Chiller Plant Operation Optimization with Minimum Up/Down Time Constraints

Danxu Zhang, *Student Member, IEEE*, Peter B. Luh, *Life Fellow, IEEE*, Junqiang Fan, *Senior Member, IEEE* and Shalabh Gupta, *Member, IEEE*

***Abstract*—A chiller plant includes chillers, cooling towers and pumps. Chiller minimum up/down times (MUDTs) are relatively long among a plant but short as compared with the time interval. MUDT constraints are generally ignored in optimization but handled heuristically. With the numbers of active units and nonlinear heat exchange, chiller plant optimization is a mixed-integer nonlinear problem. Complexity of the problem will increase if MUDT constraints are considered. In this paper, potential energy savings and complexity increase caused by considering such constraints are explored. To obtain near-optimal solutions efficiently, a method is developed based on a recent decomposition and coordination approach. Chiller subproblems are solved in two steps, first without MUDT constraints to establish stage-wise costs. Then possible state transitions are developed based on MUDT constraints from the states with feasible costs before Dynamic Programming (DP) is used. For practical problems, MUDT constraints are rarely violated. A second method is developed with DP replaced by local search to reduce computational effort. Numerical testing shows that our second method is faster and without much performance degradation as compared with the first one. However, energy savings are small with complexity increased significantly by using both methods as compared with those where MUDTs are handled after optimization. Therefore, there is no need to consider MUDT constraints in optimization.**

***Index Terms*—chiller plant optimization, decomposition and coordination, dynamic programming, local search, minimum up/down times.**

I. INTRODUCTION

A typical chiller plant includes chiller, cooling tower, primary pump and condenser pump subsystems as shown in Fig. 1. Chiller minimum up/down times (MUDTs) are relatively long among a chiller plant but short as compared with the time interval. MUDT constraints are generally ignored in operation optimization but handled heuristically. With the numbers of active units and nonlinear heat exchange, chiller plant optimization is a mixed-integer nonlinear problem which is combinatorial. Complexity of the problem will increase if MUDT constraints are considered since on/off statues of chillers will depend on not only current cooling load but also previous statues of chillers. To explore potential energy savings and complexity increase caused by considering such constraints, in this paper, chiller plant optimization with MUDT constrains is studied by using a plant with identical units in each subsystem. The results are compared with those where such constraints are handled heuristically. As suggested by industry partners, 20 min and 10 min are used as chiller minimum up and down times, respectively. Since cooling requirements often change slowly, 10 min is used as a time interval. With short MUDTs, the problem is solved by looking ahead one hour.

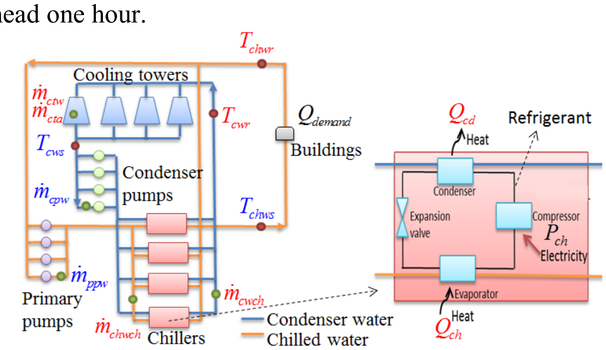

Fig. 1. Schematic of a chiller plant (Decision variables are in blue and depended variables are in red)

In Section II, papers for optimization with chiller MUDT constraints are reviewed first, followed by problems with similar characteristics. In Section III, a formulation with MUDT constraints is established based on our previous work.

In Section IV, to solve the problem efficiently and obtain near-optimal solutions, a novel method is developed by using a recent decomposition and coordination approach (i.e., surrogate augmented Lagrangian relaxation) combined with modified Dynamic Programming (DP). Since chillers are identical, with chillers coupled across time, the numbers of active chillers at current and previous time are used as augmented states. Stage-wise costs are established without MUDT constraints. Possible state transitions across time are developed based on MUDT constraints from the states with feasible costs before DP is used. For practical problems where MUDT constraints are rarely violated, a second method is developed with DP replaced by Local Search (LS) to reduce computational effort.

In Section V, two examples are tested. Example 1 demonstrates that the performance of our two methods is

Manuscript received: February, 15, 2017; Revised April, 22, 2017; Accepted June, 12, 2017. This paper was recommended for publication by Editor Kevin Lynch upon evaluation of the Associate Editor and Reviewers' comments. This work was supported in part by the United Technologies Corporation Institute for Advanced Systems Engineering (UTC-IASE) at the University of Connecticut under the Supervisory Control Synthesis project. Any opinions expressed herein are those of the authors and do not represent those of the sponsor.

