Dynamic Programming Based Algorithm for the Unit Commitment of the Transmission-Constrained Multi-Site Combined Heat and Power System

A. Rong, P. B. Luh, R. Lahdelma

Abstract—High penetration of intermittent renewable energy sources (RES) such as solar power and wind power into the energy system has caused temporal and spatial imbalance between electric power supply and demand for some countries and regions. This brings about the critical need for coordinating power production and power exchange for different regions. As compared with the power-only systems, the combined heat and power (CHP) systems can provide additional flexibility of utilizing RES by exploiting the interdependence of power and heat production in the CHP plant. In the CHP system, power production can be influenced by adjusting heat production level and electric power can be used to satisfy heat demand by electric boiler or heat pump in conjunction with heat storage, which is much cheaper than electric storage. This paper addresses multi-site CHP systems without considering RES, which lay foundation for handling penetration of RES. The problem under study is the unit commitment (UC) of the transmission-constrained multi-site CHP systems. We solve the problem by combining linear relaxation of ON/OFF states and sequential dynamic programming (DP) techniques, where relaxed states are used to reduce the dimension of the UC problem and DP for improving the solution quality. Numerical results for daily scheduling with realistic models and data show that DP-based algorithm is from a few to a few hundred times faster than CPLEX (standard commercial optimization software) with good solution accuracy (less than 1% relative gap from the optimal solution on the average).

Keywords—Dynamic programming, multi-site combined heat and power system, relaxed states, transmission-constrained generation unit commitment.

I. INTRODUCTION

INCREASING concerns for environmental impacts of energy production and use as well as fossil fuel depletion promote the improvement of energy efficiency in energy production and use as well as utilization RES such as wind, solar, geothermal and biofuel. CHP production is an important energy efficient technology. In CHP, useful heat and electric power are produced simultaneously in a single integrated process, i.e., power and heat production are interdependent. The energy efficiency of the traditional condensing power plant is less than 40% while the CHP plant can offer higher

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efficiency (over 90%) because it can utilize otherwise wasted heat in the process. For a fossil fuel based CHP plant, improved energy efficiency means the reduction of both fuel consumption and CO_2 emissions. For a renewable biofuel based CHP plant, improved energy efficiency can use scare biofuel more efficiently.

Both CHP and RES are pillars for the European Union (EU)'s energy policy for future low carbon energy systems [1]. Currently, in some European countries such as Denmark, Finland and the Netherlands, CHP has already accounted for 30-50% of national power production though penetration of CHP technology is not as extensive as it would be expected on the world scale (accounting for 10% global power generation) [2]. It is expected CHP systems will be prevalent globally in the future [2] due to various support mechanisms for deployment of CHP adopted by different countries [3].

In this paper, we deal with the transmission-constrained multi-site CHP system as shown in Fig. 1. At each site (Fig. 1 (a)), a power node can be treated as a regional power system and a heat node as a regional heat distribution system. The power distribution system operates separately from the heat distribution system and coupling of power and heat only occurs in CHP production process. It means that the power distribution for the CHP system is the same as that for the power-only system. The power-only system can be treated as a special case of CHP system without considering heat generation and heat demand. This means that multi-site CHP systems can be treated as an extension of multi-site power-only systems, see e.g., [4] and [5].

Nowadays, interconnected regional energy systems [6] are common because interconnection can reduce overall cost and improve the overall efficiency of the system. Thus, multi-site CHP systems can be interpreted in a broader context. A regional energy system can be any local energy system. In this paper, we focus on connections at a relatively high level, e.g. municipal level or national level, where the number of transmission connections between regions is moderate.

Large-scale integration of intermittent RES such as solar and wind power requires coordination of power scheduling and power transmission between regions to achieve high utilization of RES [6]. As compared with power-only systems, CHP systems can provide additional flexibility for achieving power grid balance. First, power generation in the CHP plant can be influenced by adjusting heat generation level [7]. Secondly, electric power can be utilized to satisfy heat demand via heat pump (a facility for providing heat efficiently

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using electricity) in conjunction with heat storage to avoid starting up CHP plants or reducing power generation from CHP plants [8] when there is large RES production. One

should notice that heat storage is much cheaper than electric storage [9].
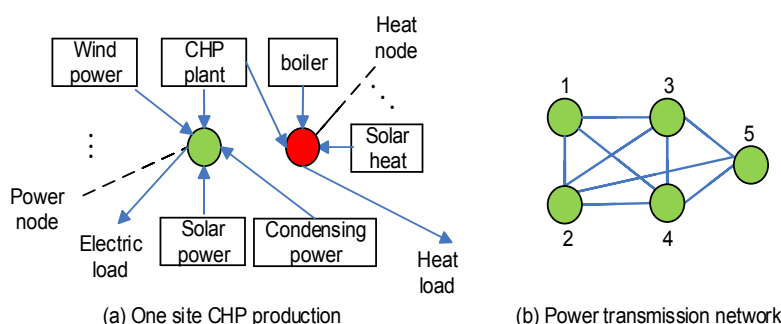

Fig. 1 A transmission constrained multi-site CHP system

In this paper, we consider UC of the transmission-constrained multi-site CHP system without storage and RES. We attempt to coordinate power and heat generation at different sites as well power transmission between different sites simultaneously. This will be the underlying problem for handling uncertainty of RES.

