}}{\sum_{i=1}^j C_{u,t}} \quad (19)$$

where  $M_{R,u,1}$  is the ratio of true production cost (sum of  $C_{u,t}$ ) and the value of heat ( $Q_{u,t}$ ) and power ( $P_{u,t}$ ) production based on the marginal heat production cost ( $\lambda_{q,i,t}$ ) and power production cost ( $\lambda_{p,i,t}$ ) associated with heat and power balance.

Note that the relaxed-state problem is an LP problem. The  $\lambda_{q,i,t}$  and  $\lambda_{p,i,t}$  can be obtained simultaneously with the optimal solution in our specialized Simplex algorithm [9] for solving the relaxed-state problem. The larger the  $M_{R,u,1}$ , the less efficient the plant. The reason why this measure works reasonably well is that the system does not generate surplus heat or power when all plants are set at relaxed states because the least efficient plants will generate no energy by operating at points  $(0, 0, 0)$ . However, it is difficult to determine the relative efficiency of plants when the system has surplus heat or power with all plants set at ON-state [34].

The plants in category two are ranked first according to a non-decreasing order of cold start-up costs. Then only the plants with the smallest start-up costs are kept and the other plants are excluded. Next, the retained plants are ranked based on a non-increasing order of measure  $M_{R,u,2}$ ,

$$M_{R,u,2} = C_{\max,u} / E_{\max,u} \quad (20)$$

where  $E_{\max,u}$  is the maximum energy production (heat plus power) for plant  $u$  and  $C_{\max,u}$  is the cost corresponding to  $E_{\max,u}$  for plant  $u$ . Thus,  $M_{R,u,2}$  is interpreted as a unit cost for

heat and power production of plant  $u$ . The larger the  $M_{R,u,2}$ , the less efficient the plant.

*C. Improved dynamic regrouping and associated MDRDP-RSC procedures*

To compensate for heuristic natures of ranking measures, dynamic regrouping are introduced to improve the solution quality after MDP-RSC [14], similar to [35]. The corresponding DP algorithm is called MDRDP-RSC. Regrouping can be implemented according to a new sequence derived from the original sequence as shown in Fig 3. In the figure the “original” row corresponds to the sequence of MDP-RSC in Fig 3 (a) and the sequence by removing some plants from sequence of MDP-RSC in Fig 3 (b). “1 (2) bit” row corresponds to the sequence by shifting the first (two) plant(s) in the “original” sequence to the last (two) positions.

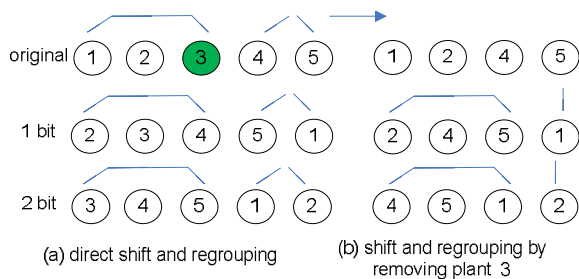


Fig.3. Variants of dynamic regrouping schemes  $|G|=3$

In [35] for the single-site UC, the new sequence was formed by using the sequence of “1 bit” row in Fig. 3(a). Two modifications are introduced here. First, the sequence of “1 bit” is extended to “ $m$  bit” ( $m \geq 1$ ) (by shifting the first  $m$  plants to the last  $m$  positions) strategy. Second, the basis sequence for shifting is formed by removing the plants whose states are ON (except forced OFF-states) according to MDP-RSC [14] procedures as shown in the corresponding “original” row of Fig.3 (b). These removed plants are the most efficient ones and thus can be committed permanently. Also, more radical change for plant combinations in the subsequent sequential commitment can be resulted from this action. This will help to address plant ranking challenges more effectively. MDRDP-RSC based on regrouping strategies in Fig.3 (a) and Fig.3 (b) are called MDRDP-RSC1 and MDRDP-RSC2, respectively.

