Hydrothermal Scheduling Via Extended Differential Dynamic Programming and Mixed Coordination*

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Abstract

This paper addresses short-term scheduling of hydrothermal systems by using extended differential dynamic programming and mixed coordination. The problem is first decomposed into a thermal subproblem and a hydro subproblem by relaxing the supply-demand constraints. The thermal subproblem is solved analytically. The hydro subproblem is further decomposed into a set of smaller problems that can be solved in parallel. Extended differential dynamic programming and mixed coordination are used to solve the hydro subproblem. Two problems are tested and the results show that the new approach performs well under a simulated parallel processing environment, and high speedup is obtained. The method is then extended to handle unpredictable changes in natural inflow by utilizing the variational feedback nature of the control strategy. A quick estimate on the impact of an unpredictable change on total cost is also obtained. Numerical results show that estimates are accurate, and unpredictable change in natural inflow can be quickly and effectively handled.

Key words: hydrothermal generation scheduling, extended differential dynamic programming, mixed coordination.

1 Introduction

Hydrothermal systems are operated by economical scheduling to produce minimum cost for thermal generation, subject to hydraulic and thermal constraints, and the demand for electrical energy. Hydraulic and thermal constraints may include operational limits on hydro and thermal generation, reservoir storage, water discharge and spillage. The short-term scheduling horizon is normally one day to one week, and the time unit is one hour, with complete river flows and load demand generally assumed. This paper addresses the short-term hydrothermal generation scheduling problems.

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Different methods have been proposed for the solution of these problems in the past. Variational methods [7], Pontryagin's maximum principle [8], general mathematical programming [1] [9] and dynamic programming [2] [19] [20] have been used to solve the problem in different formulations. Among all the methods, dynamic programming seems to be a popular approach because it is usually satisfactory to find the economic dispatch at discrete load steps rather than at continuous load levels. A serious drawback with dynamic programming, however, is that computational requirements grow drastically as the dimension of the problem increases.

Beyond obtaining the optimal solution, a very important consideration is the handling of unpredictable changes in natural inflow. If a schedule has to be re-calculated from scratch whenever a change occurs, it is inefficient and may be impractical. How to estimate the impact of a change on total cost, and to develop a quick and effective way to handle the change are of great concern to practitioners. These issues, however, have seldom been addressed.

Based on these reasons, this paper proposes to solve the short-term hydrothermal scheduling problem by using the extended differential dynamic programming (EDDP) and mixed coordination approach. The problem is decomposed into a thermal subproblem and a hydro subproblem. The thermal subproblem is solved analytically and the hydro subproblem is further decomposed into a set of smaller problems that can be solved in parallel. EDDP and mixed coordination method are used to solve the hydro subproblem. Advantages offered by our approach are

(1) The dimension problem associated with dynamic programming is avoided by using EDDP.

(2) With the choice of EDDP and mixed coordination method, and by taking advantage of the cost-effective parallel processing technologies, substantial speedup can be obtained under a parallel processing environment.

(3) The proposed approach can provide a quick and accurate estimate on the impact of a change in natural inflow on the total cost, and can also effectively handle the change by exploiting the variational feedback nature of the control strategy.

In section 2, the problem formulation is presented. EDDP and the mixed coordination are discussed in section 3. In section 4, a system with four reservoirs and two thermal plants, and another system with ten reservoirs and two thermal plants, are tested. In Section 5, the handling of unpredictable changes in natural inflow is addressed together with illustrative examples. Conclusions are then given in Section 6.

2 Problem Formulation

A typical short-term hydrothermal generation scheduling and predispatch problem can be formulated as follows [5] [13] [16]:
(P):

$$min\ J,\ with\ J \equiv \sum_{t\in T}\sum_{i\in I}\Psi_i(g_{it}), \tag{1}$$

subject to

(a) the power balance equation

$$\sum_{i\in I} g_{it} + \sum_{j\in J} h_{jt} = d_t, \tag{2}$$

(b) bounds on thermal generation

$$\underline{g_i} \le g_{it} \le \overline{g_i}, \tag{3}$$

(c) the hydro generation

$$h_{jt} = \Phi_j(u_{jt}, x_{jt}), \tag{4}$$

(d) the reservoir dynamics

$$x_{j(t+1)} = x_{jt} + \sum_{l\in N^j} u_{l(t-\tau_{lj})} - u_{jt} + \sum_{l\in N^j} s_{l(t-\tau_{lj})} - s_{jt} + w_{jt} \tag{5}$$