To authors' knowledge, there is no research addressing transmission-constrained multi-site CHP systems. There are at least three reasons for this. First, current deployment of CHP technology is not as wide as expected as just mentioned, and thus CHP is often ignored when dealing with large-scale systems. Second, CHP systems are more complicated than power-only systems due to the interdependence of power and heat generation in the CHP plant. For this reason, the marginal power production cost of a CHP plant depends on heat generation. As a result, it is difficult to rank CHP plants because there is no single measure that can determine the relative efficiency of CHP plants in all cases. It means that so called merit order for power-only systems does not hold for CHP systems. Finally, when power transmission is involved, the coordination between heat and power generation as well as power transmission becomes more challenges.

Since there is no research addressing the UC of multi-site CHP systems, in the following, we mainly review some general-purpose solution approaches that can be applied to solve the current problem. We roughly group solution methods into three categories. The first one applies decomposition based techniques such as Lagrangian relaxation (LR) [6], [10] and DP [11]. The second one applies different heuristics including priority listing [4], sequential method [12], system of system engineering approach [13], and meta-heuristics [14]. The last one is mixed integer linear programming (MILP) approach [15]-[17]. For the UC of single-site CHP systems, LR [18], DP [19]-[22] and different heuristics [23], [24] have been applied.

In this paper, we attempt to extend our previous DP approach [20], [22] for dealing with single-site CHP systems into the multi-site scheduling context, called MDP-RSC (multi-site DP using relaxed state and sequential commitment scheme). This means that we applied time-oriented

decomposition approach to circumvent the coordination challenges for the multi-site CHP system, different from the common approach for dealing with the multi-site power-only system where the site-oriented decomposition was used [6], [10].

The DP scheme is based on relaxed ON/OFF states of plants and sequential commitment of subsets of plants based on pre-determined order of the plants to reduce the dimension of problem. Each time, only the ON/OFF states for fewer plants were considered simultaneously and the plants whose states have not been determined were set to the relaxed ON/OFF states. When the plant is at relaxed state, the ON/OFF state variable can be temporarily excluded from the set of decision variables.

Using DP-based approach, the hourly multi-site CHP system was treated as an entity and the solutions for the hourly model were coordinated from hour to hour based on recursive equations for the Bellman DP principle [25] by using efficient algorithm [26] for solving the underlying economic dispatch (ED) sub-problems. This helps to improve the solution accuracy.

The paper is organized as follows. In Section II, we describe the individual plant model and the corresponding relaxed state as well as power network model. In Section III, we formulate the UC of the transmission-constrained multi-site CHP system. In Section IV, we describe the DP-based solution approach. In Section V, we report numerical results based on realistic data.

II. CHP PLANT MODEL, RELAXED STATES AND POWER NETWORK MODEL

A. CHP Plant Model and Relaxed States

Here we adopt the same extreme point formulation for modeling CHP plant and the corresponding relaxed state as described in [22]. The plant characteristic can be either convex or non-convex. For simplicity, we assume that plant characteristic is convex. The convex plant characteristic can be represented as an LP model [27], [28] and the non-convex characteristic as an MILP model [29], [30]. The convexity assumption means that we can solve underlying ED more

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efficiently using both general LP solvers and a specialized efficient LP solver [26]. However, the same DP principle would work also for non-convex plants together with an MILP solver.

It is worth mentioning that convexity assumption is not as limiting as it may seem. Some energy companies operate CHP plants according to a fixed power to heat ratio. This is a special case of convex characteristics. The operating regions of many simple backpressure plants are indeed convex. It is also possible to operate a non-convex plant for fractions of hour in different modes. This will result in the convex hourly average operating region.

Fig. 2 illustrates an operating region of the convex plant. When the plant is at ON-state, the operating region is defined by the extreme points (c_j, p_j, q_j) $(j=1,\ldots,5)$, where c_j is mainly determined by fuel cost for generating p_j and q_j. When the plant is at OFF-state, it does not consume fuel because it does not generate power and heat. It is equivalent to operating at point $(0, 0, 0)$.
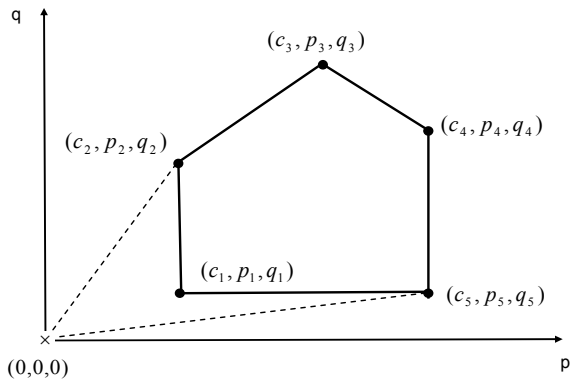

Fig. 2 The characteristic operating region of a convex CHP plant in the power-heat plane. Extreme points (c_j, p_j, q_j) $(j=1,\ldots,5)$ define the operating region in the ON-state and $(0,0,0)$ corresponds to the OFF-state. p=power, q=heat, c=cost (from [22])

To accommodate the UC, we introduce an artificial relaxed ON/OFF state for the plant. In the relaxed ON/OFF state, the ON/OFF state of the plant can be temporarily excluded for consideration. In the relaxed state, the plant is allowed to operate in the continuum between the ON- and OFF-states, as illustrated by the dash lines shown in Fig. 2. If the ON-state operating region includes $(0, 0, 0)$, then the relaxed operating region coincides with the ON-region.