Algorithm 1. MDRDP-RSC2 procedures

- Step 1.** Applying MDP-RSC procedures to solve multi-site UC.
- Step 2.** Identifying the plants whose states are set at ON over the entire horizon, calculating the start-up costs for these plants.
- Step 3.** Removing the plants determined in Step 2.
- Step 4.** Applying “ $m$  bit” ( $m \geq 1$ ) strategy to form the new sequence.
- Step 5.** Applying the similar sequential commitment procedure of MDP-RSC to re-determine ON/OFF states of the plants, where the plants whose states are not re-determined are kept at the states of the previously committed states, including the plants in Step 2.
- Step 6.** The total system cost is computed by adding the cost determined in Steps 5 and possible start-up cost in Step 2.

V. COMPUTATIONAL EXPERIMENTS

To assess the performance of MDRDP-RSC2, MDRDP-RSC1, MDP-RSC [14], CPLEX [48] were used as benchmarks. CPLEX solved an efficient MILP model by converting time dependent start-up and shut-down costs as well as non-linear relations (11) and (12) into linear relations according to the formula in [47]. MDP-RSC, MDRDP-RSC1 and MDRDP-RSC2 were implemented using C++ in Microsoft Visual Studio 2013 environment. Numerical experiments were conducted on a 2.67 GHz personal computer (RAM 4 GB) under the Windows 7 Operating system. The algorithms were applied to solve three real-life daily scheduling problems in Finnish energy companies.

A. Test problem

The UC of the transmission-constrained multi-site problem is hard to solve. In literature for power-only systems, usually 3-site [12], [17], [19], [23] and 4-site [10], [11] were used to test the performance of the algorithm. According to computational results for our current multi-site CHP system, CPLEX can only solve all instances for the 2-site problem optimally (with 0.1% gap). It is not easy for CPLEX to solve some 4-site instances with 0.1% gap. Thus, three test problems were generated: 2-site, 3-site and 4-site problems by taking the first two, three and all sites in Table I, respectively.

TABLE I  
HEAT AND POWER DEMAND, TYPES OF GENERATION FACILITIES AND HEAT AND POWER CAPACITY FOR THE 4-SITE CHP SYSTEM.

Site	$ U_i $	$ U_{p,i} $	$ U_{q,i} $	$P_i$	$Q_i$	$P_{G,i}$	$Q_{G,i}$	$UT_u$	$DT_u$	$CT_u$
1	13	0	2	815	967	940	2316	[1,5]	[1,5]	[1,10]
2	13	1	1	395	339	940	2313	[1,5]	[1,5]	[1,10]
3	13	3	1	1065	1305	719	1305	[1,5]	[1,5]	[1,10]
4	18	5	0	450	434	978	1831	[1,5]	[1,5]	[1,10]

$|U_i|$ : number of all plants at site  $i$ ;  $|U_{p,i}|$  ( $|U_{q,i}|$ ) number of power- (heat-) only plants at site  $i$ ;  $P_{G,i}$  ( $Q_{G,i}$ ): power(heat) generation capacity in MW at site  $i$ ;  $UT_u$ ,  $DT_u$ ,  $CT_u$  columns show range of  $UT_u$ ,  $DT_u$ ,  $CT_u$ ;  $Q_i$  ( $P_i$ ) maximum heat (power) demand in MW at site  $i$ .

Table I shows the CHP system configuration of four regional energy companies in Finland regarding heat and power demand, types of generation facilities and the corresponding heat and power capacity, as well as the parameters concerning start-up and shut-down characteristics of plants such as minimum up, down periods and cold start-up periods. The plant had between 3-12 extreme points including artificial point (0, 0, 0). In the table, power capacity are power generation corresponding to maximal heat. Each site had sufficient heat generation capacity to satisfy its own demand. Table II shows the transmission capacity between sites.

TABLE II  
TRANSMISSION CAPACITY (MW) BETWEEN SITES.

Site	1	2	3	4
1	0	210	290	140
2	200	0	280	230
3	150	200	0	170
4	190	230	290	0

There are two reasons why the asymmetric transmission capacity is considered. First, numerical experiments [9] showed that the asymmetric case is more difficult to solve

than its symmetric counterpart is. Second, the asymmetric power transmission capacity is possible because the transmission companies may consider leaving the safe margin for the capacity to ensure the spare capacity is available when there is a failure for another part of network.

Heat and power demands were hourly history demand data for a whole year (8760-hour). The transmission cost was applied only when the electricity flow was approaching the capacity limit of the transmission line based on the practice of Nordic power market [49].

