Incentives in Production: A Case Study

YU-CHI HO, FELLOW, IEEE, PETER B. LUH, MEMBER, IEEE, YING-PING ZHENG, JONG-MING WU, QING-YU WANG, AND LIANG CHEN

Abstract—**Increasing productivity while minimizing cost is a management goal that needs no elaboration. Using incentive systems to encourage production workers to achieve this goal is often an integral part of good management. In principle, a good incentive system should encourage individuals to do what is right by rewarding them for carrying out desirable actions. In reality, however, there are many practical considerations that have to be carefully thought out for an incentive system to be effective. This paper describes the motivation, rationale, and experiences of implementing an incentive scheme in a large petrochemical plant in the People's Republic of China for the purpose of conserving the consumption of cooling water. Experimental implementation in a section of a plant is described in detail. A few incentive-induced features uncovered after analyzing the experimental data are reported. A mathematical model with several variations is presented. A better predictive property of the incentive scheme in comparison to a more conventional method is proved analytically. The effectiveness of the scheme is shown by the amount of water conserved, and by the management's decision on the plant-wide adoption of the scheme. Practical issues encountered in experimental implementation, extensions of the scheme to more general situations, and new theoretical problems inspired by the experiment are discussed.**

I. Introduction

As succinctly pointed out by Radner [6] there are three major problems that concern the incentive theory literature: misrepresentation, moral hazard, and free rider. They have easily understood everyday interpretations in human endeavors. Misrepresentation is concerned with the problem that subordinates or followers (*Fs*) often have incentives to misrepresent important information which the leader (L) needs for an efficient economic decision. For example, everyone is always interested in obtaining more of a scarce resource than what is optimal from the group's point of view, and might be tempted to misrepresent one's production capability in order to receive more of the resource. Moral hazard deals with the fact that in the absence of society's effective inspection and control, one might, by considering one's self-interest, behave in a manner that is bad from the society's viewpoint. Finally, the free rider problem is simply another form of "eating out of the same big rice bowl," i.e., trying to enjoy fruits of others' labor without bearing any cost. These problems arise because the leader either does not have the necessary information for a decision or cannot directly control certain decision variables. He must rely on the *Fs* who have the necessary information or who can directly implement the decision variables. Incentive theory is involved with the design of reward/punishment policy to indirectly control or influence *Fs* to behave in a desirable manner out of their self interest. When used in manufacturing companies, it is an important factor in increasing productivity and in conserving scarce resources. While the principle of incentives is obvious, its quantitative application to real world situations is not that simple. In fact, the economics literature on incentives are remarkably scarce in applications. The real world does not always neatly conform itself to assumptions of the theory. A host of practical issues must be dealt with. The purpose of this paper is to report on a case study of applying incentive theory in a real situation involving real followers with real rewards. The emphasis is not on the development of complicated incentive schemes with provable properties under various assumptions. Rather, it is on the application of a simple scheme, and on the analysis of the scheme in a real world environment. We shall see how practical constraints in budgets, human psychology, and acts of nature intrude into and impose limitations on the analysis. We shall also discuss real benefits of experimentation, namely, validating mathematical models and results, and inspiring the formulation of new theoretic problems of practical importance. It is difficult to imagine that some of the issues we encountered would have occurred to us if we were merely doing "AS IF" analysis. Finally, the general applicability of the case study to other production environments will be discussed. The readers should be warned ahead of time that this paper will not be presented as a neatly wrapped gift package. Successes as well as shortfalls will be disucssed and not glossed over.

Conceptually, consider a water conservation problem in a manufacturing company. The leader (L) would like to know how much has to be spent per month on bonuses in order to achieve, say, one ton of cooling water conservation per unit of production for a follower whose action L is trying to influence. This information clearly depends on at least two factors, the effort F has to spend in conservation, and the economic well-being F derives from water conservation. It is reasonable to assume that F trades off effort for bonus payments. In other words, we postulate that F has a utility function $U(b, x)$ where

b = the amount of bonus given,

x = the effort spent on water conservation.

We shall assume

$$\partial U/\partial b>0,\ \partial U/\partial x<0,\ \partial^2U/\partial b^2\le 0,$$
$$\partial^2U/\partial x^2\le 0,\ \partial^2U/\partial x\partial b\le 0. \tag{1.1}$$

The point $U(0, 0) = 0$ is the reference point which corresponds to the no-effort no-reward status quo. Any combination of b and x which yields $U > 0$ will induce water conservation. Conversely, $U < 0$ will not induce F to conserve. If we further postulate a one-to-one relationship between x, the effort spent, and q, the amount of water conserved, as

$$q=G(x) \text{ and } x=G^{-1}(q), \text{ with } dG/dx(x)>0 \tag{1.2}$$

we then can rewrite $U(b, x) = U(b, G^{-1}(q))$ as $V(b, q)$. Finally, for any given incentive policy of the form

$$b=f(q) \tag{1.3}$$

we can calculate the optimal amount of water conserved, q^*

Manuscript received December 29, 1986; revised August 26, 1987. Paper recommended by Past Associate Editor, P. R. Belanger. This work was supported in part by the National Science Foundation under Grant ECS-82-13680 and by Grant INT-82-17595 of the U.S./China Cooperative Research Program.

Y.-C. Ho is with the Division of Applied Sciences, Harvard University, Cambridge, MA 02138.

P. B. Luh is with the Department of Electrical Engineering and Computer Science, University of Connecticut, Storrs, CT 06268.

Y.-P. Zheng, J.-M. Wu, Q.-Y. Wang, and L. Chen are with the Academia Sinica, Beijing, People's Republic of China.

IEEE Log Number 8718713.

Author corrections were not available at the time of publication.

0018-9286/88/0300-0227$01.00 © 1988 IEEE

(optimal from F's viewpoint), by maximizing $V(f(q), q)$ over q. The bonus paid, $b^* = f(q^*)$, can also be computed. Design of the incentive policy then simply means the choice of $f(\cdot)$, This is all relatively straightforward (see, for example, Ho-Luh-Olsder [2]) if U and G are accurately known to the leader L. In real situations, however, U may not even be clearly known to F much less to L. Attempts to solicit information may also be met by strategic responses from F, e.g., misrepresenting the $G(\cdot)$ in (1.2). An incorrectly designed incentive policy not only is inefficient and ineffective, but may also lead to worker dissatisfaction because of its more or less arbitrary nature. Futhermore, the relationship between q and x in (1.2) is often nondeterministic. Learning and stochastic optimization on the part of F are involved. Finally, in the real world, problems rarely arise in a well-isolated environment. Decisions and experiments involving one person or one group often have unintended consequences on others. All of these considerations will manifest themselves in the following discussions.

II. A Real World Problem and a Particular Incentive Scheme

The Yan-Shan Petrochemical Corporation, located in the suburb of Beijing, is the largest petrochemical company in China. Hourly water consumption by the company for industrial and residential use amounts to 9000 tons. With the projected doubling of production by the year 1990 under the modernization program of China, water shortage will become a serious problem.[1] Thus, water conservation is receiving increasing attention by management as well as the government. Concurrently, the Institute of Automation of the Chinese Academy of Sciences and the Division of Applied Sciences, Harvard University, concluded an agreement under the U.S./China Science Cooperative Research Project for the study of applied incentive optimization in organizations. Yan-Shan Petrochemical Corporation agreed to become an experimental site of the study. The particular test project involves the posttreatment section of the synthetic rubber production of the Sheng-Li Plant of the Company. Four teams of workers rotate among three shifts to continuously control the use of cooling water for the production of synthetic rubber. Previous water usage was at the rate of 13 tons per ton of rubber produced. It was felt that if workers carefully control various inlet/outlet valves, the consumption rate could be reduced considerably. The idea of using an incentive system to induce workers to conserve water received management approval. The U.S./China research team wanted to test the quantitative application of incentive strategy in the real world. Thus, the stage was set for experimentation.