Due to convexity, the operating costs $C_{u,t} = C_{u,t}(P_{u,t}, Q_{u,t})$ of plant u as well as heat and power production of the plant can be represented as a convex combination [31] of extreme points $(c_{j,t}, p_{j,t}, q_{j,t})$:

$$
\begin{aligned}
C_{u,t} &= \sum_{j \in J_u} c_{j,t} x_{j,t} \\
P_{u,t} &= \sum_{j \in J_u} p_{j,t} x_{j,t} \\
Q_{u,t} &= \sum_{j \in J_u} q_{j,t} x_{j,t} \\
\sum_{j \in J_u} x_{j,t} &= 1 \\
x_{j,t} &\geq 0, \quad j \hat{I} J_u,
\end{aligned}
\tag{1}
$$

For the extreme point formulation (1), usually we assume that the plant has the same number of points $|J_u|$ in each hour, but the shape of characteristic may change. If a plant operates at fewer points in some hours, extra points can be disabled by fixing the corresponding $x_{j,t} = 0$.

If the OFF-state point j_uOFF is included in J_u, then (1) corresponds to the relaxed ON/OFF state of the plants. The plant can be forced to the ON-state when x-variable corresponding to point j_uOFF is set to zero and the OFF-state when set to one (it implies x-variables for all other points are forced to zero based on convex combination, the last formula in (1)). In addition, the heat- and power-only plants can be modelled as a special case of CHP plants with zero p- and zero q-component, respectively.

B. Power Network Model

In literature, there are two approaches for modeling the transmission-constrained power network in the context of multi-site power scheduling. One is based on the DC model [10], [12] and the other on the network flow model [32], as used in [5]. We adopt the latter approach. Considering the geography dispersion for different sites and high level interconnections, only the sites close to each other have a direct transmission line (*arc*) connection but the subsystems for all individual sites can form an interconnected system. In the model, only power nodes in Fig. 1 (a) are explicitly considered and heat nodes are simply represented as the heat balance of the physical site. The power transmission network in the current study is bidirectional, i.e., if there is an arc from i to j, there is also an arc from j to i.

III. TRANSMISSION CONSTRAINED UC FORMULATION

The CHP system may include CHP plants, power-only and heat-only plants. The UC of the transmission-constrained problem is modelled by integrating plant model and power network model as well as relaxed states described in Section II. The model is given below:

$$
\min \sum_{t=1}^{T} \sum_{u \in U} \left(\sum_{j \in J_u} c_{j,t} x_{j,t} + s_u(w_{u,t-1}, y_{u,t}) \right) + \sum_{t=1}^{T} \sum_{(i,k) \in A} c_{i,k,t} z_{i,k,t} +
$$

$$
\sum_{t=1}^{T} \sum_{i \in \Phi} (c_{i,q+,t} x_{i,q+,t} + c_{i,q-,t} x_{i,q-,t} + c_{i,p+,t} x_{i,p+,t} + c_{i,p-,t} x_{i,p-,t})
\tag{2}
$$

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s.t.

$$\sum_{j \in J_u} x_{j,t} = 1, \qquad u \in U, t=1,...,T \tag{3}$$

$$\sum_{j \in J_i} q_j x_{j,t} - x_{i,q+,t} + x_{i,q-,t} = Q_{i,t}, \; i \in \Phi, \; t=1,...,T \tag{4}$$

$$\sum_{j \in J_i} p_j x_{j,t} + \sum_{\{k|(k,i)\in A\}} z_{k,i,t} -$$
$$\sum_{\{k|(i,k)\in A\}} z_{i,k,t} - x_{i,p+,t} + x_{i,p-,t} = P_{i,t}, \quad \substack{i\in \Phi, t=1,...,T,} \tag{5}$$

$$x_{j,t} \geq 0, j \in J, \; t=1,...,T \tag{6}$$

$$x_{i,q\pm,t}, \, x_{i,p\pm,t} \geq 0, i \in \Phi, \; t=1,...,T \tag{7}$$

$$0 \leq z_{i,k,t} \leq g_{i,k}, (i, \, k) \in A, \quad t=1,...,T \tag{8}$$

$$x_{j,t} \leq y_{u,t}, \; u \in U, j \in J_u \setminus \{ j_u^{OFF} \}, t=1,...,T \tag{9}$$