D. Zhang, P. B. Luh and S. Gupta are with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269 USA (e-mail: danxu.zhang@uconn.edu; Peter.Luh@uconn.edu; shalabh.gupta@uconn.edu).

J. Fan is with United Technologies Research Center, Farmington, CT 06032. USA (e-mail: Junqiang.Fan@carrier.utc.com).

Digital Object Identifier (DOI): see top of this page.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LRA.2017.2723467, IEEE Robotics and Automation Letters

good, and SALR+LS is fast without much performance degradation as compared with SALR+DP for problems where MUDT constraints are rarely violated. Based on a practical cooling load profile, Example 2 shows that energy savings by using our two methods are small as compared with those where MUDT constraints are handled heuristically. Therefore, there is no need to consider such constraints in optimization.

II. Literature Review

In this section, papers for operation optimization with chiller MUDTs are reviewed first. Since there are only a few papers focusing on such problems, problems with similar characteristics (e.g., mixed-integer, nonlinear and with MUDT constraints) are also reviewed.

For chiller plant optimization, MUDT constraints are generally ignored [1-5] and only a few papers considering such constraints [6-7]. For example, a chiller plant with four chillers where two of them are always online was studied in [6]. Each chiller was equipped with a condenser pump and a primary pump. Once a chiller is online, the corresponding pumps are online. Stage-wise costs were obtained by ignoring MUDT constraints. Then Dijkstra was used by selecting the paths that satisfy MUDT constraints. Since only on/off statues of two chillers and cooling towers need to be determined, the problem is much simpler than our problem. Computational time is not presented and energy savings caused by considering MUDT constraints is not shown.

Similarly, a chiller sequencing problem with different chillers was studied in [7]. Chillers were the only units considered in the paper and chilled/condenser water supply temperatures were assumed fixed. Chiller models were simple quadratic functions of cooling loads. The problem was solved based on a decomposition and coordination approach, Lagrangian Relaxation (LR), to reduce computational effort. Stage-wise costs are obtained without MUDT constraints and possible state transitions were generated based on MUDT constraints for DP to be used. Since chillers are the only units in the system, the problem is much simpler as compared with our problem. Computational time was not presented and no conclusion is made on energy savings caused by considering MUDT constraints. In addition, standard LR suffers from complexity difficulties, zigzagging and slow convergence, and optimal dual values are needed in the optimization process. As an improvement of LR, surrogate Lagrangian relaxation (SLR) overcomes such difficulties by using surrogate subgradients and convergence was proved in [8]. To further accelerate the speed of convergence and reduce coupling constraint violations, Surrogate Augmented Lagrangian Relaxation (SALR) was developed recently [9]. The key idea is adding quadratic penalty terms for the coupling constraints that are difficult to be satisfied. In this paper, SALR is used to efficiently obtain near-optimal solutions for our problem.

Since there are not many papers focusing on chiller plant optimization with MUDT constraints, problems with similar characteristics such as with MUDT constraints [18-20] and mixed-integer nonlinear problems [16-17] are reviewed. For example, with long MUDTs, MUDT constraints are commonly considered in unit commitment problems [18-20]. The problems are generally mixed-integer linear problems which are different from our problem which is with short MUDTs and highly nonlinear models. In addition, a chiller plant with multiple chillers was studied in [16]. Chilled/condenser water supply temperatures are assumed constant and the chiller model becomes a quadratic function of cooling loads. Complexity of the problem is much reduced as compared with our problem.

III. Problem Formulation

In this section, with chiller MUDTs considered, a problem formulation based on our previous work [10] is presented. Cooling requirements are assumed to be provided by load forecasting every 10 min with good performance.

A. Chiller Model

A commonly used chiller model for optimization is adopted and presented as follows [11-12].

Chiller power consumption

Chiller power consumption is derived by rated power consumption at a reference condition with adjustments as

$$P_{ch}\left(T_{cws},T_{chws},Q_{ch}\right)=P_{ch,ref}\times CAPFT\times EIRFT\times EIRFPLR, \quad (1)$$

$$P_{ch,ref}=Q_{ch,ref}\times EIR, \quad (2)$$

$$\begin{aligned}CAPFT=&a_1+a_2T_{chws}(t)+a_3T_{chws}^2(t)+a_4T_{cws}(t)+a_5T_{cws}^2(t)\\&+a_6T_{chws}(t)T_{cws}(t),\end{aligned} \quad (3)$$