Each team is treated as a follower. The water usage is monitored every shift, and computed on the per-team basis. Since four teams are essentially the same and operate almost independently, we consider a one-leader one-follower incentive model. Data collected from the teams are treated on an ensemble basis. While there have been many theoretical incentive schemes each possessing various provable properties under various assumptions in the literature, almost all of them assume the existence of very sophisticated Fs. It is unlikely to find such sophisticated Fs in projects of the type we are contemplating. Therefore, the emphasis of this project is on simple schemes which are easily understood by the public, and on the performance of a scheme in a real world environment (the poll results in Appendix E support this decision). More specifically, we visualize the following variant of the so-called Soviet Incentive Policy [1], [7], [5]. Let $\hat{q}(t)$ = the follower's estimate of the amount of water conservation (in tons of water conserved per ton of rubber production) to be achieved by the follower during period t (which is a month),

$q(t)$ = the actual amount of water conservation realized by the follower during period t,

$b(t)$ = the bonus to be paid for the conservation effort of period t,

then the incentive payment is given by

$$\begin{aligned} b(t) &= \beta\hat{q}(t) + \alpha[q(t) - \hat{q}(t)], \quad \text{if } q(t) \geq \hat{q}(t), \\ &= \beta\hat{q}(t) + \gamma[q(t) - \hat{q}(t)], \quad \text{if } q(t) < \hat{q}(t) \end{aligned} \tag{2.1}$$

where $0 < \alpha < \beta < \gamma$ are parameters to be chosen by L. For simplicity, we let $\alpha = 0.7\beta$ and $\gamma = 1.3\beta$. The follower F has two decision variables: $\hat{q}(t)$ to be chosen at the beginning of period t, and $q(t)$ to be actually achieved [or equivalently $x(t)$, the effort to achieve $q(t)$] during period t. The property of policy (2.1) can be easily explained by Fig. 1. In the figure several level curves of U = constant are shown. The vertical axis ($q = 0$) is placed at, by definition, the point corresponding to water usage with no effort on conservation. Thus, the level curve which passes through the origin represents the demarcation line above which bonus payment will induce conservation. The point P, with coordinate $(b_\beta q_\beta)$, is the tangent point between the line $b(t) = \beta q(t)$ and a level curve of U = constant. It is the choice that maximizes the F's utility under the simple incentive policy of

$$b = \beta q. \tag{2.2}$$

It is the optimal point (b^*, q^*) mentioned in the paragraph following (1.3). One can also verify easily that under policy (2.1) it is optimal for F to choose $\hat{q}(t) = q_\beta$ and $q(t) = q_\beta$. Suppose F estimated $\hat{q}(t) = q' > q_\beta$ at the beginning of period t, then to maximize U, F would then end up with the point P'. Similarly, if F estimated $\hat{q}(t) = q'' < q_\beta$ at the beginning of period t, F would end up with the point P''. Neither P' nor P'' yields higher utility than P. This result can also be proved analytically by deriving F's necessary conditions for optimality. Thus, comparing to policy (2.2), the incentive policy (2.1) has the advantage of encouraging F to estimate q_β accurately first, and then to strive to achieve the reported target. Several other derived benefits are also evident.

1) F has more of a sense of control over efforts and rewards, and is not told to meet a certain quota or else. This is psychologically uplifting and also eliminates the necessity of bargaining over targets and quotas.

2) This scheme encourages F to learn the work, to become an expert on the job, and to utilize the expertise.

3) In the case where F is not that certain about G and U, policy (2.1) provides F with feedbacks so that a better job can be done the next time.

4) Conservation bottlenecks can be identified as F tries to maximize U. With the management's help in lifting these bottlenecks, productivity will increase.

Note that (1)-(4) combined can be called incentive induced learning, i.e., the learning on the part of F induced by the incentive scheme and assisted by the management. Note also that (3) pertains particularly to the incentive policy (2.1).

5) It is not necessary for L to fully understand F's production function $G(x)$ and/or utility function $U(b, x)$ to implement the scheme. As more data are collected, L will gradually get to know G and U, and can iterate on incentive parameters to achieve L's long-term goals.

6) L gets prediction on water usage at the beginning of each period. This information can be valuable for L in carrying out system planning.

Many of them will become apparent in the following discussions.

While policy (2.1) presents the essence of the incentive scheme, the actual bonus payment for a team in dollar amount $B(t)$ is slightly more complicated. It is given by

$$B(t) = b(t)y(t) - P(t) \tag{2.3}$$

where $y(t)$ is the rubber production in tons at month t by the team.

[1] In 1983, five of the nine deep wells for water supply ran dry during the summer season. The average water table dropped by 4 m from 1974 to 1983.

The last term in (2.3) is a penalty term to prevent F from using an overzealous effort to secure a bonus at the risk of plant safety and product quality. Plant safety and product quality are monitored continuously by automatic monitoring devices and periodically by the plant management. A team will be penalized by 1 RMB for a minor violation of plant safety requirement, by 10 RMB for a major violation of safety requirement, or for poor product quality (average worker salary is about 60 RMB per month). Since the penalty term imposed depends on the occurrence of violations but not directly on $\hat{q}(t)$ and $q(t)$, the effect of $P(t)$ is assumed to be small, and is ignored in the analysis for the sake of simplicity.

Because of the very real possibility that F may not know G and U very well, we visualize that it will take several periods of operation for $q(t)$ to gradually approach $\hat{q}(t)$ which in turn approaches q_β. As L gains information on the relationship between β and q, L may wish to change the value of β in order to achieve some long-term goals. Therefore, a super iteration in parameters (α, β, γ) is also visualized. In fact, part way into the experiment, the management did institute an adjustment of incentive parameters (α, β, γ). These matters are discussed in Sections III and IV below with experimental data presented in Appendix D.

III. Behavior Models for the Follower

Partial results of the experiments, from August 1984 to June 1985, are summarized in Table I and shown graphically in Fig. 2. We observe that $\hat{q}(t)$ more or less converges to 7 tons after the fifth period (December 1984), whereas $q(t)$ fluctuates around $\hat{q}(t)$. This says that $q(t)$ might have already converged. This phenomenon conflicts with our previous analysis in which we have $\hat{q}(t) = q(t) = q_\beta$. Careful examination revealed that this was caused by the existence of a fair amount of randomness. There are many random factors influencing $q(t)$, such as the amount of rubber produced per shift, starting up or shutting down of a production line, etc. These factors in general are not under the control of F. In view of this, the production function of (1.2) is modified to include uncertainties as follows:

$$q(t) = G(x(t), \eta(t)) \tag{3.1}$$

where $\{\eta(t)\}$ is a sequence of independent, identically distributed random variables. We assume that for any given $\eta(t)$, the partial inverse mapping of G from $q(t)$ to $x(t)$ exists, and is denoted as

$$x(t) = \Gamma(q(t); \eta(t)). \tag{3.2}$$

We also assume that F knows the distribution but not the exact value of $\eta(t)$ when he reports $\hat{q}(t)$; while in selecting $x(t)$ [or equivalently $q(t)$], he knows the value of $\eta(t)$. Although it is questionable to what extent this assumption is correct, it nevertheless captures the essence of the information structure observed in the experiment. This assumption also facilitates modeling and analysis processes. Under these suppositions, F's problem can be written as

$$\max_{\hat{q}(t)} E\{\max_{x(t)} U[b(G(x(t), \eta(t)), \hat{q}(t)), x(t)]\}. \tag{3.3}$$

That is, for a given $\hat{q}(t)$, F selects the best $x(t)$ knowing $\eta(t)$. This generates the optimal policy for the selection of $x(t)$. This policy is in turn used in finding the best $\hat{q}(t)$ to be reported at the beginning of month t. Thus, it is a nested optimization problem and, in general, is very difficult to solve. We shall next present a specific model which has nice physical interpretations and also gives closed-form solutions.

Consider the following additive production function:

$$G(x(t), \eta(t)) = G(x(t)) + \eta(t) \tag{3.4}$$

where $\eta(t)$ is uniformly distributed over $[-\eta_1, \eta_1]$, $\eta_1 > 0$. The deterministic portion of the production function $G(x(t))$ is given by

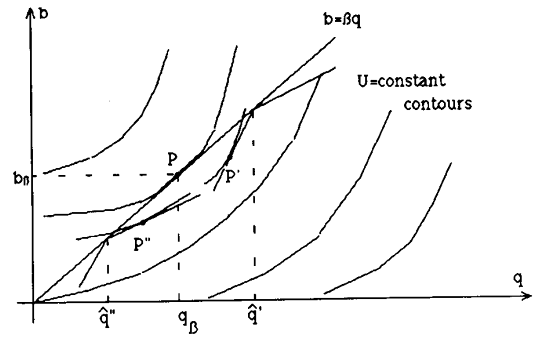


Fig. 1.

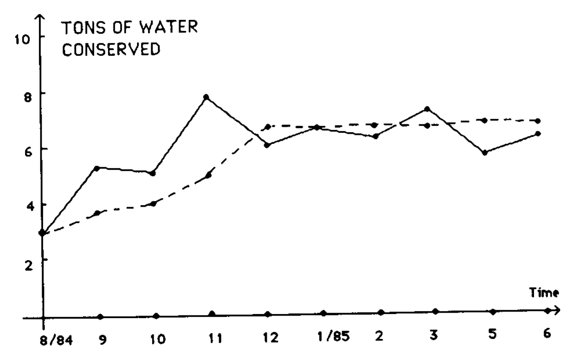


Fig. 2. Water conservation record.

$$G(x(t)) = r\{[1 - \exp(-hx(t))] + \underline{G}\} \tag{3.5}$$

where r, h, and $\underline{G}$ are constants. The constants r and h are scaling parameters. The value $r\underline{G}$ represents the output of conservation requiring almost no effort (with $x = \epsilon^+$ a small positive number). As $x(t)$ increases, $G(x(t))$ increases, and saturates at the value $r(1 + \underline{G})$. Note that this saturation phenomenon *is a must* in any conservation effort, since it is impossible to conserve more than the conservation potential. F's utility is assumed to be linear and additive:

$$U(b(t), x(t)) = b(t) - gx(t) \tag{3.6}$$

where g is a scaling constant.