$$x_{j_u^{OFF},t} \leq 1 - y_{u,t}, u \in U, t=1,...,T \tag{10}$$

$$y_{u,t} = \begin{cases} 1, & \text{if } 1 \leq w_{u,t-1} < UT_u \\ 0, & \text{if } -1 \geq w_{u,t-1} > -DT_u \; , \\ \{0,1\}, & \text{otherwise} \end{cases} \; u \in U, t=1,...,T \tag{11}$$

$$w_{u,t} = \begin{cases} \max(0, w_{u,t-1}) + 1, & \text{if } y_{u,t} = 1 \\ \min(w_{u,t-1}, 0) - 1, & \text{if } y_{u,t} = 0 \end{cases} \; u \in U, t=1,...,T \tag{12}$$

$$y_{u,t} = 0, u \in U, \, t \in Y_{u,\text{OFF}} \tag{13}$$

$$y_{u,t} = 1, u \in U, \, t \in Y_{u,\text{ON}} \tag{14}$$

$$w_{u,t} \text{ integer}, u \in U, \, t=1,...,T \tag{15}$$

$$y_{u,t} \in \{0,1\}, u \in U, \, t=1,...,T \tag{16}$$

Objective (2) is to minimize total costs. Total costs include production cost, startup and shut-down cost and transmission cost as well as possible penalty for heat surplus/slack and power surplus/slack. Constraints (3)-(8) are the constraints for the ED problem no matter the plants are at ON-states, OFF-states and relaxed ON/OFF states because the active points are implicitly reflected in the plant characteristics as described in Section II A. It means that this formulation facilitates handling the ED with different ON/OFF states when the DP-based algorithm is applied for solving the UC problem because the ED has uniform formulation. Constraints (3) and (6) form the convex combination. Constraints (4) state that heat demand in each site is satisfied by local production. Constraints (5) state that power demand in each site is satisfied by local production

plus power transmission between sites. Constraints (8) enforce the capacity for transmission lines. Constraints (9)-(16) are the constraints related to the UC decision. Constraints (9) and (10) establish the link between ED and UC. Constraints (11) and (12) enforce constraints for minimum up and down periods. Constraints (13) and (14) enforce must-on and must-off requirements for plants respectively. Must-off requirements are related to maintenance and must-on are related to technical constraints or generation requirements. When plants are at relaxed states, constraints (16) are not enforced. Accordingly, constraints (9)-(12) are also not enforced.

IV. SOLUTION APPROACHES

DP is a general optimization method that decomposes a complex problem into multiple simpler sub-problems. It interprets an optimization problem into a multi-stage decision process. Each stage consists of many states. The state is a way to describe the solution of the sub-problem, which contains sufficient information to determine the state of the future according to previously determined states. The states are generated based on the recursive equations. There are different ways to represent the states.

In the UC context, each time period t is a stage and the number of stages is T. There are two ways to represent the state. One is based on the combination of $w_{u,t}$ (general integer variables) [20], [22] and the other is based on the combination of $y_{u,t}$ (binary variables) [33]. Here, we adopt the latter one. The advantage of the latter representation is reduced number of states. Let (t,i) denote the state in stage t and $(0,k_0)$ is the initial state. The recursive equations are given below:

$$R(t,i) = \min\{R(t-1,k) + C_{ED}(t,i) + SC(t-1,k:t,i), k \in S_{t-1}\}, \; i \in S_t,$$
$$t = 1,...,T, |S_t| \leq 2^{|G|}, 1 \leq |G| \leq |U| \tag{17}$$

$$R(0,k_0) = 0 \tag{18}$$

$C_{ED}(t,i)$ can be obtained based on the algorithm in [26]. To implement sequential commitment scheme, it requires a measure to determine the relative efficiency of plants [12]. According to [20]-[22], the solution with less heat surplus and power surplus can provide a relatively good measure to rank the plants. LR was used in [21] and relaxed state was used in [20], [22] to reduce heat and power surplus. Here, we adopt the latter one. The corresponding DP scheme is called MDP-RSC (multi-site DP using relaxed state and sequential commitment scheme).

The ranking measures are derived from the solution of the relaxed-state problem where all plants are set to relaxed states. According to the solution of the relaxed-state problem, the plants can be divided into two groups: group one includes plants generating energy over the planning horizon and group two includes plants generating no energy over the planning horizon. Group two operates less efficiently than group one. The plants in group one are placed first followed by the plants in group two. For group one, we use a non-increasing order of measure $M_{R,u,1}$ to rank the plants,

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$$M_{R,u,1} = \frac{\sum_{t=1}^{T}\sum_{i\in\Phi}\sum_{u\in U_i} C_{u,t}}{\sum_{t=1}^{T}\sum_{i\in\Phi}\sum_{u\in U_i}(\lambda_{p,i,t}P_{u,t} + \lambda_{q,i,t}Q_{u,t})} \qquad (19)$$

$M_{R,u,1}$ is a dimensionless measure, where the numerator is the true cost of all plants and the denominator the value of produced energy based on the marginal heat and power cost associated with heat and power balance.

For group two, we first rank the plants based on a non-decreasing order of the cold startup costs. Only the plants with the lowest startup costs were selected and the remaining plants were set at OFF-states. Then, we rank the selected plants according to a non-increasing order of measure

$$M_{R,u,2}, M_{R,u,2} = C_{max,u} / E_{max,u} \qquad (20)$$

$M_{R,u,2}$ can be viewed as unit energy production cost of plant u.