$$\begin{aligned}EIRFT=&b_1+b_2T_{chws}(t)+b_3T_{chws}^2(t)+b_4T_{cws}(t)+b_5T_{cws}^2(t)\\&+b_6T_{chws}(t)T_{cws}(t),\end{aligned} \quad (4)$$

$$EIRFPLR=d_1+d_2\times PLR(t)+d_3\times PLR^2(t), \quad (5)$$

$$PLR(t)=\frac{Q_{ch}(t)}{Q_{ch,ref}\times CAPFT}, \quad (6)$$

where $P_{ch,ref}$ is power consumption at a reference condition, $Q_{ch,ref}$ is cooling capacity at a reference condition, EIR is energy input to cooling output ratio, $CAPFT$ is a cooling capacity adjustment for Q_{ch}. $EIRFT$ and $EIRFPLR$ are adjustments for EIR, and a_i, b_i and d_i are coefficients.

In this work, condenser water supply temperature T_{cws} is a parameter obtained based on the wet-bulb temperature T_{wb} according to a conventional strategy and chiller water supply temperature T_{chws} is a decision variable. As the formulation shows, the model is complex and highly nonlinear.

Heat exchange of chillers

As Fig. 1 shows, heat is absorbed by the evaporator and rejected by the condenser in a chiller. Heat exchange for the evaporator is

$$Q_{ch}(t)=C_p\left[\dot{m}_{chchw}(t)\left(T_{chwr}(t)-T_{chws}(t)\right)\right], \quad (7)$$

where C_p is water specific heat, T_{chwr} is chilled water return temperature, and $\dot{m}_{chchw}$ is chiller chilled water mass flow rate. Heat rejected by the condenser Q_{cd} is similar to Q_{ch} and is not repeated. According to energy balance, we have

$$P_{ch}(t)+Q_{ch}(t)=Q_{cd}(t). \quad (8)$$

Cooling load provided by a chiller is assumed proportional to chiller capacity [13]. With identical chillers used, cooling provided by chillers are the same, and should satisfy the building cooling requirements $Q_{demand}(t)$ as

$$x(t)\times Q_{ch}(t)=Q_{demand}(t), \tag{9}$$

where $x(t)$ is the number of active chillers at time t.

Since cooling provided by a chiller has lower and upper bounds, the number of active chillers should also satisfy

$$x(t)\times Q_{ch,\min}\le Q_{demand}(t)\le x(t)\times Q_{ch,\max}, \tag{10}$$

where $Q_{ch,min}$ and $Q_{ch,max}$ are the minimum and maximum cooling provided by a chiller, respectively.

Minimum up/down Time Constraints

For our problem, minimum up time is 20min and minimum down time is 10min. Since the time interval and minimum down time are the same, there is no violation when turning on chillers. Violation occurs when turning off chillers that are online for less than two time intervals. With identical chillers used, to satisfy MUDT constraints, we have

$$x(t-1)-x(t)\le x(t-2), \tag{11}$$

where the left side denotes the number of chillers turned off at time t and the right side denotes the number of active chiller at time t-2.

B. Cooling Tower Model

A typical cooling tower has a fan producing air and heat exchange occurs between the air and the condenser water. An empirical model from [14] is used and shown as follows.

Cooling tower power consumption

$$P_{ct}=P_{ct,nom}\left(\frac{\dot{m}_{cta}(t)}{\dot{m}_{cta,\text{nom}}}\right)^3, \tag{12}$$

where $P_{ct,nom}$ is the nominal power consumption and $\dot{m}_{cta,nom}$ is the nominal air mass flow rate.

The nonlinear heat exchange model, York cooling tower model, based on the approach temperature is used. The model can be found at Modelica building library [14] *Builings.Fluid.HeatExchangers.CoolingTowers.Correlations.yorkCalc*. Details are not presented.

C. Variable-speed Pump Model

Pumps are the units that circulate water and the amount of water is controlled by pressures in practice. In optimization, pumps are often modeled by using mass flow rates of the water directly for simplicity. Based on [15], we have

$$P_{pp}=\frac{k_{pp}X_{ppm}^3}{\left(1-e^{-A_{pp}X_{ppm}}\right)\eta_{\text{var}}\left(X_{ppm}\right)}, \tag{13}$$

$$X_{ppm}=\frac{\dot{m}_{ppw}(t)}{\dot{m}_{ppw,nom}}, \tag{14}$$

$$\eta_{\text{var}}\left(X_{ppm}\right)=k_1+k_2X_{ppm}+k_3X_{ppm}^2+k_4X_{ppm}^3, \tag{15}$$

where $\dot{m}_{ppw,nom}$ is the nominal primary pump mass flow rate, $\dot{m}_{ppw}$ is primary pump mass flow rate, k_{pp}, A_{pp} and k_i are coefficients.