with x_{j1} and $x_{j(T+1)}$ given,

(e) and bounds on reservoir storage, water discharge, and spillage

$$\underline{x_j} \le x_{jt} \le \overline{x_j},\ \underline{u_j} \le u_{jt} \le \overline{u_j},\ and\ \underline{s_j} \le s_{jt} \le \overline{s_j}, \tag{6}$$

where $i \in I$, $j \in J$, $t \in T$, and

g_{it}: thermal generation of plant i at hour t;
Ψ_i: operation cost of thermal plant i;
t,T: time index and number of hours;
i,I: thermal index and number of thermal plants;
j,J: hydro index and number of hydro plants/reservoirs;
d_t: load demand at hour t;
h_{jt}: hydro generation of plant j at hour t;
Φ_j: hydro generation function of plant j;
x_{jt}, u_{jt}, s_{jt}: storage, water discharge, and spillage of reservoir j at hour t;
τ_{lj}: water delay time between reservoir j and its upstream neighbor l;
w_{jt}: natural inflow to reservoir j at hour t;
N^j: the set of immediate upstream reservoirs of reservoir j;
$\underline{g}_i$, $\overline{g}_i$: lower and upper bounds of thermal generation of plant i;
$\underline{x}_j$, $\overline{x}_j$, $\underline{u}_j$, $\overline{u}_j$, $\underline{s}_j$, $\overline{s}_j$: lower and upper bounds of storage, water discharge, and spillage of reservoir j.

Using Lagrange multipliers $\{\lambda_t\}$ to relax constraint (2) as suggested in [13], one obtains

$$L = \sum_{t\in T}\left[\sum_{i\in I}\Psi_i(g_{it}) - \lambda_t\sum_{i\in I} g_{it} + \lambda_t d_t - \lambda_t \sum_{j\in J} h_{jt})\right]. \tag{7}$$

In this paper, Ψ_i and Φ_j are assumed to be convex as suggested in [13]. The water discharge with time delays from upstream reservoirs in constraint (5) are defined as state variables. Spillage is assumed to be zero for simplicity. Given Lagrange multipliers $\{\lambda_t\}$, problem (P) can now be decomposed into two subproblems.

Thermal subproblem (PT):

$$\min_{g_{it}} L_T, with L_T \equiv \sum_{t\in T}\left[\sum_{i\in I}\Psi_i(g_{it}) - \lambda_t \sum_{i\in I} g_{it} + \lambda_t d_t\right], \tag{8}$$

subject to (3).

Hydro subproblem (PH):

$$\min_{u_{jt},s_{jt}}\ L_H,\ with\ L_H \equiv \sum_{t\in T} L_{Ht} = \sum_{t\in T}\left[-\lambda_t \sum_{j\in J}\Phi_j(u_{jt}, x_{jt})\right], \tag{9}$$

subject to (5)-(6).

3 Solution Methodology

3.1 Solution of the Thermal Subproblem

To solve thermal subproblem (PT), note that once Lagrange multipliers $\{\lambda_t\}$ are given, the third term, $\lambda_t d_t$, in (8) is a constant. By taking the derivative of (8) with respect to g_{it}, one obtains

$$\frac{d\Psi_i}{dg_{it}} - \lambda_t = 0. \tag{10}$$

Therefore the solution to (PT) is

$$g^*_{it} = min\ \left\{\overline{g}_i,\ max(\underline{g}_i,\ \tilde{g}_{it})\right\}, \tag{11}$$

where $\tilde{g}_{it}$ satisfies (10).

3.2 Solution of the Hydro Subproblem

Since the major computational load is on (PH), it is further decomposed into smaller independent problems and parallel processing can be applied to speedup the computation. To facilitate further decomposition, one more subscript, n, is added to the notation in (PH). In the new notation, n is the subproblem index and t is the time index within each subproblem. Problem (PH) can now be written as

Hydro subproblem (PH'):

$$\min_{u_{jt}}\ L_H, with L_H \equiv \sum_{n\in N}\sum_{t\in T'} L_{Htn} \tag{12}$$

subject to (5) and (6) with corresponding subscript change and

$$x_{j(T'+1)n} = x_{j1(n+1)} \tag{13}$$

where N is the total number of subproblems, T' is the number of hours within each subproblem, and $T = N \cdot T'$. Equation (13) implies that the initial state of a subproblem equals the terminal state of the preceding subproblem so that problem (PH') is equivalent to (PH). Using Lagrange multipliers $\{\nu_{jn}\}$ to relax (13) and penalty function $\{P_{tm}\}$ to relax bound constraint (6), one has