*B. Computational results*

Based on yearly data, for each test problem, 14 daily (24-hour) scheduling instances were solved. The differences for the starting periods for two consecutive instances were 672 hours (4 week, one month), beginning from 0 and ending at 8736. Table III shows the setting for relaxed-state based DP algorithms (MDP-RSC [14], MDRDP-RSC1, and MDRDP-RSC2) and CPLEX. Let  $z_b$  and  $z_s$  denote the objective function values of the benchmark and subject algorithms, respectively, then the solution quality of the algorithm is assessed according to relative gap (GAP),

$$GAP=100(z_s-z_b)/z_b \quad (21)$$

TABLE III  
SETTING FOR DP-BASED ALGORITHMS AND CPLEX

Problem	G	$m_1$	$m_2$	Cgap (%)
2-site	6	1	2	0.1
3-site	6	1	3	0.5
4-site	6	1	1	0.5

|G|: number of plants that are committed simultaneously for all DP-based algorithms;  $m_1$  ( $m_2$ ): number of bits shifted for MDRDP-RSC1 (MDRDP-RSC2); Cgap (%) relative gap for CPLEX solver

TABLE IV  
RELATIVE GAP (GAP) FOR RELAXED-STATE BASED DP ALGORITHMS AGAINST CPLEX (WITH 0.1% GAP) AND SOLUTION TIME (CPU TIME) OF BOTH RELAXED-STATE BASED DP ALGORITHMS AND CPLEX FOR 2-SITE DAILY SCHEDULING INSTANCES

sample time	CPU (s)				GAP (%)		
	CPLEX	DP1	DP21	DP22	DP1	DP21	DP22
0	85.6	0.119	0.162	0.138	0.27	0.12	0.00
672	1467.6	0.155	0.261	0.282	0.23	0.21	0.21
1344	2935.5	0.196	0.377	0.288	0.34	0.23	0.01
2016	54.9	0.152	0.294	0.295	0.01	0.00	0.00
2688	251.8	0.118	0.182	0.207	0.08	0.08	0.08
3360	881.3	0.110	0.188	0.190	0.02	0.02	0.01
4032	153.7	0.136	0.241	0.256	0.79	0.23	0.22
4704	170.0	0.082	0.150	0.142	0.21	0.18	0.18
5376	818.1	0.099	0.185	0.215	0.57	0.45	0.27
6048	512.8	0.109	0.186	0.190	0.77	0.77	0.23
6720	1178.5	0.111	0.204	0.193	0.44	0.41	0.23
7392	1442.0	0.105	0.192	0.161	0.30	0.19	0.16
8064	102.5	0.104	0.205	0.194	0.27	0.27	0.05
8736	1904.6	0.095	0.175	0.175	0.38	0.38	0.11
AVG	854.2	0.121	0.214	0.209	0.34	0.25	0.13

DP1: MDP-RSC; DP21: MDRDP-RSC1; DP22: MDRDP-RSC2

Table IV shows the relative gap (GAP) for the relaxed-state based DP algorithms against CPLEX (with 0.1% gap) and solution time for all algorithms for 2-site daily scheduling

instances. Based on the table, in terms of solution quality, MDRDP-RSC2 is about 0.1% better than MDRDP-RSC1, which is in turn more than 0.1% better than MDP-RSC. Dynamic regrouping in most cases decreases both the worst case gap and average gap as compared with MDP-RSC. The solution of MDRDP-RSC2 is close to the optimal solution. In terms of solution time, MDRDP-RSC2 is from more than one hundred to more than a ten thousand (on average a few thousand) times faster than CPLEX.

Table V reports the relative gap (GAP) for MDRDP-RSC2 against CPLEX (with 0.5% gap) and solution time for MDRDP-RSC2 for 3-site and 4-site daily scheduling instances. The results of MDP-RSC and MDRDP-RSC1 are not shown in the table due to space limits. For 3-site instances, on the average, the solution quality of MDRDP-RSC2 is 0.1% better than MDP-RSC and 0.02% better than MDRDP-RSC1. For 4-site instances, the solution quality of MDRDP-RSC1 and MDRDP-RSC2 is almost the same and MDRDP-RSC2 is 0.1 % better than MDP-RSC. Based on the table, the solution gap of MDRDP-RSC2 for 3-site and 4-site instances with the optimal solution is close to 0.5%. In terms of solution time, MDRDP-RSC2 is from a few to more than one hundred (on average a few dozen) times faster than CPLEX.