Condenser pumps are similar as primary pumps, and are not presented here.

D. Coupling Constraints between Subsystems

As Fig. 1 shows, chilled water of primary pumps flows into chillers, and condenser water of condenser pumps flows into chillers and then flows into cooling towers. Based on mass balance, we have

$$N_{App}(t)\dot{m}_{ppw}(t)=x(t)\dot{m}_{chchw}(t), \tag{16}$$

$$N_{Acp}(t)\dot{m}_{cpw}(t)=x(t)\dot{m}_{chcw}(t), \tag{17}$$

$$N_{Act}(t)\dot{m}_{ctcw}(t)=x(t)\dot{m}_{chcw}(t), \tag{18}$$

where N_{App}, N_{Acp} and N_{Act} are the numbers of active primary pumps, condenser pumps and cooling towers, respectively.

To separate chillers and cooling towers, water return temperatures for both subsystems are introduced. For the equations related to chillers, T_{cwr_ch} is used to replace T_{cwr} and for the equations related to cooling towers, T_{cwr_ct} is used to replace T_{cwr}, and we assume that

$$T_{cwr\_ch}(t)=T_{cwr\_ct}(t). \tag{19}$$

E. The Objective Function and the Optimization Problem

The objective is to minimize chiller plant power consumption for a chiller plant with identical units in each subsystem with MUDT constraints. The problem is shown as:

$$\min_{\left(x,N_{Act},N_{App},N_{Acp},T_{chws},\dot{m}_{ppw},\dot{m}_{cpw}\right)} J,$$

$$\text{with } J\equiv\sum_{t=1}^{T}\Big[x(t)\times P_{ch}(t)+N_{Act}(t)\times P_{ct}(t)$$
$$+N_{Act}(t)\times P_{ct}(t)+N_{Acp}(t)\times P_{cp}(t)\Big] \tag{20}$$

subject to heat exchange constraints, lower and upper bounds, coupling constraints and MUDT constraints. Since the problem is solved by looking ahead one hour and the time interval is 10min, T is 6 in our study.

IV. Solution Methodology

With identical chillers and short MUDTs, a novel method is developed by using a decomposition and coordination approach. Chiller subproblems are solved by first ignoring MUDT constraints to obtain stage-wise costs. Then possible state transitions are developed based on such constraints for DP to be used. The relaxed problem and subproblems are presented in Subsection A. The dual function and feasible solutions are presented in Subsection B. For problems where MUDT constraints are rarely violated, a second method is developed with DP replaced by local search in Subsection C.

A. SALR + DP

The Relaxed Problem

A recent decomposition and coordination method, Surrogate Augmented Lagrangian Relaxation (SALR) [9], is used to efficiently obtain near-optimal solutions. Coupling constraints (16-19) are relaxed by using Lagrangian multipliers λ and a penalty term for (16) which is difficult to be satisfied is added. The relaxed problem is

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$$\min_{(x, N_{Act}, N_{App}, N_{Acp}, T_{chws}, \dot{m}_{ppw}, \dot{m}_{cpw})} L,$$
$$\text{with } L \equiv \sum_{t=1}^{T} \Big[J(t) + \lambda_{\text{pp}}^{k}(t)\big(N_{App}(t)\dot{m}_{ppw}(t) - x(t)\dot{m}_{chchw}(t)\big) + \lambda_{\text{cp}}^{k}(t)\big(N_{Acp}(t)\dot{m}_{cpw}(t) - x(t)\dot{m}_{chcw}(t)\big) + \lambda_{\text{ct}}^{k}(t)\big(N_{Act}(t)\dot{m}_{ctcw}(t) - x(t)\dot{m}_{chcw}(t)\big) + \lambda_{\text{T}}^{k}(t)\big(T_{cwr\_ch}(t) - T_{cwr\_ct}(t)\big) + 0.5c^{k}\big(N_{App}(t)\dot{m}_{pp}(t) - x(t)\dot{m}_{chchw}(t)\big)^{2} \Big], \quad (21)$$

subject to heat exchange constraints, lower and upper bounds and MUDT constraints.