Feasible solutions exist for the UC when all plants generating energy were included according to the solution of the relaxed-state problem because relaxed states maintain the generation capacity of plants. The reason why the subset of plants generating no energy is selected is to increase the scheduling flexibility and accommodate peak demand. Numerical experiments showed that the frequency of rescheduling (switching on the previously shut-down plants discussed in the next paragraph) will increase in the sequential DP process without including the subset of plants generating no energy.

Based on the ranking using measures (19) and (20) as just mentioned, plants are sequentially committed according to predetermined |G|, each time only the ON/OFF states of |G| plants need to be determined (the number of plants in the last subset may be less than |G| when |G| >1). The plants whose ON/OFF states have not determined are temporarily set to relaxed states. When ON/OFF states for the latter subset of plants are determined, it is possible that some previously shut-down plants in some sites need to be switched on due to the requirements for either heat or power or both). Heat demand is checked first and then the power demand. When the heat demand in some sites is not satisfied, the previously shut-down plants with heat generation capacity in the corresponding sites need to be switched on. When the power demand in some sites is not satisfied, the previously shut-down plants with power generation capacity in the corresponding sites or in the sites that can transfer power to the corresponding sites need to be switched on. The priority is given to the plants in the sites where the power demand is not satisfied.

V. COMPUTATIONAL EXPERIMENTS

To evaluate the effectiveness of the MDP-RSC algorithm we implemented the algorithm in C++ in Microsoft Visual Studio 2013 environment. We used ILOG CPLEX 12.5 [34] for solving the MILP formulation for the model (2)-(16),

where constraints (11) and (12) governing the minimum up and down periods, are transformed into linear constraints according to [15], as benchmark. All test runs were carried out on a 2.67 GHz Core CPU with 4 GB RAM under the Windows 7 Operating system.

A. Test Problem

We generated a 5-site realistic test problem based on the data from some Finnish Energy companies. Each site has its own heat and power generation facilities as well as heat and power demand. The number of plants varied from 13 to 18. CHP models were a slightly perturbed version of real plant models. Table I shows the system configuration of five sites in terms of generation facilities, capacity, power and heat demand as well as the minimum up, down time periods (including cold startup time periods) for plants.

TABLE I
GENERATION FACILITIES, CAPACITY, POWER HEAT DEMAND FOR A 5-SITE CHP SYSTEM

Site	$\|U_i\|$	$\|U_{p,i}\|$	$\|U_{q,i}\|$	P_i	Q_i	$P_{G,i}$	$Q_{G,i}$	UT_u	DT_u	CT_u
1	13	0	2	815	967	940	1997	[1,5]	[1,5]	[1,10]
2	13	1	1	395	339	940	2313	[1,5]	[1,5]	[1,10]
3	13	3	1	1065	1305	719	1310	[1,5]	[1,5]	[1,10]
4	18	5	0	450	434	978	1831	[1,5]	[1,5]	[1,10]
5	16	3	1	500	600	900	2113	[1,3]	[1,4]	[1,7]

$|U_i|$: number of all plants in site i; $|U_{p,i}|$ ($|U_{q,i}|$) number of power- (heat-) only plants in site i; $P_{G,i}$ ($Q_{G,i}$): power(heat) generation capacity in MW in site i; UT_u, DT_u, CT_u columns show range of UT_u, DT_u, CT_u; Pi (Qi) maximum power (heat) demand in MW in site i.

Heat and power demand are based on the history data. Demand data were on hourly basis for a full year (8760-hour). The hourly power demand varied from 75 MW to 1065 MW and the hourly heat demand from 130 MW to 1305 MW. The ratio of heat demand and power demand ranged from 0.25 to 3.25 on the hourly basis and from 0.99 to 1.1 on the yearly basis. Based on Table I, the regional energy companies can operate independently based own-facilities. To form an interconnected system, we assume that two sites i and j have direct transmission line connections when $1\le |i\text{-}j| \le 3$ as shown in Fig. 1 (b). For the transmission cost, we adopt the practice of Nordic power markets [35], i.e., the transmission cost is imposed only when the power flow over the transmission line close to the capacity of the transmission line. Transmission cost and capacity were generated according to uniform distribution. The range of transmission capacity is [100,300] MW, and the range of the transmission cost is [5, 30] €/MW when the transmission cost is imposed.

B. Computational Results

To capture demand patterns for the full-year, we solved 13 daily scheduling instances spanning evenly over the year. The

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starting period (sampling time) for daily scheduling was chosen as 0, 672, 1344, 2016, 2688, 3360, 4032, 6704, 5376, 6048, 6720, 7392, and 8064. The instances were solved by both MDP-RSC and CPLEX. For CPLEX. Because it is difficult for CPLEX to obtain solutions for some instances even for gap 0.5%, we set relative gap at 1% and recorded corresponding CPU time (solution time). For MDP-RSC, we chose 1, 2, 3, 4, 5 and 6 as $|G|$ values. Based on the numerical experiments, the quality of the solution has tendency to improve as $|G|$ increases though the improvement is not monotonically with respect to the increase of $|G|$. This is due to the fact that there is no single measure that can rank CHP plants properly in all cases. The quality of the solution is evaluated based on relative gap (GAP),

$$GAP=100(z_s-z_b)/z_b \ (\%) \tag{21}$$

where z_s and z_b are the objective function values of the subject and benchmark algorithms, respectively.