Hydro subproblem (PH"):

$$\max_{\nu_{jn}}\ \min_{x_{j1(n+1)}}\ \min_{u_{jtn}}\ L'_H,\ with\ L'_H \equiv$$

$$\textstyle\sum_{n\in N}\left[\sum_{t\in T'} L_{Htn} + \nu_{jn}(x_{j1(n+1)} - x_{j(T'+1)n})\right.$$
$$\left. + \sum_{m=1}^{4J}\frac{1}{C_{tm}}\cdot P_{tm}(C_{tm}q_{tm}, \mu_{tm})\right]. \tag{14}$$

with $\nu_{jN} \equiv x_{j1(N+1)} \equiv 0$, subject to (5). In (14), μ_{tm} is the nonnegative Lagrange Multiplier, C_{tm} is the nonnegative penalty coefficient, and q_{tm} represents the bound constraints. The penalty function selected is taken from [6] and has the following form:

$$P_{tm}(C_{tm}q_{tm}, \mu_{tm}) = \begin{cases} \mu_{tm}C_{tm}q_{tm} + \mu_{tm}C_{tm}^2 q_{tm}^2 & if\ q_{tm} \geq 0 \\ \frac{\mu_{tm}C_{tm}q_{tm}}{1-C_{tm}q_{tm}} & otherwise \end{cases} \quad (15)$$

Selecting $\{\nu_{jn}\}$ and $\{x_{j1(n+1)}\}$ as high level coordinating variables, problem (PH") can be decomposed into the following N subproblems:

(PH-n), n=1,2,...,N:

$$\min_{u_{jtn}} L'_{Hn}, \quad with \quad L'_{Hn} \equiv \sum_{t\in T'} [L_{Htn} - \nu_{jn}x_{j(T'+1)1} + \sum_{m=1}^{4J} \frac{1}{C_{tm}} \cdot P_{tm}(C_{tm}q_{tm}, \mu_{tm})], \quad (16)$$

subject to (5). Note that subproblems (PH-n), n=1,2,...,N are completely decoupled and can be solved in parallel. Let $\{u^*_{jtn}(\nu_{jn},\ x_{j1n})\}$ denote the optimal controls of (PH-n) for the given coordinating variables $\{\nu_{jn}\}$ and $\{x_{j1n}\}$, and $L^*_{Hn}(\nu_{jn},\ x_{j1n})$ the corresponding cost. The high level is to find optimal $\{\nu_{jn}\}$ and $\{x_{j1(n+1)}\}$, i.e.,

(PH-H):

$$\max_{\nu_{jn}} \min_{x_{j1(n+1)}} L_{HH},\ with\ L_{HH} \equiv \sum_{n\in N} \left[L^*_{Hn} + \nu^T_{j(n-1)}x_{j1n}\right], \quad (17)$$

and $\nu_{j0} \equiv \nu_{jN} \equiv 0$. It has been shown in [15] that this two-level approach is equivalent to the original (PH).

EDDP is used to solve (PH-n). Differential dynamic programming (DDP) is a successive approximation technique for solving optimal control problems ([6], [11], [12], [17], [18]). The advantage of using DDP is that it can avoid the dimension problem associated with dynamic programming, and it also has quadratic convergence near optimum. The DDP of [18] is extended in [15] to represent variational cost-to-go functions explicitly in terms of all relevant coordination variables. It has been shown that the optimal variational feedback control is of the following form:

$$\delta u^*_{jtn} = \alpha_{jtn} + \beta_{jtn}\delta x_{jtn} + \gamma_{jtn}\nu_{jn}, \quad (18)$$

where $\alpha_{jtn}, \beta_{jtn}, \gamma_{jtn}$ are coefficients of appropriate dimensions. With the optimal variational feedback control given by (18), the resulting optimal variational cost-to-go function is of the following form:

$$V_{jtn}(\nu_{jn}, \delta x_{jtn}) = \delta x^T_{jtn}Q_{jtn}\delta x_{jtn} + \nu^T_{jn}R_{jtn}\delta x_{jtn} + \nu^T_{jn}S_{jtn}\nu_{jn} + U^T_{jtn}\delta x_{jtn} + Y^T_{jtn}\nu_{jn} + \Theta_{jtn}, \quad (19)$$

where $Q_{jtn}, R_{jtn}, S_{jtn}, U_{jtn}, Y_{jtn}$ are coefficients of appropriate dimensions, and Θ_{jtn} is the sum of all other terms not containing ν_{jn} or δx_{jtn}. Detailed derivations are found in the Appendix. Since (19) is the optimal variational cost-to-go at time t of subproblem n, the optimal variational cost for (PH-n) is $V_{j1n}(\nu_{jn}, \delta x_{jtn})$. As a result, first and second order derivatives of $L^*_{Hn}(\nu_{jn},\ x_{j1n})$ with respect to coordination variables ν_{jn} and x_{j1n} are readily available from coefficients of V_{j1n}.