Chiller subproblem

By collecting all the terms related to chillers and replacing variables of other subsystems (i.e., N_{App}^{k-1} and $\dot{m}_{ppw}^{k-1}$) by using solutions of previous iterations, chiller subproblem at k^{th} iteration is obtained as:

$$\min_{(x, T_{chws})} L_{ch},$$
$$\text{with } L_{ch} \equiv \sum_{t=1}^{T} \Big[N_{Ach}(t)P_{ch}(t) - \lambda_{\text{ct}}^{k}(t)\big(x(t)\dot{m}_{chcw}(t)\big) - \lambda_{\text{pp}}^{k}(t)\big(x(t)\dot{m}_{chchw}(t)\big) - \lambda_{\text{cp}}^{k}(t)\big(x(t)\dot{m}_{chcw}(t)\big) + \lambda_{\text{T}}^{k}(t)T_{cwr\_ch}(t) + 0.5c^{k}\big(N_{App}^{k-1}(t)\dot{m}_{ppw}^{k-1}(t) - x(t)\dot{m}_{chchw}(t)\big)^{2} \Big], \quad (22)$$

subject to (7-11).

As the formulation shows, the subproblem is complicated with highly nonlinearity and coupled across time. Since identical chillers are used, based on (11), the numbers of active chiller at time *t-1* and *t*, ($x_1(t-1)$, $x_2(t)$), are used as states, and stage-wise costs are established by using a commonly used nonlinear method, Sequential Quadratic Programming (SQP), without considering MUDT constraints first. With (10) considered, if the solution is not feasible (i.e., the number of chillers cannot satisfy cooling requirements), the cost of the state is assumed to be infinite. From the states with feasible costs, possible state transitions are created based on MUDT constraints. Then, DP is used to obtain the final solution of the subproblem.

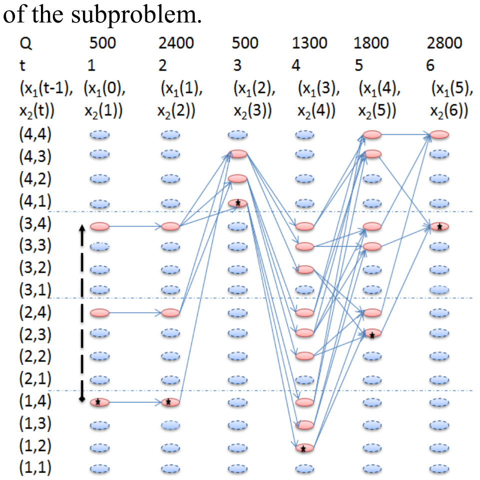

Fig. 2. Possible state transition for a chiller plant with four identical chillers

Figure 2 shows possible state transitions for a chiller plant with four identical chillers. Minimum and maximum cooling load provided by the chiller is 140 KW and 700 KW, respectively. Cooling requirements Q are [500 KW, 2400 KW, 500 KW, 1300 KW, 1800 KW, 2800 KW]. Blue dots represent the states with infeasible costs where the numbers of active chillers cannot satisfy cooling load requirements. For example, since minimum cooling provided by four chillers (i.e., 560 KW) is higher than the requirement 500 KW at t=1, the solution is infeasible when $x_1(1)$ is 4. The state (4, 1) at t=2 is shown by a blue dot. State transition is created from pink dots based on MUDT constraints.

Cooling tower subproblem and pump subproblem

By collecting all the terms related to cooling towers, cooling tower subproblem at k^{th} iteration is obtained as:

$$\min_{(N_{Act})} L_{ct}, \text{with } L_{ct} \equiv \sum_{t=1}^{T} \big[l_{ct}(t) \big], \quad (23)$$

$$l_{ct}(t) = N_{Act}(t)P_{ct}(t) - \lambda_{\text{ct}}^{k}(t)N_{Act}(t)\dot{m}_{ctcw}(t) - \lambda_{\text{T}}^{k}(t)T_{cwr\_ct}(t), \quad (24)$$

subject to heat exchange and lower and upper bound constraints. Since cooling tower subproblems are not coupled across time, with identical cooling towers, SQP is used to solve (24) by considering all the possible cases where the number of active cooling tower satisfies the cooling requirement. The final solution of the subproblem is the one with minimum cost.

Pump subproblems are obtained and solved similarly as the cooling tower subproblem. Details are not presented.

