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Head Dependence of Pump-Storage-Unit Model Applied to Generation Scheduling

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***Abstract*—This paper presents a head-dependent model for pump storage units (PSUs) in a power system for short term generation scheduling over one week. A hydraulic system with upper and lower reservoirs each having their own in and out river flows is considered. Hydraulic conditions as well as head effects are explicitly modeled for both generation and pumping modes of the PSUs. The problem is solved by branch-and-cut method to obtain a near-optimal solution. Test results of hourly generation schedule including comparisons of pump efficiencies, hydraulic conditions, and operating costs of the PSUs for a real power system are presented.**

***Index Terms*—Discharge, generation scheduling, head effect, mixed-integer linear programming, pump storage unit, reservoir**

Nomenclature

Sets:

$J=\{1,\cdots,\bar{j}\}$ Set of thermal units

$N=\{1,\cdots,\bar{n}\}$ Digitized head range for pump-storage units (PSUs)

$S=\{1,\cdots,\bar{s}\}$ Digitized generation segments for units

$T=\{1,\cdots,\bar{t}\}$ Set of time periods considered

$X=\{1,\cdots,\bar{x}\}$ Set of PSU x

Parameters:

C_{jst} Generation cost of thermal unit *j* in segment s in time period t ($j\in J$, $s\in S$, $t\in T$)[NTD/MW]

F_{sn} Step ratio parameter of generation and corresponding discharge of a PSU for corresponding segment s and head range n($s\in S$, $n\in N$).

$\overline{G_{xt}}$, $\underline{G_{xt}}$ Maximum and minimum limits of PSU x in the generation mode in time period t ($x\in X$, $t\in T$)[MW]

H_n Head range n of the PSU($n\in N$)

K_{xn} Coefficient of unit x in the pumping mode and is equal to the ratio of consumed power and pumped water for head range n

L_t Load demand in period t ($t\in T$)[MW]

$\overline{p_{xnt}}$, $\underline{p_{xnt}}$ Maximum and minimum power consumption limits of PSU x in the pumping mode, head range n, and period t ($x\in X$, $t\in T$)[MW]

$\overline{q_t}$, $\underline{q_t}$ Volume of upper and lower reservoirs in time period t ($t\in T$)[Mm^3]

$\overline{r_n}$, $\underline{r_n}$ Maximum and minimum limits of volume difference between the upper and lower reservoirs of PSU x for head range n ($n\in N$)[Mm^3]

$\overline{R^+}$, $\overline{R^-}$ In and out river flows of the upper reservoir [m^3]

$\underline{R^+}$, $\underline{R^-}$ In and out river flows of the lower reservoir [m^3]

Y_{jt} Startup cost of thermal unit j in time period t ($j\in J$, $t\in T$)[NTD]

Z_{jt} Shutdown cost of thermal unit j in time period t ($j\in J$, $t\in T$)[NTD]

ε Maximum resolution to measure the volume of a reservoir and is 0.01 in this research [Mm^3]

Variables:

$\overline{\beta^-}$, $\overline{\beta^+}$ Volume limits of the upper reservoir [Mm^3]

$\underline{\beta^-}$, $\underline{\beta^+}$ Volume limits of the lower reservoir [Mm^3]

d_{xt} Equivalent hourly water discharge from PSU x in time period t ($x\in X$, $t\in T$) [Mm^3]

e_{xt} Electric power from PSU x in time period t ($x\in X$, $t\in T$)[MW]

g_{jst} Generation of thermal unit j in segment s in time period t ($j\in J$, $s\in S$, $t\in T$)[MW]

g_{xst} Generation (in the generation mode) of PSU x in segment s in time period t ($x\in X$,$s\in S$ $t\in T$)[MW]

p_{xt} Power consumption (in the pumping mode) of the PSU x in time period t ($x\in X$, $t\in T$)[MW]

Binary variables:

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h_{nt} Head index in time period t, h_{nt} is one when n matches the digitized head range number, otherwise h_{nt} is zero (n∈N, t∈T)

u_{jt} Status of thermal unit j in time period t (j∈J, t∈T) [1 = on; 0 = off]

v_{xt} Status of PSU x in the generation mode in time period t (x∈X, t∈T)[1 = on; 0 = off]

w_{xt} Status of PSU x in the pumping mode in time period t (x∈X, t∈T)[1 = on; 0 = off]

I. Introduction and Literature Review

Consider a power system with multi-unit pump-storage hydro plants to be scheduled for a specified time period, typically one day or one week, on an hourly basis. Pumped-storage hydro plant (PSP) is designed to save fuel costs by serving the peak load with hydro energy and then pumping the water back up into the reservoir at light load periods. Normally, PSP is operated unless the added pumping cost exceeds the savings in thermal cost. However, the added pumping cost is relative and not definite, and is affected by the water head of the PSP. In addition, PSP typically can only provide limited hours (normally four to eight or ten hours) of continuous operation as a generator or pump due to the limited storage capability of upper/lower storage reservoirs. It is important to model the PSP correctly to make sure all the hydraulic constraints are considered and the total cost is at a minimum. Otherwise, the scheduling results will cause significant discrepancy from a real situation.