The modified Newton's method [15] is selected to solve (PH-H) because of its quadratic convergence near optimum, and the readily available gradient and Hessian information provided by EDDP. The overall convergence of the algorithm can be concluded from the convergence of the multiplier method [3] and the convergence of the mixed coordination method [15]. This new algorithm for the hydro subproblem is denoted as COnstraint Relaxed MIXed coordination, or CORMIX for short, and is summerized below.

The CORMIX Algorithm.

Step 0 Set $\mu^0_{tm}=C^0_{tm}=E^0_{tm}=0$ for all t and m, where E_{tm} is a bound constraint violation indicator.

Step 1 Solve (PH-n) in parallel using EDDP and solve (PH-H) using the modified Newton's method.

Step 2 Check the violation of constraint q_{tm}. If $q_{tm} \leq \varepsilon$ for all t and m, stop. Otherwise go to step 3.

Step 3 Update μ_{tm} and C_{tm}.

- If $q^k_{tm} \leq 0$ and $E^k_{tm} = 0$, then μ^{k+1}_{tm}, C^{k+1}_{tm} and E^{k+1}_{tm} remain to be zeros.
- If $q^k_{tm} > 0$ and $E^k_{tm} = 0$, select $\mu^{k+1}_{tm} > 0$, $C^{k+1}_{tm} > 0$ and $E^{k+1}_{tm} = E^k_{tm} + 1$.
- If $q^k_{tm} \leq 0$ and $E^k_{tm} \neq 0$, then $\mu^{k+1}_{tm} = \nabla P_{tm}(C^k_{tm}q^k_{tm}, \mu^k_{tm})$, $C^{k+1}_{tm} = C^k_{tm}$ and E^k_{tm} remains the same.
- If $q^k_{tm} > 0$ and $E^k_{tm} \neq 0$, then $\mu^{k+1}_{tm} = \nabla P_{tm}(C^k_{tm}q^k_{tm}, \mu^k_{tm})$, $C^{k+1}_{tm} \geq C^k_{tm}$ and E^k_{tm} remains the same.
- Go to step 1.

Note that there is no need to increase C_{tm} to a very large number [3]. The ill-conditioning effect associated with large penalty coefficients can thus be eliminated or at least reduced.

After solving (PT) and (PH), $\{\lambda_t\}$ is updated using the steepest descent gradient method. This iterative process continues until $|L^{k+1} - L^k|/L^k \leq \zeta$ is satisfied.

4 Numerical Results

In this section, two problems are tested. Because of the nonlinear objective function adopted in the test problems, CORMIX is compared to the documented CORDDP algorithm presented in [6]. Both algorithms are written in FORTRAN and tested on a VAX 11/785 under a simulated parallel processing environment in the absence of a parallel processing system. It is assumed that the number of processors equals N, and communication times between processors are negligible. Synchronous processing is also assumed, i.e., (PH-H) will not begin the next iteration until all the (PH-n) problems are solved. The performance measure adopted is the speedup [4], $S_p \equiv \frac{T_s}{T_p}$ where T_s is the CPU time by using CORDDP and T_p by using CORMIX. The stopping criterion is $\zeta = 0.000001$.

Problem P_1. A four reservoir, two thermal plant system.