TABLE V  
RELATIVE GAP OF DP22 (MDRDP-RSC2) AGAINST CPLEX (WITH GAP 0.5%) AND THE CPU TIME (SOLUTION TIME) OF DP22 AND CPLEX FOR 3-SITE AND 4-SITE DAILY SCHEDULING INSTANCES

sample time	CPU(s)				GAP (%)	
	CPLEX		DP22		3-s	4-s
	3-s	4-s	3-s	4-s		
0	3.5	10.7	0.612	1.568	0.00	0.11
672	11.8	120.8	0.659	1.759	-0.14	-0.08
1344	4.0	85.2	0.816	2.014	-0.02	0.00
2016	11.4	13.3	1.051	2.593	-0.19	0.06
2688	3.5	8.2	0.745	2.087	-0.10	-0.18
3360	4.1	63.9	0.667	1.830	-0.06	0.00
4032	28.0	232.9	0.770	2.188	-0.24	0.04
4704	2.9	7.0	0.588	1.618	-0.05	0.06
5376	4.4	10.7	0.711	1.970	-0.08	0.29
6048	8.0	14.9	0.673	1.982	-0.10	0.21
6720	63.8	102.3	0.728	2.306	-0.07	-0.02
7392	54.0	18.9	0.540	1.595	0.10	0.29
8064	3.8	291.8	0.659	2.104	-0.08	0.04
8736	121.0	154.6	0.654	1.951	-0.04	0.20
AVG	23.1	81.1	0.705	1.969	-0.08	0.07

3-s: 3-site; 4-s: 4-site

These results mean that the performance of MDRDP-RSC2 is good with respect to both solution speed and quality. This is highly relevant for dealing with uncertainty of the intermittent RES based on stochastic programming approach, where numerous scenarios of intermittent RES need considering [15], [16], especially when Monto Carlo simulation were used [50]. Each scenario corresponds to a deterministic multi-site UC instance that requires solving efficiently to handle the integration of intermittent RES into CHP systems.



### VI. CONCLUSION

This paper has developed a dynamic programming (DP) based approach to handle the deterministic unit commitment (UC) of the transmission-constrained multi-site CHP system efficiently. Using the standard optimization software (CPLEX) as a benchmark, the DP-based approach can get near-optimal solution thousands of times faster for the small system, and on the average a few dozen times faster with comparable solution accuracy for the large system. This will lay foundation dealing with stochasticity of intermittent RES such as wind and solar power. To increase further operational flexibility of the system for accommodating more RES, one important extension is to consider power ramping ability of CHP plants in the UC context. This remains to be challenging since power ramp ability depends on heat production level in the CHP plant [51].

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Chief of the IEEE Transactions on Robotics and Automation, and the Founding Editor-in-Chief of the IEEE Transactions on Automation Science and Engineering.

**Aiying Rong** received the B.S. degree in Electrical Engineering from Northeastern University, Shenyang, China, in 1989, the M.S. degree in Industrial Engineering and Engineering Management from Hong Kong University of Science and Technology, China, 2002, and the PhD degree in Computer Science from University of Turku, Finland, in 2006. She is currently working in Aalto University, Finland as a researcher. Her research interests include planning, scheduling and optimization of large scale systems. She has expertise in dealing with polygeneration energy systems (combined heat and power (CHP) is the basic form of polygeneration), energy market optimization and integration of intermittent renewable energy (solar and wind) into polygeneration systems.

**Peter B. Luh** (S'77–M'80–SM'91–F'95) received the B.S. degree in Electrical Engineering from National Taiwan University, Taipei, Taiwan, in 1973, the M.S. degree in aeronautics and astronautics from the Massachusetts Institute of Technology, Cambridge, MA, USA, in 1977 and the Ph.D. degree in Applied Mathematics from Harvard University, Cambridge, in 1980.

He has been with the University of Connecticut, Storrs, CT, USA, since 1980, and is the SNET Professor of Communications and Information Technologies. His current research interests include smart power systems—Smart Grid, design of auction methods for electricity markets, effective renewable (wind and solar) integration to the grid, electricity load and price forecasting with demand response, and micro grid.

Dr. Luh was the vice President of Publication activities for the IEEE Robotics and Automation Society, an Editor-in-