With F's model described by (3.4)-(3.6) and the incentive policy described by (2.1), F's nested optimization problem (3.3) can be solved to give the following results (derivations are provided in Appendix A):

$$\hat{q}(t) = r(1 + \underline{G}) - 1.03k/\beta, \tag{3.7}$$

$$E[q(t)] = r(1 + \underline{G}) - 1.10k/\beta + 0.022(k/\beta)^2/\eta_1, \tag{3.8}$$

where

$$k \equiv g/h. \tag{3.9}$$

In deriving (3.7) and (3.8), $\alpha = 0.7\beta$ and $\gamma = 1.3\beta$ are used. A certain condition on the magnitude of η_1 is also assumed. For details, see Appendix A. Note that although $E[q(t)] \neq \hat{q}(t)$, they are very close since the third term in (3.8) is small. The difference is caused by the different information F has in reporting $\hat{q}(t)$ and in selecting $q(t)$.

The stochastic optimization model presented above still does not fully explain the experimental data since the data show a drastic improvement during the first few months as presented in

Fig. 2. Detailed examination of the experimentation revealed that there were a lot of learning activities going on during that period of time. On the one hand, teams tried to figure out how to effectively control water valves to conserve water without violating safety/quality requirements. Experiences were shared among the teams. On the other hand, conservation bottlenecks were identified, and some of the bottlenecks were lifted with the assistance from the management. To model this *incentive induced learning* process, we modify (3.4) and (3.5) to let the parameter r capture the learning feature

$$q(t)=r(t)\{[1-\exp(-hx(t))]+\underline{G}\}+\eta(t) \quad (3.10)$$

where the variable $r(t)$ is described by

$$r(t)=r_M(1-\rho^t) \quad (3.11)$$

and ρ is the learning parameter satisfying $0<\rho<1$. By following the same procedure that led to (3.7) and (3.8), we obtain

$$\hat{q}(t)=r(t)(1+\underline{G})-1.03k/\beta \quad (3.12)$$

and

$$E[q(t)]=r(t)(1+\underline{G})-1.10k/\beta+0.022(k/\beta)^2/\eta_1,$$
$$=q_M(1-\rho^t)-1.10k/\beta+0.022(k/\beta)^2/\eta_1 \quad (3.13)$$

where $q_M \equiv r_M(1+\underline{G})$. Under a certain condition on the magnitude of η_1, the term $-1.10k/\beta+0.022(k/\beta)^2/\eta_1$ in (3.13) is less than zero. Consequently, $E[q(t)] \le q_M$, and q_M has the meaning of being the conservation potential. It is the largest $E[q(t)]$ that can be expected. The learning parameter ρ describes how fast F learns. Small ρ implies fast convergence of $E[q(t)]$ to its final value. The parameter $k \equiv g/h$ is a scaling constant. It relates effort $x(t)$ and conservation $q(t)$ to bonus $b(t)$ [refer to (3.6) and (3.10)], and represents the effectiveness of effort. The parameter η_1 represents the level of randomness. Large η_1 (large variance) implies high uncertainty, and would result in small $E[q(t)]$. Finally, the larger the incentive rate β, the higher the $E[q(t)]$ would be. *Consequently, there are five key parameters in the conservation process, namely conservation potential q_M, learning parameter ρ, effectiveness of effort k, level of uncertainty η_1, and incentive rate β.* It is believed that they are key parameters to many other conservation/production related incentive efforts as well, and possess a certain degree of generality.

The fitting of the first six months' data to the above learning model results in the following set of parameters:

$$q_M=8.42 \text{ tons}, \ \rho=0.52,$$
$$k/\beta=1.69 \text{ tons}, \ \eta_1=2.0. \quad (3.14)$$

Derivation of these numbers is presented in Appendix D. This set of parameters does have a predictive feature as shown in Table V. Note that with $\rho = 0.52$, the learning process essentially ends at the fifth month.

The sensitivity of $E[q(t)]$ with respect to β is

$$S \equiv \partial E[q(t)]/\partial\beta \cdot \beta/E[q(t)]$$
$$=[1.10k/\beta-0.044(k/\beta)^2/\eta_1]/E[q(t)]. \quad (3.15)$$

With parameters given by (3.14), we have $S \simeq 27$ percent for $t \ge 5$. This low sensitivity was observed in January when β was increased from 10 to 15 percent while $\hat{q}$ and q remain essentially unchanged. This suggests that the practical limit of conservation might have been reached. It also suggests that a moderate bonus would induce F to learn how to conserve, and to actually conserve. After a certain point has been reached, however, it would be difficult for F to conserve more, even with a higher bonus.

IV. Adjustment of the Bonus Coefficient and the Origin of the q Axis

For the choice of β and the origin of the q axis, the desire of L is threefold: to gain knowledge about the relationship between water conservation and bonus paid; to optimize some long-term economic objective; and to adjust upwards the goal of water conservation as time goes by. The last objective anticipates future water supply and demand projections and the increase in F's ability to control the water consumption. There are several considerations. First, by varying β, the leader gains some knowledge about the relationship between β and q. However, it is bad psychologically to decrease β. For this reason, it is better to start out with a small β and gradually increase it. But if β is to be gradually increased, then anticipating this the follower may decide to hold off the conservation effort until β becomes sufficiently large in order to gain more reward. This form of strategic manipulation can be thought of as the inverse of the well-known *ratchet principle* [8]. In this case, the worker's reasoning for not working hard initially is not the fear of *tightening the screw* by the management, but is an attempt to outsmart the management. On the other hand, shifting up the origin of the q axis, a, means imposing a minimum conservation quota. This does represent a tightening of the screw and may jeopardize F's future cooperation. Thus, it was decided that if F conserves the same amount of water before and after the change in β and a, F should receive the same bonus. To promote additional conservation effort, higher (lower) levels of conservation should lead to even higher (lower) levels of reward than if there were no change in β and a.

Specifically, suppose $\hat{q}(t) = q(t) = q_1$ before the change. Let

$$\beta' = \text{value of } \beta \text{ after the change} = \beta/(1-\lambda)$$
$$\alpha'=(\beta'/\beta)\alpha,$$
$$\gamma'=(\beta'/\beta)\gamma, \quad (4.1)$$

and

$$b=\beta'(\hat{q}-a')+\alpha'(q-\hat{q}) \quad \text{if } q\ge\hat{q}$$
$$=\beta'(\hat{q}-a')+\gamma'(q-\hat{q}) \quad \text{if } q<\hat{q} \quad (4.2)$$

where

$$a'=a+\lambda(q_1-a)=\text{value of the origin after the change.} \quad (4.3)$$

This is illustrated in Fig. 3(a) and (b) (for $a = 0$). That is, L increases β and also shifts the origin of the q axis. The two adjustments are done in a way so that if F achieves the same amount of conservation (q_1) before and after the change, F would receive the same reward. However, since the new incentive coefficient β' is higher, F is encouraged to conserve more. By the same token, the reward is decreasing faster than before for underachieving. In this sense, there is a gradual tightening of the screw. But in no case will F be punished for keeping the status-quo, and will be rewarded for doing more. Appendix B includes an analysis of this adjustment scheme under the special assumption that F is a bonus maximizer (i.e., $U(b, x) = b$), and L carries out one adjustment on β and a. It shows that if $\lambda < 1/2$, then the scheme possesses another interesting property, i.e., even if F is aware of the impending changes in β and a, it is still in F's best interest to maximize F's current bonus payment. In other words, F will not engage in the type of strategic manipulation mentioned in the opening paragraph of this section. In fact, it is easy to generalize this result to the case of n iterations of β and a. Under the condition that $\lambda < 1/(n+1)$, F will attempt to maximize the bonus payment for each period.