The Dual Problem and Feasible Solutions

The solutions of the subproblems are coordinated through the iterative updating of multipliers to maximize the high-level dual function. The dual function is shown as:

$$\max_{(\lambda)} q, \text{with } q \equiv L_{ch} + L_{ct} + L_{pp} + L_{cp}. \quad (25)$$

Multipliers are updated based on (26-28) after solving one subproblem, and then used for next subproblem until the dual function is maximized [8, 9].

$$\lambda^{k+1}(t) = \lambda^{k}(t) + s^{k}(t)\tilde{g}\big(x^{k}(t)\big), \quad (26)$$

$$s^{k}(t) = \alpha_{k} s^{k-1}(t)\frac{\big\|\tilde{g}\big(x^{k-1}(t)\big)\big\|}{\big\|\tilde{g}\big(x^{k}(t)\big)\big\|}, \quad (27)$$

$$\alpha_{k} = 1 - \big(M \cdot k^{p}\big)^{-1},\ 0 < p < 1,\ M > 1,\ k = 1, 2, ..., \quad (28)$$

where s is the stepsize, M and p are constants, k is the number of iterations and $\tilde{g}$ is the augmented surrogate subgradient.

Solutions obtained above may not be feasible for the original problem with coupling constraints relaxed. To obtain feasible solutions, the numbers of active units obtained are checked and used directly or with modification by increasing or decreasing the number of active units. Then, SQP is used to obtain continuous variables for the entire problem.

B. SALR + Local Search

For practical problems, MUDT constraints are rarely violated. To reduce computational effort, a second method is developed with DP replaced by Local Search (LS). The first two steps including establish state costs and developing

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possible transitions are the same as those of SALR+DP. Then, the states with minimum costs are marked by stars and checked. If there are MUDT constraint violations, the related states are moved up or down to new states until MUDT constraints are satisfied. If more than one new state is obtained, the one with minimum cost is the solution. For example, according to Fig. 2, violation occurs in the first transition. The new state is obtained by moving the first state up from (1, 4) to (3, 4). Computational effort of SALR+LS is reduced as compared to SALR+DP. Solution quality may not be as good as SALR+DP, while the difference is small for the problem where MUDT constraints are rarely violated as implies in our results.

C. Baseline

In practical, chiller plants are often operated based on rules. For studies focusing chiller plant optimization, MUDT constraints are generally ignored but handled heuristically after the optimization process. Based on our previous work on chiller plant optimization [10], a baseline is defined with following strategies: SALR combined with sequential quadratic programming is used without considering MUDT constraints; the solutions are checked to see whether MUDTs are satisfied. If not, the on/off statuses of a chiller at previous time will be used.

V. Numerical Testing

The methods presented above have been implemented in MATLAB 2013a on a Core i7 2.8 GHz laptop with 16 GB memory. Two examples are tested. Example 1 is to show the ideas and intuition of our methods. In Example 2, we resolve the problem of Example 1 by using a 5 hours' cooling load profile to show energy savings by using our methods as compared with the baseline.

Example 1

In this example, a chiller plant with 4 units in each subsystem is used. The time interval is 10 min since cooling load requirements slowly in practice. With short minimum up/down times (i.e., 20min/10min), the chiller plant optimization problem with MUDT constraints is solved by looking ahead 1 hour using our methods. The rated capacity of the chiller is 703 KW. Model coefficients of chillers are from Modelica standard library and pumps are from [15]. Cooling requirements tested are [465.40 KW, 1463.5 KW, 464.64 KW, 452.66 KW, 455.04 KW, 459.84 KW] based on a profile of UConn's chiller plant. Penalty coefficient c is updated as $c=c_0\times\beta^k$. Stopping criteria is $\|\lambda^k-\lambda^{k-1}\|\leq\varepsilon|\lambda^k|$ with ε=0.001.

As mentioned earlier, lower bounds are obtained after convergence. With coupling constraints relaxed, the solutions may not be feasible. To obtain feasible solutions, discrete variables obtained by solving the relaxed problems are used and SQP is applied to solve the entire problem. Results for lower bounds, feasible costs (i.e., plant power consumption), gaps between feasible costs and lower bounds, CPU and the number of iterations are shown in Table I. According to the results, power consumption at t=1 by using SALR+DP and SALR+LS are the same. The numbers of active chillers, cooling towers, primary pumps and condenser pumps are [2, 3, 1, 1, 1, 1], [2, 4, 2, 2, 2, 2], [2, 4, 1, 1, 1, 1] and [3, 4, 3, 3, 3, 3], respectively. Chilled water supply temperatures are [4.44 ^{0}C, 5.63 ^{0}C, 5.04 ^{0}C, 4.76 ^{0}C, 4.82 ^{0}C, 4.93 ^{0}C]. Mass flow rates of primary pumps and condenser pumps are [8.10 kg/s, 9.29 kg/s, 11.10 kg/s, 10.53 kg/s, 10.64 kg/s, 10.87 kg/s] and [8.33 kg/s, 19.19 kg/s, 8.33 kg/s, 8.33 kg/s, 8.33 kg/s, 8.33 kg/s], respectively. MUDT constraints are satisfied.