TABLE II
RELATIVE GAP (GAP) FOR MDP-RSC AGAINST CPLEX (GAP 1%) AS WELL AS SOLUTION TIME FOR BOTH MDP-RSC AND CPLEX FOR DAILY SCHEDULING

Sampling time	CPU(s)			GAP1(%) (%)	GAP2(%)
	CPLEX	MDP-RSC1	MDP-RSC2		
0	13.1	2.74	1.12	0.06	0.14
672	297.4	3.20	1.32	-0.35	-0.35
1344	5.4	3.52	1.36	-0.50	-0.50
2016	8.5	3.97	1.59	-0.40	-0.40
2688	130.8	3.16	1.26	-0.56	-0.54
3360	243.8	3.24	1.43	-0.13	-0.13
4032	241.1	3.35	1.34	-0.45	-0.45
4704	118.8	3.98	1.11	0.05	0.05
5376	187.6	3.10	1.15	0.41	0.53
6048	359.5	3.38	1.32	0.29	0.29
6720	225.6	3.71	1.40	-0.19	-0.19
7392	352.3	2.64	1.04	0.24	0.24
8064	409.5	3.19	1.24	-0.02	0
AVG	199.5	3.25	1.28	-0.12	-0.10

Table II gives the relative gap (GAP) for MDP-RSC against CPLEX as well as solution time for both MDP-RSC and CPLEX. In the table, GAP1 is the best gap we get for all $|G|$ values we tested. For our test instances, it seems that the best results occur when $|G|$= 5 or 6. It means that we need to do tests for $|G|$= 5 and 6 to get the better results. Consequently, we recorded the solution time for MDP-RSC (MDP-RSC1 column) as the sum of the CPU time for $|G|$= 5 and 6. GAP2 is GAP for $|G|$=5. It is the best result in terms of both average result (AVG row) and the sample standard deviation for individual test instances for $|G|$= 5 and 6 if we evaluate the result for individual $|G|$. Accordingly, MDP-RSC2 column reported the CPU time for $|G|$=5.

We observed that GAP2 is close to GAP1. The average GAP is less than 1% and the sample standard deviation is 0.34. This means that the worst GAP is less than 2% (averge+3*standard deviation). Here, we compare the solution time in MDP-RSC2 column against that for CPLEX. MDP-

RSC is from a few to a few hundred times faster than CPLEX with average 155. It means that MDP-RSC shows advantage over CPLEX when the uncertainties of the intermittent RES such as wind and solar power need to be considered. In this situation, numerous scenarios of intermittent RES need considering [36]. In each scenario, a deterministic UC of the multi-site CHP problem needs to be solved. The final schedule can be obtained by combining the results of individual scenarios. It implies that the solution time for the deterministic problem must be fast enough to facilitate evaluation.

VI. CONCLUSION

To address power grid balance issues caused by large penetration of intermittent RES such as solar and wind power, into the energy system, CHP systems have additional mechanisms to increase the RES utilization when compared with power-only systems. On one hand, power generation in the CHP plant can be influenced by heat generation level due to the interdependence between power and heat generation. On the other hand, the operations of CHP systems are usually driven by heat demand. When there is a large scale RES generation, RES power can be used to satisfy heat demand to avoid the startup of CHP plants. However, CHP systems are more complicated than power-only systems. The scheduling and planning coordination in the CHP system needs to consider the heat and power generation and demand simultaneously and thus more sophisticated methods need to be developed.

In this paper, we have formulated UC of the transmission-constrained multi-site CHP system by integrating the special modeling technique for the UC of the single-site CHP system [22] and network flow model [32] for the power transmission [5]. We have developed DP-based algorithm for solving the problem. Numerical experiments for daily scheduling with realistic CHP models and data show that the DP-based algorithm has a good solution accuracy (less than 1% gap with the optimal solution on the average) and the fast solution speed (the DP algorithm is from a few to a few hundred times faster than CPLEX, the standard optimization software). This will lay solid foundation for addressing RES integration into the energy system for considering different scenarios under uncertainty.

LIST OF SYMBOL

A. Indices

i, k	site index
(i,k)	power arc in the power transmission network
j	extreme point index
j_u^{OFF}	index of extreme point corresponding to OFF-state of plant u, $j_u^{OFF} \in J_u$,
p, q	superscript/subscripts or prefixes for power and heat

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in the system

u plant index

t index of a period or a point in time. The period t is between points t-1 and t. In our problem, period length is one hour

B. Index Sets

Φ set of generation and demand sites (nodes)

A set of arcs $A \subseteq \{ (i, k) : i, k \in \Phi, i \neq k \}$

J set of extreme points of the operating regions for all plants

J_i set of extreme points of the operating regions for plants at site i

J_u set of extreme points of the operating region of plant u

U set of all plants

U_i set of plants at site $i \in \Phi$.