This problem is a slightly modified version of the one in [13] (see Fig. 1). For the thermal subsystem, the incremental cost of the thermal plants is

$$\frac{d\Psi_1}{dg_1} = 10 + g_1, \quad \frac{d\Psi_2}{dg_2} = -20 + 1.66g_2.$$

The bounds are (MWs)

$$10 \leq g_1 \leq 80, \quad 20 \leq g_2 \leq 80.$$

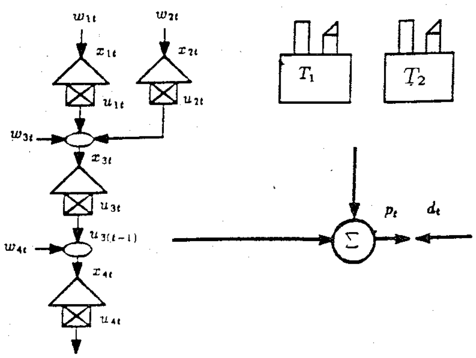

Fig.1. Test problem P_1

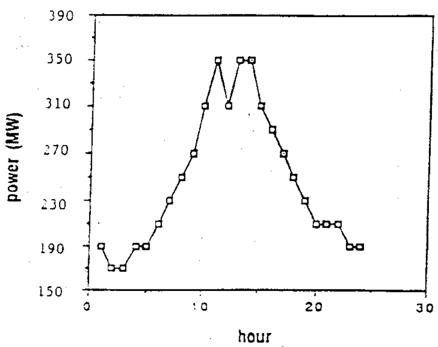

Figure 2. System load for problem P_1

For the hydro subsystem, nonzero time delays in (5) need to be considered as state variables because of the formulation of EDDP. The reservoir dynamics from Fig. 1 is

$$\begin{aligned} x_{1(t+1)} &= x_{1t} - u_{1t} + w_{1t} \\ x_{2(t+1)} &= x_{2t} - u_{2t} + w_{2t} \\ x_{3(t+1)} &= x_{3t} - u_{3t} + u_{1t} + u_{2t} + w_{3t} \\ x_{4(t+1)} &= x_{4t} - u_{4t} + u_{3(t-1)} + w_{4t} \end{aligned} \quad (20)$$

with a delay in u_3 going into reservoir 4. Defining $x_{5t} = u_{3(t-1)}$, the reservoir dynamics is modified below.

$$\begin{aligned} x_{1(t+1)} &= x_{1t} - u_{1t} + w_{1t} \\ x_{2(t+1)} &= x_{2t} - u_{2t} + w_{2t} \\ x_{3(t+1)} &= x_{3t} - u_{3t} + u_{1t} + u_{2t} + w_{3t} \\ x_{4(t+1)} &= x_{4t} + x_{5t} - u_{4t} + w_{4t} \\ x_{5(t+1)} &= u_{3t}. \end{aligned} \quad (21)$$

The hydro generation function for each plant is [13]

$$\Phi(u, x) = c_1 x^2 + c_2 u^2 + c_3 xu + c_4 x + c_5 u + c_6$$

The values of c_i for each plant are given in Table 1. The natural

Table 1: Hydro Generation Coefficients

	c_1	c_2	c_3	c_4	c_5	c_6
1	-0.001	-0.1	0.01	0.40	4.0	-30
2	-0.001	-0.1	0.01	0.38	3.5	-30
3	-0.001	-0.1	0.01	0.30	3.0	-30
4	-0.001	-0.1	0.01	0.38	3.8	-30

Table 2: Hydro Plant Data

	$\underline{x}$	$\overline{x}$	$\underline{u}$	$\overline{u}$	x_0	x_{25}
1	80	150	5	15	100	120
2	60	120	6	15	80	70
3	100	240	10	30	170	170
4	70	160	13	25	120	140

inflows are $w_t = [10810]^T$ $(\times 10^3 m^3/h)$ for all t. The scheduling horizon is 24 hours. The initial and terminal reservoir storage, and bounds on reservoir storage and water discharge are in Table 2 (units for x is $\times 10^3 m^3$ and unit for u is $\times 10^3 m^3/h$). Load demand d_t is given in Fig. 2.

CPU time and speedup are summarized in Table 3. The

Table 3: Test results for P_1

CORDDP			CORMIX			S_p
T(h)	T_s(s)	cost	$N \times T'$(h)	T_p(s)	cost	
24	37.9	$221,905	2×12	26.7	$221,905	1.41