Table I Optimized results with MUDT constraints

	SALR+DP	SALR+LS	LR+DP
Lower Bound	511.76	511.58	501.79
Feasible cost (KW)	513.12	513.12	513.12
Gap (%)	0.27	0.30	2.21
CPU(s)	36.64	24.38	359.65
Number of iterations	210	155	1000

According to Table I, gaps of our two methods are small showing that the quality of the solutions is good. SALR+LS is better than SALR+DP in terms of computational time and the number of iterations. This is reasonable because there is only one violation if MUDT constraints are not considered and fewer steps are needed to obtain solutions that satisfy MUDT constraints for SALR+LS as compared with SALR+DP.

To show the performance of our methods, LR+DP from [7] is used for comparison. With penalty for constraints that are difficult to be satisfied added and multipliers updated after solving one subproblem, CPU and the number of iterations of SALR are much less than those of standard LR. The results show that both of our methods are better than LR+DP in terms of CPU and solution quality.

To show energy savings by considering MUDT constraints, the baseline presented in Section IV Subsection C is used for comparison. Without considering MUDT constraints, the numbers of active chillers are [1, 3, 1, 1, 1, 1]. Since there is one violation, based on baseline strategies, the numbers become [1, 3, 3, 1, 1, 1]. The numbers of active cooling towers, primary pumps and condenser pumps are [2, 4, 2, 2, 2, 2], [2, 4, 1, 1, 1, 1] and [3, 4, 3, 3, 3, 3], respectively. CPU is around 4.07 s. Power consumption is 545.82 KW. According to the results, CPU is around 6 times as compared with results of the baseline and power consumption is increased by 2%. Computational time implies that complexity of the problem with MUDT constraints is increased significantly as compared with the baseline.

Example 2

Based on a cooling load profile of UConn's chiller plant, we resolve the problem in Example 1 by using our methods. The profile is shown in Fig. 3. The time interval is 10 min since cooling load requirements slowly in practice. With short minimum up/down times (i.e., 20min/10min), the problem is solved at the beginning of each time interval by looking ahead one hour. To show energy savings caused by considering MUDT constraints, the baseline presented in section IV is used for comparison.

As implies in Fig. 3, MUDT constraints are rarely violated. Power consumption with and without MUDT constraints are

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the same for most of the time as shown in Fig. 4. Energy saving caused by considering such constraints in optimization is around 1.71% as compared with the baseline. Computational time of using SALR+DP and is around 6 times of the computational time of the baseline. Since energy saving caused by considering MUDT constraints in optimization is small as compared with the baseline with computational time increased significantly, there is no need to consider MUDT constraints in optimization. The performance of our methods under uncertainties is beyond the scope of this paper. Nevertheless, the performance of our method without MUDT constraints under input uncertainties is being studied and preliminary results show that our method is robust.

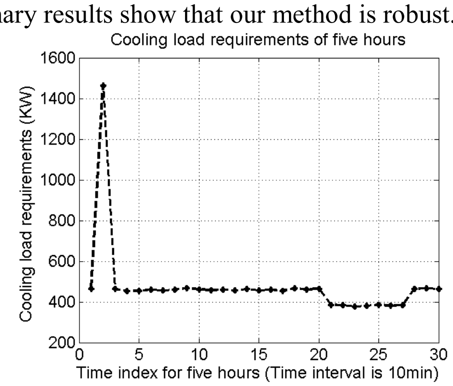

Fig. 3. Five hours' cooling load profile

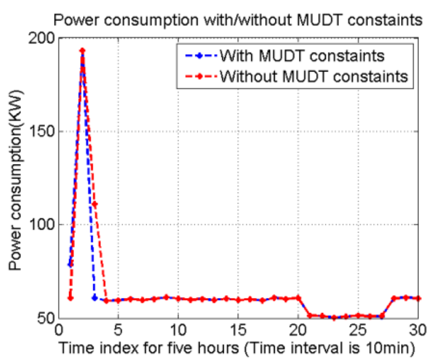

Fig. 4. Chiller plant power consumption with/without MUDT constraints

VI. Conclusion

In this paper, two methods, SALR+DP and SALR+LS, are developed for chiller plant optimization with MUDT constraints considered. Numerical testing demonstrates that near-optimal solutions are obtained by both of our methods. For practical systems, MUDT constraints are rarely violated. The second method SALR+LS is fast and without much performance degradation as compared with SALR+DP. However, energy savings caused by considering MUDT constraints are small while complexity is increased significantly as compared with those where MUDT constraints are handed heuristically. Therefore, there is no need to consider such constraints in optimization.