Such parameter adjustment was carried out in January 1985. The value of β was increased from 10 to 15 percent and the nominal water consumption was reduced from 13 to 10.5 tons (equivalent to $a = 0$ and $a' = 2.5$, respectively). Since at that

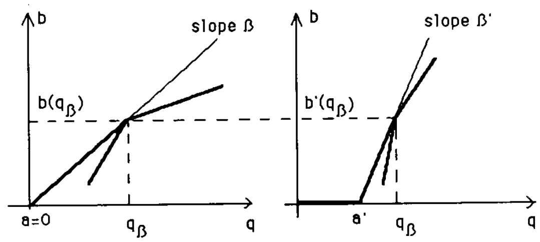


Fig. 3.

time $\hat{q} = q = 7$, we have $\lambda = 2.5/7 = 0.357$ from (4.3). Experimental results show that $\hat{q}$ and q essentially remain unchanged after the adjustment. This can be explained partly by the low sensitivity of $E[q(t)]$ with respect to β as shown at the end of Section III, and partly by the indifference of 10 and 15 percent in F's perception. It also suggests that the practical limit of conservation might have been reached.

The analysis so far assumed that F is myopic and can see at most into the finite future. A discussion of the case where F is not myopic and can see into the infinite future is in Ho [3].

V. Variance Reduction Property

A fairly common incentive scheme is for the form

$$b = \beta q. \tag{5.1}$$

That is, the bonus is proportional to the amount of conservation (or production). We shall call it the β incentive scheme, in contrast to the $\alpha\beta\gamma$ incentive scheme discussed so far. Under the β incentive scheme, F does not have to report $\hat{q}(t)$ at the beginning of a period. The bonus $b(t)$ is determined at the end of the period according to the single bonus coefficient β. This is unlike the $\alpha\beta\gamma$ incentive scheme which discourages overachieving by having a low incentive rate α for the overachieved portion, and also discourages underachieving by having a high penalty rate γ for the underachieved portion. Consequently, it is intuitively plausible that under the $\alpha\beta\gamma$ incentive scheme, the resultant conservation $q(t)$ would be close to $\hat{q}(t)$. This in turn suggests that the variance of $q(t)$ would be smaller under the $\alpha\beta\gamma$ scheme than under the β scheme.

To be more specific, consider a fairly general model having the stochastic production function of (3.1) and the following additive utility function:

$$U(b, x) = b - H(x) \tag{5.2}$$

where $U(b, x)$ satisfies conditions (1.1) and (1.2). For notational simplicity, we shall in this section temporarily drop the time argument t. Let $q^{\alpha\beta\gamma}$ and q^{β} denote, respectively, the conservations under the $\alpha\beta\gamma$ scheme and the β scheme, $\bar{q}^{\alpha\beta\gamma}$ and $\bar{q}^{\beta}$ the means of conservations; and VAR $q^{\alpha\beta\gamma}$ and VAR q^{β} their variances. It can be shown that (the proofs are given in Appendix C)

$$E[(q^{\beta} - \hat{q})^2] > E[(q^{\alpha\beta\gamma} - \hat{q})^2], \tag{5.3}$$

and

$$\text{VAR } q^{\beta} - \text{VAR } q^{\alpha\beta\gamma} = \{E[(q^{\beta} - \hat{q})^2] - E[(q^{\alpha\beta\gamma} - \hat{q})^2]\} + (\bar{q}^{\alpha\beta\gamma} - \bar{q}^{\beta})^2 + 2(\bar{q}^{\alpha\beta\gamma} - \bar{q}^{\beta})(\bar{q}^{\beta} - \hat{q}). \tag{5.4}$$

On the right-hand side of (5.4), the term in the pair of braces is strictly positive from (5.3), and the next term is nonnegative. The last term would be zero if $\bar{q}^{\alpha\beta\gamma} = \bar{q}^{\beta}$ or $\bar{q}^{\beta} = \hat{q}$, and would be small if $\bar{q}^{\alpha\beta\gamma} \simeq \bar{q}^{\beta}$ or $\bar{q}^{\beta} \simeq \hat{q}$. Thus, in general, we have

$$\text{VAR } q^{\beta} > \text{VAR } q^{\alpha\beta\gamma} \tag{5.5}$$

except for the pathological case where the last term on the right-hand side of (5.4) is extremely negative. In Appendix C, we also show that inequality (5.5) holds for the specific model described by (3.4)–(3.6).

Consequently, beyond the advantage of having $\hat{q}(t)$ at the beginning of each period, *the $\alpha\beta\gamma$ incentive scheme also enjoys the advantage of having smaller variance on the actual conservation $q(t)$ for each period.* This is valuable to system planning.

VI. Conclusions

From August 1984 to June 1985, water conserved by the posttreatment section amounts to 152 775 tons. With the cost of water at 0.20 RMB/ton, the money saved is 30 555 RMB. Among them, 3208 RMB (10.5 percent) were used as a bonus for workers. These numbers are somewhat deceiving, however. The reason is that the cooling water used at the experimental station was further used by another production unit downstream. Since the other production unit was not on any incentive program, water consumption remained high. Consequently, it has to import water elsewhere as the experimental unit was not discharging enough water for its use. Thus, the net water conservation of the plant is somewhat less than 152 775 tons. From this it is clear that to achieve the goal of water conservation, every production unit has to conserve. In view of this and also because of the success of the experiment, the management decided to extend the inventive experiment to cover all major water consuming units of the Sheng-Li Plant.

The extension of the incentive scheme started October 1985, after a three-month stoppage of the experiment caused by a major industrial accident at another section of the plant. On the one hand, the production environment of the experiment section was changed and a new group of workers joined the four teams as results of the accident. On the other hand, the incentive scheme was also extended to cover all other major water consuming production units. It was observed that experiences were shared among old workers and newly joined workers, and new skills were developed for changed/new environments. The situations were quite complicated, and results are reported in Luh *et al.* [4]. We shall next sketch a generic problem encountered in the plant-wide implementation of the incentive scheme.

The multistation problem concerns water conservation of up-stream and down-stream production units. Because the two units are connected in series, conservation effort by one unit is not sufficient to guarantee overall savings. On the other hand, because production environments of units are not necessarily the same, their conservation potentials differ accordingly. A worker in a unit can and will argue that compared to others they have to spend more effort in conserving the same amount of water. Consequently, they should be rewarded at a higher rate. This leads to familiar incentive problems: how to induce truthful reporting, and how to allocate rewards such that the distribution is fair. This is a very difficult and challenging theoretical issue. Details of the problem formulation and some of the desired properties of the incentive scheme are presented in Luh *et al.* [4].

The entire project from initial negotiation to completion spanned a period of three years. Although none of the principal investigators from either side worked full time on the project, it is doubtful that progress can be made must faster otherwise. For example, the major industrial accident at another section of the plant in the summer of 1985 caused the stoppage of the experiment for three months as mentioned before. Other matters such as budget constraints, negotiation with workers, etc., all took time. Nevertheless, we believe that a number of things concerning the application of incentive theory to a real world problem have been accomplished.

1) Actual Conservation of Water Usage: From August 1984 to June 1985, water conserved by the posttreatment section amounts to 152 775 tons resulting in 10.5 percent reduction in the cost of production. Extension of the incentive-conservation scheme for plant-wide implementation has been underway. Plans are now being made to further extend it for the conservation of other energy resources, such as steam usages. The parent company is also planning to adopt the $\alpha\beta\gamma$ scheme to increase productions of individual plants by letting plants estimate productions first and then sharing profits among the parent company and plants. To this extent, the project is practically successful.

2) Implementation and Generalizability Issues: The experiment showed that workers are generally myopic and unsophisticated. It is doubtful that a more elaborate incentive scheme can be practiced. On the other hand, they have an intuitive fear for the so-called ratchet effect. Adjustment of β with the shift of origin as discussed in Section IV required a considerable amount of convincing. Second, there was definite incentive induced learning in the experiment. Skills in conserving water were acquired and shared in the course of the project. This was further evidenced in the fall of 1985 when the production environment of the posttreatment section was changed and a new group of workers joined the four teams after the industrial accident. Experiences were shared among old members and new members, new skills for the changed environment were developed, and data also show an apparent learning process taken place. Finally, one has to contend with a great deal of natural and man-made uncertainties and noise. Accidents, weather, changes in management tactics and production goals, safety rules, and workers rights all intrude into the conduct of the experiment and corrupt the data. Completely controlled experiments are difficult to carry out. However, we do believe that the model developed and discussed here has generalizing possibilities. The five key parameters, conservation potential q_M, learning parameter ρ, effectiveness of effort k, uncertainty level η_1, and the incentive rate β are basic to the practical implementations of incentive schemes of this type. Results from the plant-wide adoption of the scheme further supports this argument, and can be founded in Luh *et al.* [4].

3) New Theoretical Analysis: The experiment also brought forth several new issues in incentive theory. The variance reduction property of the $\alpha\beta\gamma$ scheme and the formulation of the multistation problem are direct results inspired by working in the real world.