This is a unit commitment (UC) problem which determines the commitment and generation levels of resources to minimize the operating cost under nonlinear security and hydraulic constraints. Many mathematical programming approaches of the UC problem have been discussed and various system-wide constraints are considered for years [1][2]. In addition, system operators are required to optimize reliability and economy of a power system with hydro units because water is a renewable and scarce resource. The mathematical models for hydro units become important in a UC problem [3]. Each approach has its own advantages and disadvantages. Lagrangian relaxation (LR) provides an efficient way to solve the hydrothermal scheduling problem [4]. However, LR has the disadvantage of a laborious computational implementation, requiring qualified staff with knowledge of several fields of optimization. The mixed-integer nonlinear programming (MINLP) also requires good knowledge of the problem and the performance is affected by the starting points. Compared to LR and MINLP, mixed-integer linear programming (MILP) models are much easier to modify. The MILP algorithm may obtain good results with less computational time in nonlinear cases with acceptable gap [5]. Recently, the solvers of MILP have significantly improved their performances, so that the UC problem can be solved by MILP with reasonable accuracy in running times compatible with actual operational use. The nonlinear relationship between the generation and water flow rate (water discharge) under a specified head position of a hydro unit can be addressed by using piecewise linear approximation with an MILP model [6]-[9]. Moreover, MILP model for hydro units along cascaded and head-dependent reservoirs to meet hourly power demand for one day or one week is considered [10]-[13].

PSUs play an important role in many power systems [14]. Compared with traditional hydro units, a PSU is more complex and has three operating modes: generating, off, and pumping. The power that can be produced is the rate of water flow multiplied by a conversion coefficient that takes into account the head multiplied by the conversion efficiency of the turbine generator [15]. Thus, the head effect is one of the important elements for a PSU and should be considered in the model. Reference [16] presented a practical MILP model for a PSU in the generation mode with a head-dependent reservoir. Unfortunately, the head effect for PSU in pumping mode is not considered. Furthermore, head effects on both generation and discharge for both pumping and generation modes of the PSU must be considered to enhance the accuracy.

This paper establishes an MILP-based PSU model from characteristic curves to consider head effects not only for generation but also for discharge of the PSU in both generation and pumping modes, and presents head indices which are defined as binary variables to handle the generation and discharge of a PSU in both generating and pumping modes.

The power consumption of a PSU in the pumping mode is not fixed and will be changed by the controller inside the PSU. The controller, usually a proportional–integral–derivative controller, is designed to operate the PSU to follow the trajectory of best efficiency in the pumping mode and adjusts the power consumption when the PSU pumps. If the power consumption is fixed, the efficiency of the PSU in the pumping mode will decrease due to the increase of the water head by the pumped water from lower reservoir to upper reservoir. Thus, the controller will reduce the power consumption to decrease the water flow rate of the PSU in the pumping mode to reduce the hydraulic losses which is related to the square of the water flow rate. That means both pumped water and power consumption will decrease with the increase in the water head. This head-affected power consumption in the pump mode will also be included in the MILP model in addition to the generation mode in this paper.

II. Mathematical Model

A. Overview

Figure 1 shows a hydraulic system of a traditional pumped storage plant (PSP) with upper and lower reservoirs. This paper establishes an MILP-based PSU model based on characteristic curves with the hydraulic system to overcome the nonlinearity difficulty to handle head dependence in both generation and pumping modes and is organized as follows. Subsection B presents the head indices which are defined as binary variables to handle the generation and discharge of a

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PSU in both generating and pumping modes. The ratio parameters and discharge parameters are also established from the characteristic curves of PSUs. Subsection C presents the objective function and power balance constraint to connect the thermal units and PSUs. Subsection D presents constraints related to PSUs such as unit status, generation limits, and power consumption. Subsection E describes the equations related to water discharge constraints of a PSU. Subsection F presents the constraints related to the hydraulic system including the reservoirs of the PSU.

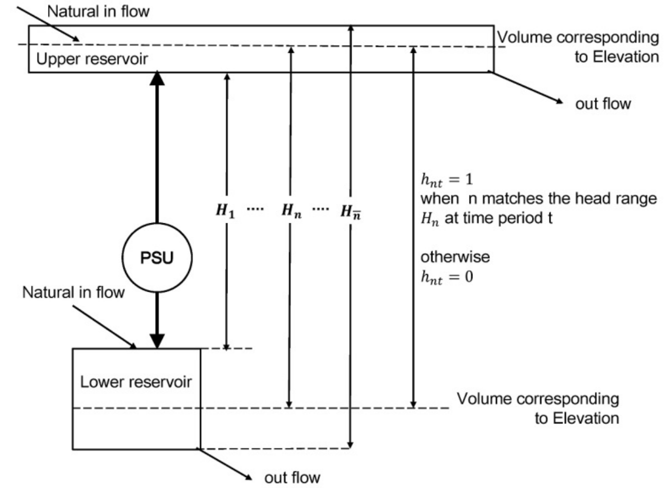

Fig. 1. A traditional PSP with upper and lower reservoirs.

B. The Head Index and Parameters from Characteristic Curves

The water head which is the height for the water to fall is the difference of water elevation between the upper and lower reservoirs of a PSP and is divided into $\overline{n}$ head ranges in the proposed model; n equals to one represents the lowest head range, and n equals to $\overline{n}$ means the highest head range. The head index is represented as h_{nt} and is equal to one when n matches the head range H_n in the time period t, otherwise h_{nt} equals to zero.

The discharge which is the water flow rate of the PSU and is modeled with $\overline{s}$ segments of generation and the corresponding head range n based on the characteristic curves of the PSU. When s equals to one, the discharge corresponds to minimum generation, and when s equals to $\overline{s}$, it represents the maximum generation for each head range.