C. Parameters

$c_{i,k,t}$ transmission cost on arc (i, k)

$g_{i,k}$ capacity of arc (i, k)

$(c_{j,t}, p_{j,t}, q_{j,t})$ extreme point $j \in J_u$ of operating region of plant u (cost, power, heat) in period t

$c_{i,p\pm,t}$ power surplus/slack penalty cost at site i in period t

$c_{i,q\pm,t}$ heat surplus/slack penalty cost at site i in period t

$s_u(w_{u,t-1}, y_{u,t}, y_{u,t-1})$ start-up and shut-down cost for plant $u \in U$ at the beginning of time period t

$P_{i,t}$ power demand at site i in period t

$Q_{i,t}$ heat demand at site i in period t

$Y_{u,OFF}$ set of hours when plant $u \in U$ is at forced OFF-states

$Y_{u,ON}$ set of hours when plant $u \in U$ is at forced ON-states

UT_u minimum up time for plant $u \in U$ after start-up

DT_u minimum down time for plant $u \in U$ after shut-down

CT_u cold start-up time for plant $u \in U$

T number of periods over the planning horizon

D. Decision Variables

$w_{u,t}$ state variable for plant $u \in U$, indicating number of periods that plant u has been ON or OFF at the end of period t, (negative values denote OFF time). Initial state $w_{u,0}$ is given

$x_{j,t}$ decision variables used for encoding the operating level of each plant u in terms of extreme points $j \in J_u$ of the operating region in period t,

$x_{i,p\pm,t}$ power surplus/slack quantity at site i in period t,

$x_{i,q\pm,t}$ heat surplus/slack quantity at site i in period t

$y_{u,t}$ zero-one decision variable indicating whether plant $u \in U$ is OFF/ON in period t

$z_{i,k,t}$ power flow on arc (i,k) in period t.

E. Notations Related to DP Algorithm

$R(t,i)$ least total cost to arrive at state (t,i)

$C_{ED}(t,i)$ economic dispatch (ED) cost for state (t,i)

$SC(t-1,k:t,i)$ transition cost from state $(t-1,k)$ to state (t,i)

S_t set of states at stage t.

G subset of plants that are committed simultaneously

F. Notation Used for Ranking

$\lambda_{p,i,t}$ dual value of power balance at site i in period t.

$\lambda_{q,i,t}$ dual value of power balance at site i in period t.

$P_{u,t}$ power production in plant u in period t

$Q_{u,t}$ heat production in plant u in period t

$C_{u,t}$ production cost as function of $P_{u,t}$ and $Q_{u,t}$ in plant u in period t

$M_{R,u,1}$ ranking measure for plants generating energy over the planning horizon based on the solution of the relaxed-state problem

$M_{R,u,2}$ ranking measure for plants generating no energy over the planning horizon based on the solution of the relaxed-state problem,

$E_{max,u}$ characteristic point with $\max\{p_j+q_j, j\in J_u\}$ for plant u,

$C_{max,u}$ cost corresponding to $E_{max,u}$ for plant u

REFERENCES

[1] IEA (International Energy Agency). *Cogeneration and renewables—Solutions for a lower carbon future*, 2011.

[2] IEA (International Energy Agency). *Cogeneration and district energy—Sustainable energy technologies for today and tomorrow*, 2009.

[3] M. Jaradi, S.Riffat, "Trigeneration systems: Energy policies, prime movers, cooling techniques, configurations and operation strategies," *Renewable and Sustainable Energy Reviews*, vol.32, pp.396-415, 2014.

[4] Z.Ouyang, M. Shahidehpour, "Heuristic multi-area unit commitment with economic dispatch," *IEE Proceedings-C*, vol.138 (3), pp.242-252, 1991.

[5] D. Streiffert, "Multi-area economic dispatch with tie lines constraints," *IEEE Transactions on Power Systems*, vol.10 (4), pp.1946-1951, 1995.

[6] Z. Li, M. Shahidehpour, W. Wu, B. Zeng, B. Zhang, W. Zheng, "Decentralized multiarea robust generation unit and tie-line scheduling under wind power uncertainty," *IEEE Transactions on Sustainable Energy*, vol. 6(4), pp.1377-1388, 2015.

[7] A. Rong, R. Lahdelma, "Role of polygeneration in sustainable energy system development—Challenges and opportunities from optimization viewpoints," *Renewable and Sustainable Energy Reviews*, vol.53, pp.363-372, 2016.

[8] P. Meibom, J. Kiviluoma, R. Barth, H. Brand, C. Weber, H.V. Larsen, "Value of electric boiler and heat bumps for wind power integration," *Wind Energy*, vol.10, p.p. 321-337, 2007.

[9] D. Connolly, "Heat roadmap Europe—A low carbon heat and cooling strategy for Europe," *Summer School for District Heating and Cooling*, Helsinki, August, 2014, Finland.