Appendix A
Proofs for Section III

In this Appendix, the model of (3.4)–(3.6) is assumed. With F's two-stage optimization process described by (3.3), we first solve the inner optimization problem in finding the optimal $x(t)$ [or equivalently $q(t)$] given $\hat{q}(t)$ and $\eta(t)$. Using this result, we then solve the outer optimization problem in finding the optimal $\hat{q}(t)$. Lastly, the above results are combined to yield $E[q(t)]$. For notational simplicity, the time argument t will be suppressed in this Appendix.

1. Determination of x and q Given $\hat{q}$ and η

Given $\hat{q}$, there are three exhaustive and mutually exclusive cases for the realization of q: $q > \hat{q}$, $q < \hat{q}$, and $q = \hat{q}$. The overachieving case ($q > \hat{q}$) is called the α mode since the incentive rate α is ineffective for the overconserved portion. Similarly, the underachieving case ($q < \hat{q}$) is called the γ mode, and the case where q equals $\hat{q}$ is called the β mode.

Consider first the α mode with $q > \hat{q}$. In this case,

$$\begin{aligned} U &= \alpha q + (\beta-\alpha)\hat{q} - gx \\ &= \alpha r[1+\underline{G}-\exp(-hx)] + \alpha\eta + (\beta-\alpha)\hat{q} - gx. \end{aligned} \tag{A.1.1}$$

First-order necessary condition is given by

$$\partial U/\partial x = \alpha hr \exp(-hx) - g = 0. \tag{A.1.2}$$

With $k \equiv g/h$, this yields

$$x_\alpha = -[\ln(k/\alpha r)]/h \tag{A.1.3}$$

and

$$q_\alpha = r(1+\underline{G}) - k/\alpha + \eta \tag{A.1.4}$$

where subscript α denotes that they are optimal values in the α mode. The condition that $q > \hat{q}$ requires

$$\eta > [\hat{q} - r(1+\underline{G}) + k/\alpha]. \tag{A.1.5}$$

Therefore, the probability of being in the α mode is

$$P_\alpha = \Pr\{\eta > [\hat{q} - r(1+\underline{G}) + k/\alpha]\}. \tag{A.1.6}$$

By following the same derivation, we obtain

$$x_\gamma = -[\ln(k/\gamma r)]/h, \tag{A.1.7}$$

$$q_\gamma = r(1+\underline{G}) - k/\gamma + \eta \tag{A.1.8}$$

and

$$P_\gamma = \Pr\{\eta < [\hat{q} - r(1+\underline{G}) + k/\gamma]\}. \tag{A.1.9}$$

Finally,

$$\begin{aligned} P_\beta &= 1 - P_\alpha - P_\gamma \\ &= \Pr\{[\hat{q} - r(1+\underline{G}) + k/\gamma] \le \eta \le [\hat{q} - r(1+\underline{G}) + k/\alpha]\} \end{aligned} \tag{A.1.10}$$

and

$$q_\beta = \hat{q}. \tag{A.1.11}$$

The variable q_β in (A.1.11) represents the conservation in the β mode. It is different from the q_β of Section II defined under the deterministic formulation. Substituting (A.1.11) into (3.4) and (3.5) we obtain

$$x_\beta = -[\ln(1+\underline{G} - \hat{q}/r + \eta/r)]/h. \tag{A.1.12}$$

2. Determination of $\hat{q}$

F selects $\hat{q}$ to maximize $E[U]$. Rewrite $E[U]$ as

$$E[U] = P_\alpha E[U|\alpha] + P_\gamma E[U|\gamma] + P_\beta E[|\beta]. \tag{A.2.1}$$

Since η is uniformly distributed over $[-\eta_1, \eta_1]$, (A.1.6) implies that

$$\begin{aligned} P_\alpha &= 1 \text{ if } [\hat{q} - r(1+\underline{G}) + k/\alpha] < -\eta_1, \\ &= 0 \text{ if } [\hat{q} - r(1+\underline{G}) + k/\alpha] > \eta_1, \\ &= 1 - [\hat{q} - r(1+\underline{G}) + k/\alpha + \eta_1]/2\eta_1 \text{ otherwise.} \end{aligned} \tag{A.2.2}$$

Similarly, we have

$$\begin{aligned} P_\gamma &= 1 \text{ if } [\hat{q} - r(1+\underline{G}) + k/\gamma] > \eta_1, \\ &= 0 \text{ if } [\hat{q} - r(1+\underline{G}) + k/\gamma] < -\eta_1, \\ &= [\hat{q} - r(1+\underline{G}) + k/\gamma + \eta_1]/2\eta_1 \text{ otherwise.} \end{aligned} \tag{A.2.3}$$

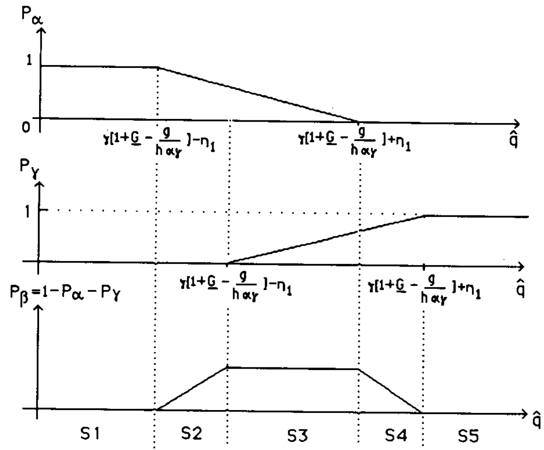


Fig. 4.

Equations (A.1.10), (A.2.2), and (A.2.3) then imply a five-segment P_β given by

$$\begin{aligned} P_\beta &= 0 \text{ if } \hat{q} \le [r(1+\underline{G}) - k/\alpha - \eta_1], \\ &= [\hat{q} - r(1+\underline{G}) + k/\alpha + \eta_1] \\ &\quad \cdot \text{ if } [r(1+\underline{G}) - k/\alpha - \eta_1] < \hat{q} \le [r(1+\underline{G}) - k/\gamma - \eta_1], \\ &= k[1/\alpha - 1/\gamma]/2\eta_1 \text{ if } [r(1+\underline{G}) - k/\gamma - \eta_1] \\ &\quad < \hat{q} \le [r(1+\underline{G}) - k/\alpha + \eta_1], \\ &= 1 - [\hat{q} - r(1+\underline{G}) + k/\gamma + \eta_1]/2\eta_1 \\ &\quad \cdot \text{ if } [r(1+\underline{G}) - k/\alpha + \eta_1] < \hat{q} \le [r(1+\underline{G}) - k/\gamma + \eta_1], \\ &= 0 \text{ otherwise.} \end{aligned} \quad \text{(A.2.4)}$$

The situation is depicted in Fig. 4, where the five segments of $\hat{q}$ are denoted by S_1 through S_5, respectively. For technical convenience and also justified by experimental data, we assume that

$$\eta_1 > 0.61k(1/\alpha - 1/\gamma). \quad \text{(A.2.5)}$$

Note that the above condition implies that S_3 is not an empty set.

To evaluate $E[U|\alpha]$, we have [by making use of (A.1.1), (A.1.3), and (A.1.4)]

$$E[U|\alpha] = (\beta - \alpha)\hat{q} + \alpha r(1+\underline{G}) - \alpha k + k \ln(k/\alpha r) + \alpha E[\eta|\alpha]. \quad \text{(A.2.6)}$$

The value of the conditional expectation $E[\eta|\alpha]$ depends on the selection of $\hat{q}$. To illustrate the idea, we shall consider only the case where $\hat{q}$ belongs to S_3, i.e.,

$$[r(1+\underline{G}) - k/\gamma - \eta_1] < \hat{q} \le [r(1+\underline{G}) - k/\alpha + \eta_1]. \quad \text{(A.2.7)}$$

In this case,

$$E[\eta|\alpha] = [\hat{q} - r(1+\underline{G}) + k/\alpha + \eta_1]/2. \quad \text{(A.2.8)}$$

Substituting (A.2.8) into (A.2.6), we obtain

$$E[U|\alpha] = (\beta - \alpha/2)\hat{q} + \alpha r(1+\underline{G})/2 - k/2 + \alpha\eta_1/2 + k \ln(k/\alpha r). \quad \text{(A.2.9)}$$

Similarly, for γ and β modes we have

$$E[U|\gamma] = (\beta - \gamma/2)\hat{q} + \gamma r(1+\underline{G})/2 - k/2 + \gamma\eta_1/2 + k \ln(k/\gamma r) \quad \text{(A.2.10)}$$

and

$$E[U|\beta] = \beta\hat{q} + k\alpha\gamma\,\{[\ln(k/\alpha r)]/\alpha - [\ln(k/\gamma r)]/(\gamma - \alpha) - k. \quad \text{(A.2.11)}$$

Consequently, $E[U]$ can be obtained by using (A.2.1).