Figure 2 shows the characteristic curves of the discharge versus head versus generation of the PSU with a head range from 340 to 400 meter from measurements. The figure shows that an increasing discharge rate due to increasing generation output causes increasing hydraulic losses. The discharge is expressed as the water flow rate and is expressed in "cubic-meter per second (CMS) or (m^3/s)".

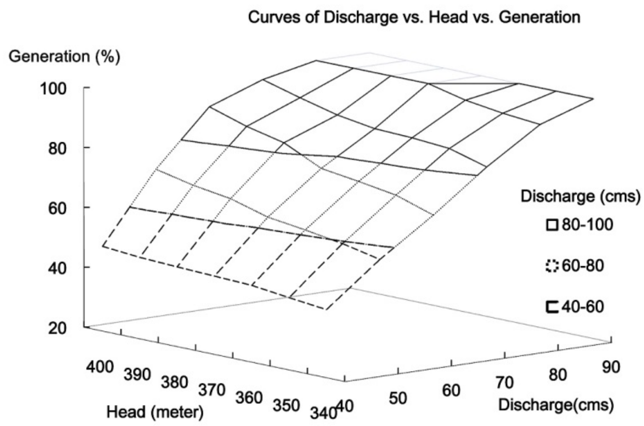

Fig. 2. Curves of the discharge versus head versus generation

From Fig. 2, an $\overline{s}$ by $\overline{n}$ dimensional matrix can be derived to define ratio parameters m_{sn} and discharge parameters q_{sn} as listed in Tables I and II, respectively.

TABLE I
$\overline{S}$ BY $\overline{N}$ DIMENSIONAL RATIO PARAMETER MATRIX

S / N	1	..	$\overline{s}$
1	m_{11}	m_{s1}	$m_{\overline{s}1}$
:	m_{1n}	m_{sn}	$m_{\overline{s}n}$
$\overline{n}$	$m_{1\overline{n}}$	$m_{s\overline{n}}$	$m_{\overline{s}\overline{n}}$

Here m_{sn} represents the ratio of generation and discharge of the PSU at head range n and generation segment s.

$$m_{s\in S,n\in N} = \frac{g_{s\in S,n\in N}}{q_{s\in S,n\in N}}. \tag{1}$$

TABLE II
$\overline{S}$ BY $\overline{N}$ DIMENSION DISCHARGE PARAMETER MATRIX

S / N	1	..	$\overline{s}$
1	q_{11}	q_{s1}	$q_{\overline{s}1}$
:	q_{1n}	q_{sn}	$q_{\overline{s}n}$
$\overline{n}$	$q_{1\overline{n}}$	$q_{s\overline{n}}$	$q_{\overline{s}\overline{n}}$

Each q_{sn} represents the discharge of the PSU at head range n and generation segment s.

In order to convert the characteristic curves of the PSU to the MILP model, a step ratio parameter $F_{s\in S,n\in N}$ (MW per m^3/s) and a step generation parameter $I_{s\in S,n\in N}$ (MW per s) can be obtained from parameters m_{sn} and q_{sn} by (2) and (3) respectively. Step ratio parameter is for the calculation of discharge of the PSU, and step generation is for the generation calculation of the PSU for the following subsection.

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$$F_{sn} = \begin{cases} m_{sn} & , s = 1, n \in N \\ \frac{(q_{sn}m_{sn} - q_{(s-1)n}m_{(s-1)n})}{(q_{sn} - q_{(s-1)n})} & , s \neq 1, n \in N \end{cases} \quad (2)$$

The step generation parameter $I_{s\in S,n\in N}$ can be obtained from (3)

$$I_{sn} = \begin{cases} q_{sn}m_{sn}, & s = 1, n \in N \\ q_{sn}m_{sn} - q_{(s-1)n}m_{(s-1)n}, & s \neq 1, n \in N \end{cases} \quad (3)$$

C. The Objective Function and Power Balance Equation

The model aims to minimize the sum of the production cost $\sum_{s\in S}(g_{jst}u_{jt}C_{jst})$, startup cost Y_{jt}, and shutdown cost Z_{jt} of all thermal units $j \in J$ over all time period $t \in T$. Equation (4) presents the piecewise linear objective function.

$$\min \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\left(g_{jst}u_{jt}C_{jst} + Y_{jt} + Z_{jt}\right) \quad (4)$$

The load requirement L_t should equal the sum of generation provided by all thermal units $\sum_{j\in J}\sum_{s\in S} g_{jst}$ and PSUs $\sum_{x\in X} e_{xt}$ for each time period t. The power balance equation is expressed as (5).

$$\sum_{j\in J}\sum_{s\in S} g_{jst} + \sum_{x\in X} e_{xt} = L_t, t \in T \quad (5)$$

D. Generation and Pump Constraints of PSUs

The power e_{xt} of PSU x in time period t is expressed as (6) and will have a positive value if the unit works as a generator $(v_{xt} = 1, w_{xt} = 0)$, a negative value if the unit works as a pump $(v_{xt} = 0, w_{xt} = 1)$, and zero if the unit is offline $(v_{xt} = 0, w_{xt} = 0)$. The combination of PSU status is expressed as (7). Only one status exists in each time period t.

$$e_{xt} = \sum_{s\in S} g_{xst}v_{xt} + p_{xt}w_{xt}, x \in X, s \in S, t \in T \quad (6)$$

$$v_{xt} + w_{xt} \leq 1, x \in X, t \in T \quad (7)$$

Each PSU x exists in one of three statuses.