[10] C. L.Tseng, X. Guan, A.J. Svoboda, "Multi-area unit commitment for large scale power systems," IEE Proceedings for Generation, Transmission and Distribution, vol. 145(4), pp. 415-421, 1998.

[11] M. Wang, B. Zhang, Y. Deng, "A novel unit commitment method considering various constraints," 2000 IEEE Power Engineering Society Meeting, vol.3, pp.1778-1783.

[12] F.N. Lee, J. Huang, R. Adapa, "Multi-area unit commitment via sequential method and a DC power flow network model," *IEEE Transactions on Power Systems*, vol. 9(1), pp. 279-287, 1994.

[13] A. Kargarian, Y. Fu, P. Liu, C. Wang, "A system of system engineering approach for unit commitment in multi-area power markets," 2014 IEEE PES General Meeting, pp. 1-5, 2014.

[14] K. Venkatesan, C.C.A. Rajan, "A simulated annealing method for solving multi-area unit commitment problem in deregulated environment," *2011 IEEE PES Innovative Smart Grid Technologies*, pp.305-310, India, December, 2011.

[15] M. Carrion, J.M. Arroyo, "A computationally efficient mixed integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21(3), pp.1371-1378, 2016.

[16] J. Ostrowski, M.F. Anjos, A. Vanneli, "Tight mixed integer linear programming frmulations for the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 27(1), pp. 39-46, 2012.

[17] G. Morales-Espana, J.M. Latorre, A. Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol.28 (4), pp. 4897-4908, 2013.

[18] E. Thorin, H. Brand, C. Weber, "Long term optimization of cogeneration systems in a competitive market environment," *Applied Energy*, vol. 81, pp. 152-169, 2005.

[19] A.L. Facci, L. Andreassi, S. Ubertini, "Optimization of CHCP(combined heat, power and cooling) systems operation strategy using dynamic programming*," Energy*, vol.66, pp. 387-400, 2014.

[20] A. Rong, H. Hakonen, R. Lahdelma, "A variant of dynamic programming algorithm for unit commitment of combined heat and power systems. *European Journal of Operational Research*, vol. 190, pp.741-755, 2008.

[21] A. Rong, R. Lahdelma, M. Grunow, "An improved unit decommitment for combined heat and power systems," *European Journal of Operational Research*, vol. 195, pp.552-562, 2009.

[22] A. Rong, H. Hakonen, R. Lahdelma, "A dynamic regrouping based sequential dynamic programming algorithm for unit commit of

World Academy of Science, Engineering and Technology
International Journal of Computer and Systems Engineering
Vol:10, No:8, 2016

combined heat and power systems," *Energy Conversion and Management*, vol.50, pp.1108-1115, 2009.

[23] N.H. Kjeldsen, M. Chiarandini, "Heuristic solutions to long-term unit commitment problem with cogeneration plants," *Computers & Operations Research,* vol.39, pp. 269-282, 2012.

[24] H. Gopalakrishnan, D. Kosanovic, " Operational planning of combined heat and power plants through genetic algorithms for mixed 0-1 nonlinear programming," *Computers & Operations Research,* vol.56, pp. 51-67,2015.

[25] R.Bellman. *Dynamic programming*, Princeton University Press, NJ, USA, 1957.

[26] A. Rong, R. Lahdelma, "An efficient model and algorithm for the transmission-constrained multi-site combined heat and power system," paper in submission.

[27] R. Lahdelma, H. Hakonen, "An efficient linear programming algorithm for combined heat and power production," *European Journal of Operational Research*, vol. 148, pp. 141-151, 2003.

[28] A. Rong, H. Hakonen, R. Lahdelma, "An efficient linear model and optimization algorithm for multi-site combined heat and power production, *European Journal of Operational Research*, vol.168, pp. 612-632, 2006.

[29] S. Makkonen, R. Lahdelma, "Non-convex power plant modeling in energy optimization," *European Journal of Operational Research*, vol.171, pp.1113-1126, 2006.

[30] A. Rong, R. Lahdelma, "An efficient envelope-based Branch and Bound algorithm for non-convex combined heat and power production planning," *European Journal of Operational Research*, vol. 183, pp. 412-431, 2007.

[31] G. Dantzig, *Linear programming and extension*, Princeton University Press, Princeton. NJ, 1963.

[32] R.K. Ahuja, T.L. Magnanti, J.B. Olin, *Network flows—theory, algorithms and applications*, Prentice Hall, Upper Saddle River, NJ, 1993.

[33] P.K. Singhal, R.N. Sharma, "Dynamic programming approach for solving power generating unit commitment problem," *International Conference on Computer & Communication Technology*, pp. 298-303, 2011.

[34] IBM ILOG CPLEX Optimization Studio 12.5. http://ibm-ilog-cplex-optimization-studio.software.informer.com/12.5/.

[35] Nordic power market. www.nordpoolspot.com.

[36] M.A. Hozouri, A. Abbaspour, M. Fotuhi-Firuzabad, M. Moeini-Aghtaie, "On the use of pumped storage for wind energy maximization in transmission-constrained power system," *IEEE Transactions on Power Systems,* vol.30 (2), pp. 1017-1025, 2015.