Our goal is to find the $\hat{q}$ that maximizes $E[U]$. Note that the $\hat{q}$ axis is divided into five segments, and the computation of $E[U]$ depends on which segment $\hat{q}$ belongs to. We first obtain the best $\hat{q}$ for each segment, and then select the best one from the resulting five. To find the best $\hat{q}$ in S_3, we set $\partial E[U]/\partial \hat{q} = 0$, where $E[U]$ is computed for $\hat{q} \in S_3$. After some manipulations we obtain (3.7). Condition (A.2.5) ensures that this $\hat{q}$ belongs to S_3. It turns out that this $\hat{q}$ is also the best if we include all other segments. Therefore, the optimal $\hat{q}$ is given by (3.7).

3. The Evaluation of $E[q]$

To evaluate $E[q]$, we write it as

$$E[q] = P_\alpha E[q_\alpha|\alpha] + P_\gamma E[q_\gamma|\gamma] + P_\beta E[q_\beta|\beta]. \quad \text{(A.3.1)}$$

The variables q_α, q_γ, and q_β are given by (A.1.4), (A.1.8), and (A.1.11), respectively. The probabilities P_α, P_γ, and P_β are given by (A.2.2), (A.2.3), and (A.2.4), respectively. After some manipulations we obtain

$$E[q] = r(1+\underline{G}) + 0.33k[\hat{q} - r(1+\underline{G})]/\beta\eta_1 - 1.10k/\beta + 0.36(k/\beta)^2/\eta_1 \quad \text{(A.3.2)}$$

for any $\hat{q} \in S_3$. For the optimal $\hat{q}$ given by (3.7), the above equation reduces to (3.8).

Appendix B
The Stagewise Bonus Maximizing Property of the Parameter Adjustment of Process of Section IV

In this Appendix we assume that F is a bonus maximizer under a deterministic production environment and a single adjustment of the bonus coefficient "b" and the origin (conservation quota) "a" is to be made. We wish to show a certain steady-state property of the adjustment scheme. Let us define the following:

$a(t)$ = the quota of water conserved at period t

$q(t)$ = the actual saving at period $t (= q(t)$ as steady state is postulated)

$b(t)$ = F's bonus of period $t = \beta(t)(q(t) - a(t))$ for steady state

$q_M(t)$ = the maximal achievable conservation level at period t.

Our adjustment scheme is

$$a(t) = a(t-1) + \lambda(q(t-1) - a(t-1)) \quad \text{(B.1)}$$

$$\beta(t) = \beta(t-1)/(1-\lambda). \quad \text{(B.2)}$$

It is easy to confirm that (B.1) and (B.2) satisfy the following properties:

$$\text{if } q(t) = q(t-1) \text{ then } b(t) = b(t-1)$$

$$\text{if } q(t) > q(t-1) \text{ then } b(t) = \beta(t)(q(t) - a(t)) > b(t-1) = \beta(t-1)(q(t) - a(t-1))$$

$$\text{if } q(t) < q(t-1) \text{ then } b(t) = \beta(t)(q(t) - a(t)) < b(t-1) = \beta(t-1)(q(t) - a(t-1)).$$

We next prove that under this scheme, although the quota will be increased, the follower would not limit his conservation. In other words, it is in his self-interest to strive for his maximal conservation at every period such that his bonus will be maximized. Since we are only adjusting (β, a) once, there are two possibilities for the follower in this two-stage problem. One is $q(1) = q_M(1)$, and $q(2) = q_M(2)$ [$q_{M(1)}$ may or may not be equal to $q_{M(2)}$]. The other possibility is $q(1) < q_M(1)$ and $q(2) = q_M(2)$, i.e., the follower holds back his effort until b is increased at the second stage. For the first case

$$b=\beta(1)(q(1)-a(1))+\beta(2)(q(2)-a(2))=b(1)(q_M(1)-a(1))+\beta(2)(q_M(2)-a(2)).$$

For the second case

$$b'=\beta(1)(q(1)-a(1))+b(2)(q(2)-a(2)')$$

where because $q(1) \neq q_M(1)$, $a(2)' = a(1) + \lambda(q(1) - a(1)) \neq a(2)$ from (B.1). Noticing that $\beta(2) = \beta(1)/(1 - \lambda)$, $a(2) = a(1) + \lambda(q(1) - a(1))$, we have

$$\begin{aligned} b-b' &=\beta(1)(q_M(1)-a(1))+\beta(2)(q_M(2)-a(2)) \\ &\quad -\beta(1)(q(1)-a(1))-\beta(2)(q_M(2)-a(2)') \\ &=\beta(1)(q_M(1)-q(1))+\beta(2)(a(2)'-a(2)) \\ &=\beta(1)(q_M(1)-q(1))+(b(1)/(1-\lambda))(a(1) \\ &\quad +\lambda(q(1))-a(1)-\lambda(q_M(1)-a(1))) \\ &=\beta(1)(q_M(1)-q(1))+(b(1)/(1-\lambda))\lambda(q(1)-q_M(1)) \\ &=\beta(1)(q_M(1)-q(1))(1-2\lambda)/(1-\lambda). \end{aligned}$$

Because $q_M(1) > q(1)$, as long as $\lambda < 1/2$, then $b - b' > 0$. In other words, if $\lambda < 1/2$, the follower will always maximize his stagewise bonus. This argument can be extended to the case of $n - 1$ adjustments where the analogous condition is $\lambda < 1/n$. The algebra is straightforward but messy. We omit the details.

Appendix C
Proofs Relating to the Variance Reduction Property

In this section, the general stochastic production function of (3.1) and the additive utility function of (5.2) are assumed. We first derive necessary conditions in selecting $x(t)$ [or equivalently $q(t)$] under the β incentive scheme, and then under the $\alpha\beta\gamma$ incentive scheme. In both cases we assume that F knows $\eta(t)$ in selecting $x(t)$. By examining these necessary conditions, we then show that the variance of $q(t)$ under the $\alpha\beta\gamma$ incentive is in general smaller than that under the β incentive scheme. We finally prove that the above statement is correct for the special model described by (3.4)–(3.6). For notational simplicity, the time argument t will be suppressed in this Appendix.

Consider first the selection of x under the β incentive scheme. The problem

$$\text{Max } U(b, x), \qquad \text{subject to } b=\beta q=\beta G(x, \eta) \tag{C.1}$$

bears no particular interest unless U and G are specified. On the other hand, in the (b, q)-plane with $V(b, q, \eta) \equiv U(b, \Gamma(q, \eta))$, the solution to

$$\text{Max } V(b, q, \eta), \qquad \text{subject to } b=\beta q \tag{C.2}$$

consists of those points (b^*, q^*) where the tangent lines to the level curves of $V(b, q, \eta)$ coincide with the line $b = \beta q$. The implicit equation of such tangent lines is given by the differential equation

$$dV=\partial V/\partial b db+\partial V/\partial q dq=0 \tag{C.3}$$

($d\eta = 0$ since η = constant). Thus,

$$db/dq=-(\partial V/\partial q)/(\partial V/\partial b) \tag{C.4}$$

gives the slope of such a tangent line at any point (b, q, η) where $\partial V/\partial b$ is not zero, and the equation

$$-(\partial V/\partial q)/(\partial V/\partial b)=\beta \tag{C.5}$$

defines the locus of points where this slope is β. We call such a locus the β curve. Of course, the solution set to problem (C.2) coincides with the intersection of the β curve with the line $b = \beta q$. Whenever this intersection exists and is unique, we write (b^β, q^β) for this point (the superscript β denotes that it is the solution under the β incentive scheme).

With the additive utility function of (5.2), we have

$$db/dq=[dH(\Gamma(q; \eta))/dx][\partial\Gamma(q; \eta)/\partial q] \tag{C.6}$$

which is independent of b. This implies that the β slope line in this case is a straight line parallel to the b axis.

Now consider the $\alpha\beta\gamma$ incentive scheme. Similar to the β slope line, we define the $\alpha(\gamma)$ slope line as the curve where at every point on it the slope of the corresponding level curve equals $\alpha(\gamma)$. By following the same procedure, we conclude that the $\alpha(\gamma)$ slope line is a straight line parallel to the b axis under the additive utility function of (5.2). In view of the basic assumption on $U(b, x)$ [(1.1) and (1.2)], it is clear that the γ slope line lies to the right of the β slope line, which in turn lies to the right of the α slope line. Furthermore, in α mode (i.e., the case where $q > \hat{q}$ as defined at the beginning of Appendix A), the solution $(b_\alpha^{\alpha\beta\gamma}, q_\alpha^{\alpha\beta\gamma})$ is the intersection of the α slope line and the α segment of the incentive scheme, as shown in Fig. 5. Similarly, in γ mode (i.e., the case where $q < \hat{q}$), the solution $(b_\gamma^{\alpha\beta\gamma}, q_\gamma^{\alpha\beta\gamma})$ is the intersection of the γ slope line and the γ segment of the incentive scheme. It is also clear that in β mode, $q_\beta^{\alpha\beta\gamma} = \hat{q}$, and $b_\beta^{\alpha\beta\gamma} = \beta\hat{q}$. As η varies, these slope lines shift to the right or to the left, causing the resulting b and q to change.