Status 0: unit x is offline (i.e., $v_{xt} = 0, w_{xt} = 0$), and both generation and discharge values are zero.

Status 1: unit x is in the generation mode (i.e., $v_{xt} = 1, w_{xt} = 0$), and both generation and discharge values of the upper reservoir are positive.

Status 2: unit x is in the pumping mode (i.e., $v_{xt} = 0, w_{xt} = 1$), and both generation and discharge values of the upper reservoir are negative.

Equation (8) limits the minimum and maximum generation of PSU x in Status 1 (generation mode).

$$\underline{G_{xt}}v_{xt} \leq \sum_{s\in S} g_{xst} \leq \overline{G_{xt}}, x \in X, t \in T \quad (8)$$

Equation (9) expresses the limits of the generation in each segment s of PSU x in the generation mode at head range n in time period t. The upper limit of generation will be affected by the head index h_{nt} and equals to the sum of all $I_{xsn}h_{nt}$. Note that the item I_{xsn} will be accumulated only when h_{nt} is equal to one.

$$g_{xst} \leq \sum_{n\in N} I_{xsn}h_{nt}, x \in X, s \in S, n \in N, t \in T \quad (9)$$

The values of parameter I_{xsn} is I_{sn} of PSU x and can be assumed identical for all PSUs in the same PSP due to the water head and characteristic curves being the same.

Equation (10) expresses the power consumption $\underline{p_{xt}}$ of PSU x in the pumping mode $(w_{xt} = 1)$ in time period t. The measured value $\underline{p_{xn}}$ is the power consumption for head range n of PSU x. As mentioned in the Introduction, the consumed power will be affected by the change of head.

$$p_{xt} = w_{xt}h_{nt}\underline{p_{xn}}, x \in X, n \in N, t \in T \quad (10)$$

E. Water Discharge Constraints of PSUs

In (11), the equivalent hourly water discharge (d_{xt}) from PSU x in period t is equal to 3600 times the discharge in CMS of the difference between generation and pumping modes. Equation (11) will have a positive value if the unit works as a generator, a negative value if the unit works as a pump, and zero if the unit is offline.

$$d_{xt} = 3600\left(\sum_{s\in S,n\in N}\left(\frac{g_{xst}V_{xt}h_{nt}}{F_{sn}}\right) - \frac{p_{xnt}}{K_{xn}}\right), x \in X, n \in N, t \in T \quad (11)$$

Normally, both pumped water and power consumption will be affected by the water head and will decrease with the increase in the head due to the operation of the controller in the PSU.

F. Hydraulic System Constraints

Water head which is equal to the difference of the water elevation between the upper and lower reservoirs can be modeled as a function of water volume of the smaller reservoir [12][14]. Each head range can be assigned to a binary variable as the head index h_{nt} for the head range n. The head index h_{nt} is one when n matches the digitized head range number n of the head range H_n, otherwise h_{nt} is zero. Equations (12)–(14) show the constraints of the head index h_{nt} where $\underline{r_n}$ and $\overline{r_n}$ are minimum and maximum limits of volume difference between the upper and lower reservoirs of PSU x for head range n. They can be obtained from analysis of the hydraulic system.

$$\sum_{n\in N} h_{nt} = 1, t \in T \quad (12)$$

$$\sum_{n\in N} \underline{r_n}h_{nt} \leq \left(\overline{q_t} - \underline{q_t}\right) + \varepsilon, t \in T \quad (13)$$

$$\sum_{n\in N} \overline{r_n}h_{nt} \geq \left(\overline{q_t} - \underline{q_t}\right), t \in T \quad (14)$$

Equations (15) and (16) describe the constraint of the volume limits of the upper and lower reservoirs, respectively.

$$\overline{\beta^-} \leq \overline{q_t} \leq \overline{\beta^+}, t \in T \quad (15)$$

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$$\underline{\beta^-} \le \underline{q_t} \le \underline{\beta^+}, t \in T \quad (16)$$

Equations (17) and (18) are volume calculations of the upper and lower reservoirs contributed by all PSUs and river flows in time period t respectively. In order to focus on the PSU, the in and out river flows of the upper and lower reservoirs are assumed to be constant values in this paper. In real power system operation, the river flow may not be fixed and is affected by weather and other miscellaneous reasons.

$$\overline{q_t} = \overline{q_{t-1}} + \overline{R^+} - \overline{R^-} - \sum_{x \in X} d_{xt}, x \in X, t \in T \quad (17)$$

$$\underline{q_t} = \underline{q_{t-1}} + \underline{R^+} - \underline{R^-} + \sum_{x \in X} d_{xt}, x \in X, t \in T \quad (18)$$

III. Testing Results

A. Parameter Derivation of an Example PSU from Curves

Figure 3 shows the characteristic curves of an example PSU to describe the relationships among discharge, head, and generation. H1, H2, and H3 represent the head range 380 to 390, 390 to 400, and more than 400 meters, respectively.

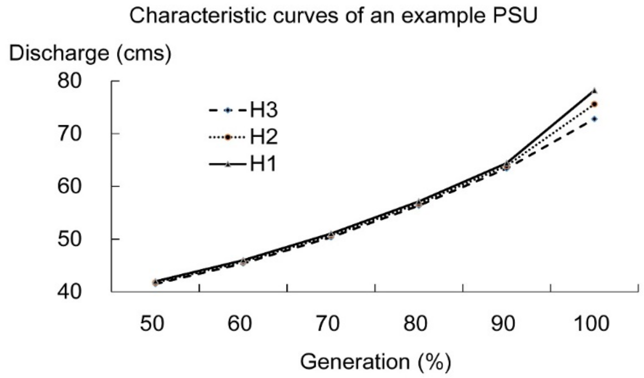

Fig. 3. Characteristic curves of an example PSU

Tables III and IV correspond to table I and II, and can be established from Fig 3 by equation (1).