Acknowledgment

The authors would like to thank Dr. Trevor Bailey and Mr. Christian Netter from United Technologies for their guidance and constructive comments on the research.

References

[1] A. R. Simpson, G. C. Dandy and L. J. Murphy, "Genetic algorithms compared to other techniques for pipe optimization," *Journal of Water Resources Planning and Management*, vol. 120, no. 4, pp. 423–443, 1994.

[2] W. Huang and H. N. Lam, "Using genetic algorithms to optimize controller parameters for HVAC systems," *Energy and Buildings*, vol. 26, no. 3, pp. 277–282, 1997.

[3] Y. C. Chang, J. K. Lin and M. H. Chuang, "Optimal chiller loading by genetic algorithm for reducing energy consumption," *Energy and Buildings*, vol. 37, no. 2, pp. 147–155, 2005.

[4] A. J. Ardakani, F. F. Ardakani and S. H. Hosseinian, "A novel approach for optimal chiller loading using particle swarm optimization," *Energy and Buildings*, vol. 40, no. 12, pp. 2177–2187, 2008.

[5] J. Sun and A. Reddy, "Optimal control of building HVAC&R systems using complete simulation-based sequential quadratic programming (CSB-SQP)," *Building and Environment*, vol. 40, no. 5, pp. 657–669, 2005.

[6] A. Torzhkov, P. Sharma, C. B. Li, R. Toso and A. Chakraborty, "Chiller Plant Optimization - An Integrated Optimization Approach for Chiller Sequencing and Control," *49th IEEE Conference on Decision and Control*, 2010.

[7] Y. C. Chang, "An Outstanding Method for Saving Energy - Optimal Chiller Operation," *IEEE Transactions on Energy Conversion*, vol. 21, no. 2, 2006.

[8] M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu and G. A. Stern, "Convergence of the surrogate Lagrangian relaxation method," *Journal of Optimization Theory and Applications*, vol. 164, no. 1, pp. 173–201, 2015.

[9] X. Sun, P. B. Luh, M. A. Bragin, Y. Chen, J. Wan, and F. Wang, "A decomposition and coordination approach for large-scale security constrained unit commitment problems with combined cycle units," in Proc. 2017 IEEE Power and Energy Soc. General Meeting, 2017.

[10] D. X. Zhang, P. B. Luh, J. Fan and S. Gupta, "Chiller Plant Optimization: A decomposition coordination-based approach for primary-only and primary-secondary systems," under review.

[11] *DOE 2 reference manual, Part 1, version 2.1*, Berkeley Lawrence Berkeley National Laboratories, CA, 1980.

[12] M. Hydeman and K. L. Gillespie, "Tools and techniques to calibrate electric chiller component models," *ASHRAE Transactions*, vol. 108, 2002.

[13] L. Lu, W. J. Cai, Y. C. Soh, L. H. Xie and S. J. Li, "HVAC system optimization-condenser water loop," *Energy Conversion and Management*, vol. 45, no. 4, pp. 613–630, 2004.

[14] M. Wetter, W. D. Zuo, T. S. Nouidui and X. F. Pang, "Modelica buildings library," *Journal of Building Performance Simulation*, vol. 7, pp. 989-999, 2014.

[15] Y. Yao, Z. Lian, Z. Hou and X. Zhou, "Optimal operation of a large cooling system based on an empirical model," *Applied Thermal Engineering*, vol. 24, no. 16, pp. 2303–2321, 2004.

[16] S. Bracco, F. Delfino, F. Pampararo, M. Robba and M. Rossi, "A dynamic optimization-based architecture for polygeneration microgrids with tri-generation, renewable, storage systems and electrical vehicles," *Energy Conversion and Management*, vol. 96, pp. 511-520, 2015.

[17] Kody. M. Powell, W. J. Cole, U. F. Ekarika and T. F. Edgar, "Optimal chiller loading system with thermal energy storage," *Energy*, vol. 50, pp. 445-453, 2013.

[18] A. Y. Saber and G. K. Venayagamoorthy, "Intelligent unit commitment with vehicle-to-grid-Acost-emission optimization," *Journal of Power Sources*, vol. 195, pp. 898-911, 2010.

[19] T. O. Ting, M. V. C. Rao and C. K. Loo, "A novel approach for unit commitment problem via an effective hybrid particle swarm optimization," *IEEE Transitions on Power Systems*, vol. 21, no. 1, 2006.

[20] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher and S. S. Oren, "Co-Optimization of generation unit commitment and transmission switching with N-1 reliability," *IEEE Transitions on Power Systems,* vol. 25, no. 2, 2010.