From the above reasoning, it is easy to see that for any η, we have

$$\hat{q}<q_\alpha^{\alpha\beta\gamma}(\eta)<q^\beta(\eta) \text{ in the } \alpha \text{ mode,} \tag{C.7}$$

$$\hat{q}>q_\gamma^{\alpha\beta\gamma}(\eta)>q^\beta(\eta) \text{ in the } \gamma \text{ mode, and} \tag{C.8}$$

$$\hat{q}=q_\beta^{\alpha\beta\gamma}(\eta) \text{ in the } \beta \text{ mode.} \tag{C.9}$$

Since (C.7)–(C.9) cover all the possible realizations of η, (5.3) then follows. To derive (5.4), we note that

$$\begin{aligned} \text{VAR } q^\beta &=E[(q^\beta-\bar{q}^\beta)^2] \\ &=E[((q^\beta-\hat{q})+(\hat{q}-\bar{q}^\beta) \\ &=E[(q^\beta-\hat{q})^2]-(\hat{q}-\bar{q}^\beta)^2. \end{aligned}$$

Similarly,

$$\begin{aligned} \text{VAR } q^{\alpha\beta\gamma} &=E[(q^{\alpha\beta\gamma}-\bar{q}^{\alpha\beta\gamma})^2] \\ &=E[(q^{\alpha\beta\gamma}-\hat{q})^2]-(\hat{q}-\bar{q}^{\alpha\beta\gamma})^2. \end{aligned} \tag{C.11}$$

Using (C.10) and (C.11), we then get (5.4) after some lengthy manipulations.

For the special model described by (3.4)–(3.6), $\hat{q}$ is given by (3.7); $q_\alpha^{\alpha\beta\gamma}$, $q_\gamma^{\alpha\beta\gamma}$, and $q_\beta^{\alpha\beta\gamma}$ are described by (A.1.4), (A.1.8), and (A.1.11), respectively; and $\bar{q}^{\alpha\beta\gamma}$ is given by (3.8). It can be easily shown that

$$q^\beta=r(1+\underline{G})-k/\beta+\eta$$

and

$$\bar{q}^\beta=r(1+\underline{G})-k/\beta.$$

After some lengthy manipulations, we obtain

$$\text{VAR } q^\beta-\text{VAR } q^{\alpha\beta\gamma}=-0.14(k/\beta)^2+0.41\eta_1 k/\beta +0.25\eta_1^2+0.0004(k/\beta)^3/\eta_1+0.0005(k/\beta)^4/(\eta_1)^2 \tag{C.14}$$

which is greater than zero under condition (3.9).

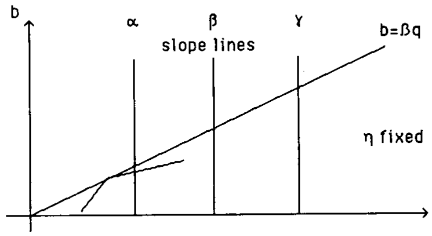


Fig. 5.

TABLE I
DATA FOR q AND q

Team		Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	May	June
1	q	3	3.5	4	5	7	7	7	7.1	6.5	7
	q	4.53	6.1	5.13	8.72	8.25	7.08	7.24	8.6	5.12	6.14
2	q	3	3.5	4	5	7	7	7	7.1	7	7
	q	4.49	6.23	6.35	8.19	7.22	7.63	6.82	7.58	6.57	6.27
3	q	3	4	4	5	7	7	7	7.1	7	7
	q	1.89	5.95	5.4	7.49	7.26	7.98	7.34	6.88	7.4	7.28
4	q	3	4	4	5	7	7	7	7.1	7	7
	q	3.31	6.15	6.06	7.67	6.86	7.34	6.53	7.7	6.51	6.26

Appendix D
Data Analyses

As we discussed in Section III, F's behavior is described by a learning model with four parameters (q_M, ρ, k, and η_1) responding to one incentive parameter β. From (3.13) we see that if we can determine the four parameters, then we can establish the model and predict the F's average behavior for future times. In this Appendix we will give the method for the determination of these parameters from experimental results. The month-by-month conservation data for all teams are shown in Table I.

A. Parameter #1, η_1

In the real world, as in our experiment, we postulate there are two kinds of uncertainties. One is dependent on F's effort x and is under F's control. Examples are F's miscalculation, errors in execution, psychological factors, misunderstanding the incentive scheme, and so on. The other is independent of F's effort x and beyond his control. Examples are the random state of nature, such as the outside temperature which affects that of the cooling water, the amount of rubber production, and even the last shift's operation. Careful observation reveals that the former kind of uncertainties is a temporary active factor. Its persistent time is short but its range is large. In contrast, the latter is a stable active factor and its range is not large. In other words, the latter has smaller variance than the former. It is this latter kind of uncertainty we wish to represent using η. Based on this consideration we propose to discriminate η from other kinds of uncertainties as follows.

Typical experimental data for a team in one month is illustrated in Table II.

The following steps are then taken.

1) Among the data for $w = W/R$ in a month, discard out-of-field points based on their standard deviation δ. For example, according to the 2δ rule, i.e., if $w > w + 2\delta$ or $w < w - 2\delta$, then w is an out-of-field point where w is the sample average of per unit water consumption.

2) After discarding such points, recompute the mean and variance for the remaining monthly data.

3) Repeat step 1) and discard out-of-field points until no more data points can be discarded.

From the data of w in Table II we can see some potential out-of-field points. After processing these data as described we found that 4 points are discarded. The results are $w = 8.680556$ instead of 8.775 which is based on 22 points and $\delta^2 = 1.953939$.

TABLE II

Team #1 Aug. '84	Tons of rubber produced R	Tons of water consumed W	Water consumption per unit of production w=W/R
1	19.75	74	3.75
2	22.875	175	7.65
3		day off	
4		day off	
5	26.625	174	6.54
6	26.125	310	11.87
7	24.125	223	9.14
8	19.5	276	14.15
.	......	.....	....
.	......	....	....
26	19.375	127	6.55
27		day off	
28		day off	
29	31.50	279	8.86
30	27.5	429	15.6

TABLE III
(a) FOR TEAM 1 (b) FOR TEAM 2 (c) FOR TEAM 3 (d) FOR TEAM 4

Month	w	δ^2	# of points before	# of points after
8/84	8.680556	1.953939	22	18
9/84	6.034118	1.38057	22	17
10/84	7.717777	2.71809	20	18
11/84	5.436,	1.604283	23	10
12/84	4.321667	0.3939862	23	12
1/85	5.7665,	1.130846	22	20
2/85	5.74533	1.044293	21	15
3/85	4.89117	0.8913715	24	17
5/85	6.448334	0.7425506	18	12
6/85	6.7395	1.832006	22	20

mean of δ_η^2=1.3692

* April is annual plant-wide equipment overhaul and cleaning month; hence no experiment were conducted for any team.

(a)

Month	w	δ^2	# of points before	# of points after
8/84	8.529473	2.048394	22	19
9/84	6.57125	1.502612	20	16
10/84	6.686471	2.104084	22	17
11/84	4.794	0.9385498	21	15
12/84	6.346429	1.965328	22	14
1/85	5.001765	1.131513	21	17
2/85	5.924376	0.9527166	22	16
3/85	6.005625	1.078942	22	16
5/85	5.813334	0.425144	17	12
6/85	6.587778	1.475847	22	18

mean of d_h^2=1.36005

(b)

Month	w	δ^2	# of points before	# of points after
8/84	11.76159	4.251003	22	22
9/84	7.979412	3.165549	21	17
10/84	7.16375	2.32791	20	16
11/84	5.679334	1.299627	22	15
12/84	5.825001	1.751245	22	14
1/85	4.719375	0.9062195	23	16
2/85	5.267857	0.7118047	21	14
3/85	5.599	1.320851	23	20
5/85	5.210769	1.334285	18	13
6/85	5.962106	1.691091	21	19

mean of δ_η^2=1.875959

(c)

Month	w	δ^2	# of points before	# of points after
8/84	8.91833	2.761764	21	18
9/84	6.494499	1.52366	23	20
10/84	6.228125	1.497596	20	16
11/84	5.647857	1.341683	24	14
12/84	5.230001	1.065581	22	15
1/85	5.092	1.026903	23	20
2/85	6.136667	1.648998	20	18
3/85	4.87375	0.5296193	24	16
5/85	5.606667	0.955872	19	12
6/85	6.395294	1.344397	20	17

mean of δ_η^2=1.369607

(d)

Following the same procedure we process all the data from August 1984 to June 1985 for team 1, the results are as shown in Table III (a). Similar results for the other three teams are shown in Tables III (b)–(d).