TABLE III
THE PARAMETER m_{sn} FOR A PSU IN DIFFERENT HEAD RANGE(METER) AND SEGMENT(S)

head range (meter)	Digitized generation segments for PSUs **(s)**					
	1	*2*	*3*	*4*	*5*	*6*
400~	3.37	3.38	3.40	3.42	3.44	3.66
390~400	3.27	3.28	3.30	3.32	3.34	3.46
380~390	3.22	3.23	3.25	3.27	3.29	3.32

TABLE IV
THE PARAMETER q_{sn} FOR A PSU IN DIFFERENT HEAD RANGE(METER) AND SEGMENT(S)

head range (meter)	Digitized generation segments for PSUs (s)					
	1	*2*	*3*	*4*	*5*	*6*
400~	41.50	45.40	50.35	56.35	63.40	72.80
390~400	41.75	45.70	50.70	56.75	63.90	75.60
380~390	42.00	46.00	51.05	57.15	64.40	78.20

Using the data in Tables III and IV, step ratio parameters and step generation parameters can be calculated as Table V and VI by equations (2) and (3).

TABLE V
THE STEP RATIO PARAMETERS FOR A PSU IN DIFFERENT HEAD RANGE(METER) AND SEGMENT(S)

head range (meter)	Digitized generation segments for PSUs (s)					
	1	*2*	*3*	*4*	*5*	*6*
400~	3.37	3.49	3.58	3.59	3.60	5.14
390~400	3.27	3.39	3.48	3.49	3.50	4.12
380~390	3.22	3.34	3.43	3.44	3.45	3.46

TABLE VI
THE STEP GENERATION(MW) FOR A PSU IN DIFFERENT HEAD RANGE(METER) AND SEGMENT(S)

head range (meter)	Digitized generation segments for PSUs (s)					
	1	*2*	*3*	*4*	*5*	*6*
400~	139.86	153.46	171.20	192.73	218.11	266.46
390~400	136.52	149.89	167.30	188.40	213.42	261.57
380~390	135.24	148.58	165.91	186.88	211.88	259.63

Assuming that the head is in the range of H2 at time t, the maximum generation of the PSU will be limited to 261.57MW as described in equation (9) and the total discharge from the PSU is 75.60 CMS as described in equation (11) for the maximum generation. The behavior of the PSU is described from equations (6) to (11) and will be affected by the head index when the head range changes.

B. Example PSU Applied to a Real Hydraulic System and Tested in a Real Isolated Power System

Figure 4 shows a real hydraulic system of a power utility consisting of two PSPs fed by a common upper reservoir with a capacity of 150×10^6 m^3 with 20-meter depth and two lower reservoirs with capacities of 12×10^6 m^3 with 28-meter depth and 8×10^6 m^3 with 20-meter depth. The model is applied to this real hydraulic system in an actual power system for testing. Normally, the upper reservoir will be operated to being nearly full of water in the morning for keeping as much storage energy as possible. The variation of water elevation of the upper reservoir is usually less than ten percent and can be ignored compared to the variation in the lower reservoirs.

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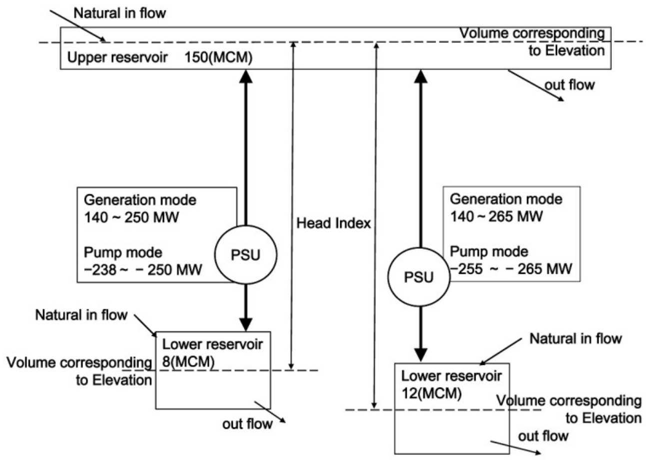

Fig. 4. Real PSP system of a power utility.

Table VII lists the system data including nuclear and thermal units [17][18]. The operating cost of a nuclear or a thermal unit is expressed as a piecewise linear function of the unit output, resulting in a stepwise incremental cost curve whose first step corresponds to the unit minimum output. The no-load cost is not included, but can be incorporated in the first step of the incremental cost curve if necessary.

The water from the hydro units in the system is constrained under the requirements of flood prevention, water supply, irrigation, tourism, town and village safety, and main transport routes. The hourly generation outputs of these hydro units are assumed to be known according to the water requirements.

The scheduled generation listed in the last row in Table VII is contributed by independent power producers (IPPs), and the generation for each hour is known as scheduled by contracts. Therefore, in this study, the scheduled generation contributed by the IPPs will be ignored by adjusting the actual load requirement down by 20%. The generation from renewable power and co-generation is statistically approximately 2% of the actual load requirement and will be modeled as a miscellaneous item. The actual load requirement for that hour will be adjusted by 2% lower for the miscellaneous item.