In our calculation we also throw out the point: $w \leq 4 = w_s$

TABLE IV
VALUES OF δ_η^2

team no.	1	2	3	4
8/84-1/85	1.530	1.614	2.283	1.536
8/84-5/85	1.369	1.360	1.876(1.288)	1.369

TABLE V

Team no.	1		2		3		4	
month	pred.	actual	pred.	actual	pred.	actual	pred.	actual
Feb.	7.504	7.255	7.367	7.08	7.464	7.732	7.678	6.863
March	7.506	8.109	7.371	7.0	7.555	7.4	7.690	8.126
May	7.507	6.552	7.372	7.19	7.603	7.79	7.695	7.43
June	7.508	6.2605	7.372	6.412	7.629	7.04	7.697	6.605

where w_s is the absolute limit of water consumption based on an independent measurement. Because of the possible meter reading errors (0.5 unit of graduation), we actually take $w_s = 3.5$. Examining the results of Tables III we note that δ_η^2 are nearly the same for all teams except team 3. Now if we apply the same 2δ rule to the monthly data of team 3 in Table III (c), discarding out-of-field points of w, then we have 7 remaining points comprising the months of November 1984 through May 1985. The average of δ^2 now equals 1.287876 more nearly equal to that of the other teams. These results are summarized in Table IV.

Since the variance of a uniformly distributed η is $\delta_\eta^2 = \eta_1^{2/3}$, then we can calculate $\eta_1 = 3\delta_\eta^2$. In Table IV if we use the data from the first six months and discard the value for team 3, then we get the average value of $\delta_\eta^2 = 1.56$ and $\eta_1 = 2.16$. On the other hand, using the ten month data we have $\delta_\eta^2 = 1.35$ and $\eta_1 = 2.01$ which is the value we determined for η_1.

Other Parameters

From (3.13) we have,

$$E[q(t)] = y - q_M \rho^t$$

where $q_M = r_M(1 + G)$ and $y = q_M - 1.1(k/b) + 0.022(k/b)^2/\eta_1$. Taking the experimental value of $q(t)$ for different t as noisy values for $E[q(t)]$, we use the least-squares method to find the minimum of $\Sigma(q(t) - y + q_M\rho^t)^2$. From the fitted y we can in turn calculate k/b. The results using the first six months data from four teams are given in (3.14).

These values are then used via (3.13) to predict the value of $q(t)$ for the next four months with the results shown in Table V.

REFERENCES

[1] J. S. Berliner, "General incentives and decision rules," in *The Innovation Decisions in Soviet Industry.* Cambridge, MA: M.I.T. Press, 1976, ch. 14.

[2] Y.-C. Ho, P. B. Luh, and G. J. Olsder, "A control theoretic view of incentives," *Automatica,* vol. 18, no. 2, pp. 167–179, 1982.

[3] Y.-C. Ho, P. B. Luh, J.-M. Wu, Y.-P. Zheng, and Q.-Y. Wang, "On incentives and their applications to production," in *Proc. 1985 Amer. Contr. Conf.,* Boston, MA, June 1985, pp. 237–244.

[4] P. B. Luh, Y.-C. Ho, Y.-P. Zheng, J.-M. Wu, Q.-Y. Wang, and L. Chen, "Incentives in production, A case study in modeling and control of socio-economic systems," in *Proc. IFAC 10th World Congress,* Munich, FRG, July/Aug. 1987.

[5] B. Holmstrom, "Design of incentive schemes and the new Soviet incentive model," *European Econom. Rev.,* vol. 17, no. 2, pp. 127–148, Feb. 1982.

[6] R. Radner, "Incentive and control in economic organizations," in *Proceeding of the 9th IFAC World Congress.* New York: Pergamon, 1984.

[7] M. Weitzman, "The new Soviet incentive model," *Bell J. Econom.,* vol. 7, no. 1, pp. 251–257, 1976.

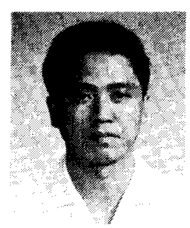
[8] ——, "The Ratchet principle and performance incentives," *Bell J. Econom.,* vol. 11, no. 1, pp. 302–308, 1980.

Yu-Chi "Larry" Ho (S'54–M'55–SM'62–F'73) received the S.B. and S.M. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1953 and 1955 and the Ph.D. degree in applied mathematics from Harvard University, Cambridge, MA, in 1961.

After graduation, he worked for three years in industry before returning to Harvard for graduate studies. He has been on the Harvard Faculty since 1961. He is currently Gordon McKay Professor of Engineering and Applied Mathematics and Chairman of the Committee on Higher Degrees in Business Studies, a joint Ph.D. program between the Harvard Business School and the Faculty of Arts and Sciences. He has published over 100 articles and two books, one of which has been translated into both Russian and Chinese.

Dr. Ho is on the Editorial Board of seven international journals. He was a Guggenheim Fellow in 1970 and USA–USSR Senior Exchange Fellow in 1973. He was elected Fellow of the IEEE in 1972, and elected to distinguished membership of the IEEE Control Systems Society in 1985, and to membership in the U.S. National Academy of Engineering in 1987.

Peter B. Luh (S'76–M'80), photograph and biography not available at the time of publication.



Ying-Ping Zheng was born in China in 1941. He graduated from the Department of Mathematics and Mechanics, Peking University, Peking, People's Republic of China, in 1963, and completed graduate study at the Institute of Automation, Academia Sinica, Peking, in 1967, specializing in automatic control theory and primarily adaptive control.

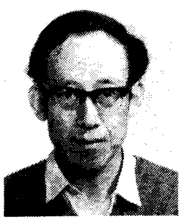
Since 1967 he has been involved in research on control theory at the Academia Sinica, where he is currently a Research Professor. He has visited and worked at the Coordinated Science Laboratory, University of Illinois, Urbana, from 1981 to 1983, and the Division of Applied Sciences, Harvard University, Cambridge, MA, in 1984. His research interests include the areas of linear system, optimal control, general system theory, differential game, especially incentive theory with applications to management decision-making, and manufacturing automation.

Dr. Zheng is currently a member of the Transactions on Theory, IFAC, Deputy Secretary General, and Vice-Chairman of the Transactions on Theory of the Chinese Association of Automation.

Jong-Ming Wu was born in Fenghua County, Zhejiang Province, People's Republic of China on January 20, 1930. He received the Engineer degree from the Department of Electrical Engineering, Harbin Polytechnic Institute, in 1955.

He was with Harbin Polytechnic Institute from 1955 to 1960. As Deputy Director of the Computer Teaching and Research Section, he was the founder of the computer speciality at HIT. From 1960 to 1971 he was in the 9th Research and Design Institute and served as Head of the Computer Department. From 1972 to 1986 he was with the Institute of Automation, Academia Sinica, Peking, where he served as Head of the Control Systems, and the Information Control Systems Laboratories. Since 1987 he has been Chairman of the Department of Computer Science and Technology, Beijing Information Technology Institute. His research interests include computer digital control, database concurrent control, database security, systems analysis, management information systems, and office automation.

Prof. Wu is very active with the Chinese Association of Automation, NOM of IFAC. He was Vice-Secretary General of CAA from 1980 to 1985, IFAC EMSCOM member from 1983 to 1987, and main organizer of the Bilateral Meeting on Control Systems in Shanghai in 1981 and the IFAC CSTD'85 in Beijing.



Qing-Yu Wang received the B.S. degree in electronic engineering from the University of Science and Technology of China, Peking, People's Republic of China, in 1968.

He participated in the Sino-American Joint Project on Incentives and Their Applications in China from 1983 to 1986. His interests are in modeling, decision, and management science.

Liang Chen received the B.S. degree in mathematics from the Beijing Institute of Communication, Beijing, People's Republic of China, in 1981, and the M.S. degree in automatic control from Peking University, Peking, in 1984.

He worked as a Research Assistant and then Research Associate at the Institute of Automation, Academia Sinica, Peking, from 1984 to 1986. Currently he is pursuing the Ph.D. degree in the Department of Industrial Engineering, University of Wisconsin at Madison. His research interests are in dynamic game theory and its applications to management and manufacturing automation.