TABLE VII
SYSTEM DATA

Fuel type	Number of units	Installed capacity (MW)
Nuclear	6	5,144
Coal	192	21,561
LNG		
Oil		
PSU	10	2,602
Hydro (reservoir + R.O.R)	72	1,617
Others (renewable+co-gen)	-	1,516
Scheduled (IPPs)	34	7,707

Under normal operation in this isolated power system, not all PSUs will be scheduled in the power utility because the remaining PSUs are part of the reserve contributors. Six PSUs participated in the seven-day generation scheduling test. The other four PSUs are operated as in reserve in case of contingencies in real-time operation.

Figure 5 shows the seven-day generation scheduling results of the PSU model applied to the actual power system and a typical seven-day load demand from Saturday to Friday. The lowest load demand occurs at the 79th hour of the seven days. The highest load demand occurs at the 111th hour of the seven days. The PSU is in the pumping mode in the valley area and in the generation mode in the peak area of the load requirement, as expected. The water discharge from the PSU of the 12-MCM lower reservoir in the generation mode are 1068, 1055, and 1054 (CMS/MW-H) corresponding to head indices one, two, and three, respectively. The water discharge from the same PSU in the pumping mode is 878 (CMS/MW-H). The pumping efficiency rates of the PSU are 82.2%, 83.2%, and 83.3%, respectively, corresponding to each head range. That makes calculation of water value possible by applying the PSU model.

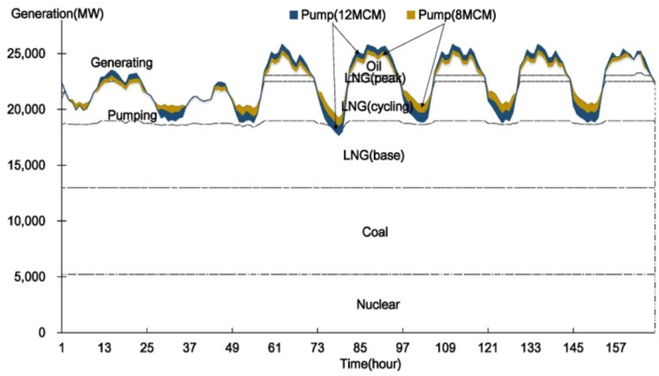

Fig.5 Seven-day Generation scheduling results of proposed PSU model applied to the real hydraulic system and tested in an actual power system.

Another advantage of applying the model is the operating cost of the PSUs will become very easy to estimate by comparing the total cost of the two cases by separately committing and dis-committing the PSUs. Table VIII lists the total fuel cost and savings in fuel cost of different PSUs' commitment combinations and head effects consideration. All solutions are implemented in GAMS with solver Gurobi, and the tests are executed on an Intel Core i7 at 2.3 GHz laptop with 8 GB RAM. The stopping criterion is an optimality gap of less than or equal to 0.5%.

There are eight case groups to consider different head effects in table VIII, and each case group consists of six cases for various PSU commitment combinations. The number of variables, number of integer, number of constraints and execution time corresponding to each case are included. The first case group which is the base case group is with conditions of $\bar{s} = 5$ and head ranges from H_1 to H_3. The other case groups were listed for comparisons with the base case group. The second case group is with conditions of $\bar{s} = 5$ and only considers the PSU characteristic in head range H_1. The first case in each case group is the base case, which operates without PSUs, and incurs the largest cost in that group. Each second case in the group shows the operating condition of the first case with one PSU being added to the 12-MCM lower reservoir and one PSU being added to the 8-MCM lower

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reservoir. The result shows that 21,507,421 NTD. (0.38%) is saved by the operation of the two PSUs in the seven-day generation schedule in case one. The largest saving amount in the base case is 53,735,408 NTD (0.95%), which occurs in case five where ten PSUs operate in the system. However, the contribution in terms of saving efficiency by each PSU decreases with the increase in the number of PSUs committed. The MILP performance is influenced by data in some cases [9], such as cases 3, 17, 20, 27, and 29. In general, the convergence is fast in cases where the unit status change is less.

The cost of providing extra service from the PSUs, such as spinning reserve or other ancillary services, can be estimated by the method to incorporate corresponding constraints in the model. Applying the PSU model in the generation scheduling can help in operation plan decision for independent system operators, utilities, and companies that own PSUs in a deregulated electricity market [14][19].

TABLE VIII
FUEL COST COMPARISON UNDER DIFFERENT OPERATIONS OF PSUS

Digitized generation segments for PSUs	Simulated characteristic in the head range for PSUs	Case Item	Unit number of 12-MCM reservoir	Unit number of 8-MCM reservoir	Variable number	Constraint number	Integer number	Execution time (sec.)	Fuel cost of seven-day generation scheduling [NTD]	Saving [NTD]	Saving [%]
$\bar{s}$ = 6	H_1 to H_3	0	0	0	44,848	167,710	24,173	3.04	5,660,132,641	-	-
		1	1	1	53,243	167,710	28,338	339.51	5,638,625,220	21,507,421	0.38
		2	2	2	61,361	167,710	31,065	1302.41	5,624,629,294	35,503,347	0.63
		3	3	3	70,440	167,710	33,761	1002.35	5,614,571,469	45,561,172	0.81
		4	4	4	78,803	167,710	36,442	12493.87	5,608,539,560	51,593,081	0.91
		5	6	4	87,268	167,710	39,152	39304.84	5,606,397,233	53,735,408	0.95
$\bar{s}$ = 6	H_1	6	0	0	44,184	119,662	24,173	2.82	5,660,132,641	-	-
		7	1	1	46,515	119,662	25,417	7.98	5,638,628,812	21,503,829	0.38
		8	2	2	49,028	119,662	26,081	4.83	5,623,991,186	36,141,455	0.64
		9	3	3	50,563	119,662	26,753	7.79	5,614,763,277	45,369,364	0.80
		10	4	4	52,757	119,662	27,425	5.61	5,606,322,040	53,810,601	0.95
		11	6	4	54,774	119,662	28,097	6.21	5,602,916,727	57,215,914	1.01
$\bar{s}$ = 6	H_2	12	0	0	44,184	119,662	24,173	3.04	5,660,132,641	-	-
		13	1	1	46,515	119,662	25,417	4.90	5,638,404,332	21,728,309	0.38
		14	2	2	49,027	119,662	26,081	4.53	5,624,233,760	35,898,881	0.63
		15	3	3	50,563	119,662	26,753	5.89	5,613,556,957	45,575,684	0.82
		16	4	4	52,757	119,662	27,425	5.92	5,606,375,514	53,757,127	0.95
		17	6	4	54,774	119,662	28,097	62.89	5,606,003,138	54,129,503	0.96
$\bar{s}$ = 6	H_3	18	0	0	44,184	119,662	24,173	2.72	5,660,132,641	-	-
		19	1	1	46,499	119,662	25,417	18.11	5,639,027,007	21,105,634	0.38
		20	2	2	49,027	119,662	26,081	4.55	5,624,520,588	35,612,053	0.63
		21	3	3	50,563	119,662	26,753	12.59	5,615,210,789	44,921,852	0.79
		22	4	4	52,757	119,662	27,425	5.68	5,606,478,190	53,654,451	0.95
		23	6	4	54,774	119,662	28,097	6.08	5,602,949,649	57,182,992	1.01
$\bar{s}$ = 3	H_1 to H_3	24	0	0	44,184	149,230	24,173	2.97	5,660,132,641	-	-
		25	1	1	49,289	149,230	28,337	92.47	5,640,647,516	19,485,125	0.34
		26	2	2	53,357	149,230	31,065	872.05	5,629,656,823	30,475,818	0.54
		27	3	3	58,395	149,230	33,761	757.36	5,622,376,872	37,755,769	0.67
		28	4	4	62,747	149,230	36,441	13116.20	5,618,603,386	41,529,255	0.73
		29	6	4	67,138	149,230	39,139	4549.14	5,616,825,999	43,306,642	0.77
$\bar{s}$ = 3	H_1	30	0	0	44,184	111,262	24,173	2.75	5,660,132,641	-	-
		31	1	1	45,640	111,262	25,409	4.07	5,639,961,827	20,170,814	0.36
		32	2	2	47,012	111,262	26,081	4.44	5,628,052,231	32,080,410	0.57
		33	3	3	47,539	111,262	26,753	6.44	5,622,189,394	37,943,247	0.67
		34	4	4	48,725	111,262	27,425	5.35	5,615,840,215	44,292,426	0.78
		35	6	4	49,734	111,262	28,097	7.55	5,616,237,309	43,895,332	0.78
$\bar{s}$ = 3	H_2	36	0	0	44,184	111,262	24,173	2.81	5,660,132,641	-	-
		37	1	1	45,639	111,262	25,409	4.21	5,640,170,479	19,962,162	0.35
		38	2	2	47,011	111,262	26,081	4.20	5,628,338,767	31,793,874	0.56
		39	3	3	47,539	111,262	26,753	5.21	5,620,307,094	39,825,547	0.70
		40	4	4	48,725	111,262	27,425	7.07	5,618,016,267	42,116,374	0.74
		41	6	4	49,734	111,262	28,097	7.73	5,616,182,074	43,950,567	0.78
$\bar{s}$ = 3	H_3	42	0	0	44,184	111,262	24,173	2.83	5,660,132,641	-	-
		43	1	1	45,640	111,262	25,409	3.98	5,640,393,156	19,739,485	0.35
		44	2	2	47,011	111,262	26,081	4.10	5,628,649,736	31,482,905	0.56
		45	3	3	47,539	111,262	26,753	5.29	5,620,580,755	39,551,886	0.70
		46	4	4	48,725	111,262	27,425	6.66	5,618,111,846	42,020,795	0.74
		47	6	4	49,734	111,262	28,097	6.99	5,616,175,382	43,957,259	0.78

C. Results of Applying Head Dependence PSU Model

Figure 6 shows the results of water elevation, head, and head index of the 12-MCM lower reservoir for the seven-day

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generation schedule. Elevation of the upper reservoir is from 728 to 748 meter. Elevation of the lower reservoir is from 345 to 373 meter. Water head is divided into three ranges. Head index equals to one represents the lowest head range, and three means the highest head range.

At hour 25 which is midnight in this case, point A(746.6 meter) and B(360 meter) represent the water elevation of the upper and the lower reservoir respectively. Water head at point C(386.6 meter) is equal to 746.6(point A) subtracted by 360(point B). Point D represents the head index and is equal to one which represents the PSU is in the lowest head range.

At hour 55, which is in the morning in this case, water is pumped to the upper reservoir. Water elevation rises to 748 meter (A1) for the upper reservoir, and reduces to 345 meter (B1) for the lower reservoir. Water head is at point C1 (403 meter) and is equal to 748(point A1) subtracted by 345(point B1). Point D1 shows that the head index is equal to three which means the PSU is in the highest head range.

In this seven-day case, the profile of load demand is different in each day. Although reservoirs of PSPs are required to be refilled daily, different operation strategy of the PSPs for each day is allowed. Points B1 (345 meter), B2 (345 meter), B3 (346.8 meter), B4 (347.7 meter), and B5 (345.6 meter) represent different lowest water elevation of the lower reservoir from Monday to Friday for economy consideration. Two situations are shown in the result and are very close to the seven-day cycle operation plan in this real power system with head effects consideration. First, the result of the points B1 to B4 is getting higher each day during weekdays which represents the stored energy in the upper reservoir is transferred to the power system in weekdays and is pumped back in the weekend to save operation cost. Second, higher water head has higher generation efficiency. As far as how deep the lower reservoir should be pumped will be decided by the load demand and generation in the next few days. The endpoint of the target volume of the lower reservoir is set to be equal to the initial volume for weekly operation of the PSUs.

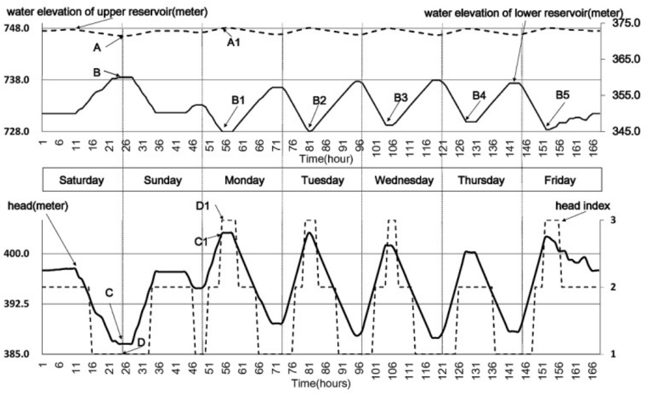

Fig.6. Results of the water elevation, head, and head index of the12-MCM lower reservoir for the seven-day generation scheduling

D.Results of Not Applying Head Dependence PSU Model

The PSU model without head dependence was applied in the same cases in subsection D for comparison. Figure 7 shows the results of water elevation, head, and head index of the 12-MCM lower reservoir for the seven-day generation schedule. Elevation of the upper reservoir is from 728 to 748 meters. Elevation of the lower reservoir is from 345 to 373 meters. Only one PSU characteristic of head range is applied in this case and the middle value, head range two, is selected.

At hour 25, which is midnight in this case, point A(746.6 meter) and B(360.2 meter) represent the water elevation of the upper and the lower reservoir respectively. Water head at point C(386.4 meter) is equal to 746.6(point A) subtracted by 360.2(point B). Point D represents the head index and is equal to two which represents only head range 2 was applied in this case.

At hour 55, which is in the morning in this case, water is pumped to the upper reservoir. Water elevation rises to 747.9 meter (A1) for the upper reservoir, and reduces to 348.2 meter (B1) for the lower reservoir. Water head is at point C1 (399.7 meter) and is equal to 747.9(point A1) subtracted by 348.2(point B1). Point D1 shows that the head index is fixed, and no head effect was applied in this case.

The lowest water elevations of the lower reservoir from Monday to Friday are B1 (348.2 meter), B2 (347.6 meter), B3 (347.7 meter), B4 (347.4 meter), and B5 (346.5 meter), respectively, and are kept around 347 meter. This is a different result, and the second consideration in subsection C is not shown in this case. This result shows that considering the head effects on both generation and pumping modes of PSUs can improve the accuracy.

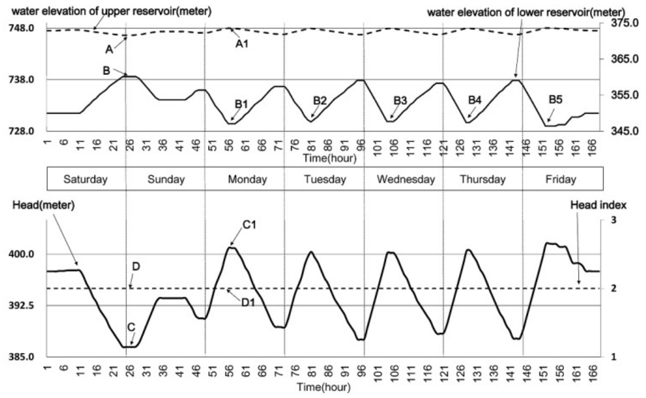

Fig.7. No-head-effect results of the water elevation, head, and head index of the12-MCM lower reservoir for the seven-day generation scheduling

IV. Conclusions

An MILP approach of a head-dependent PSU model is established in this paper and successfully applied to a real hydraulic system and tested in an actual power system for a seven-day generation scheduling. Test results showing an hourly-based generation schedule, head effects on both of the generation and pump modes of the PSU, and the operating cost savings for different number of PSUs committed in the power system are presented. The important contributions of the model are making PSU operation plans to provide the operation guidelines, enabling PSU operation cost estimation, calculating marginal water values, and determination of the target volume/elevation of the reservoirs at the end of the day/week. The flexible MILP based PSU model can be extended to scheduling applications to consider ancillary service in a deregulated environment by slightly modification on the constraints functions and the objective equation [19].

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