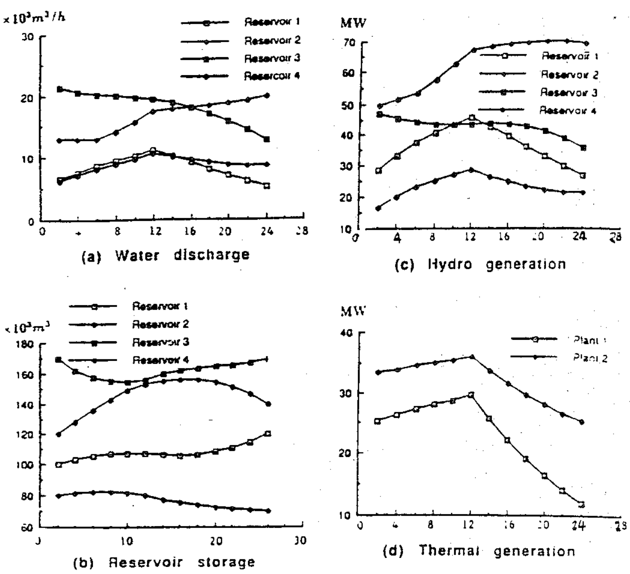

Fig.3. Test results for P_1

optimal water discharge, reservoir storage, and hydro and thermal generation are shown in Fig. 3. From Fig.3(a) it can be seen that the trajectories of water discharge for reservoirs 1 and 2 closely follow the pattern of the system load demand because they are the upstream reservoirs. Water discharge from reservoir 3 is the largest at the beginning because it has the highest reservoir storage and the hydro generation is proportional to both the water discharge and reservoir storage. The water discharge continues to be high until hour 12, when the load demand starts to drop, u_3 starts to drop in order to meet the terminal storage conditions. The water discharge from reservoir 4 continues to increase because it is down stream of reservoir 3. As a result of this, hydro generation h_4 continues to increase, keeping the thermal generation g_1 and g_2 down (Fig. 3(d)) and hence reducing cost.

Problem P_2. A ten reservoir, two thermal plant system (Fig. 4).

The system under consideration is owned and operated by the Taipower Company. However, contrived data (including system load profile, natural inflow, and cost coefficients) are used in the testing. The characteristics of the ten reservoirs are given in Table 4 with x and u having the same units as in P_1. The hydro generation function is the same as that in P_1. The system load profile for one week is shown in Fig. 5. The natural inflows are $w_t = [\ 50\ \ 0\ \ 20\ \ 0\ \ 50\ \ 0\ \ 0\ \ 20\ \ 10\ \ 20\ ]^T$ for all t ($\times 10^3 m^3/h$). Three cases are considered: two weekdays, three weekdays, and one week. The results are summarized in Table 5 . The optimal water discharge and hydro generation for the four major plants for a weekday in the third test case are also shown in Fig. 6.

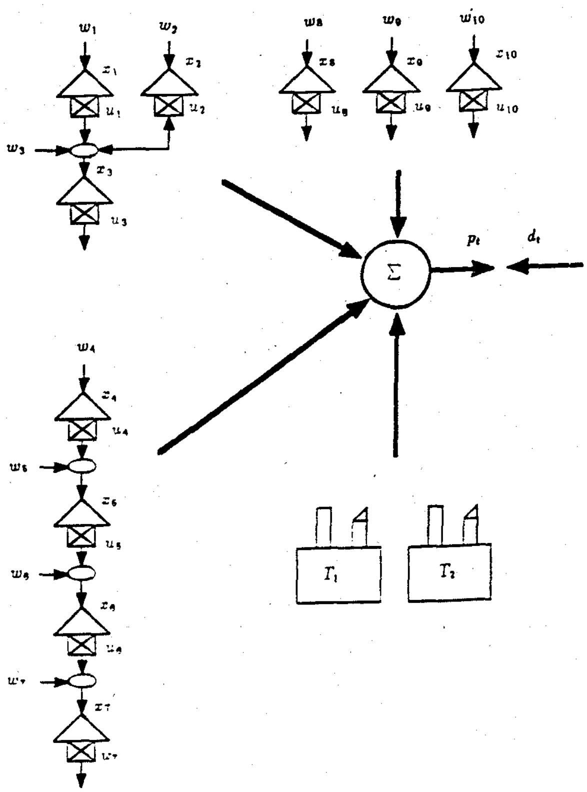

Fig.4. Test problem P_2

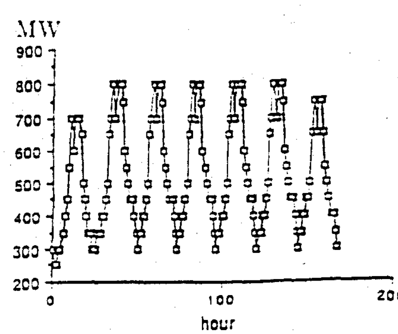

Fig.5. System load for P_2

Table 4: Characteristics of the ten reservoirs

	$\underline{x}$	$\overline{x}$	$\underline{u}$	$\overline{u}$	x_0
1	13269	155685	-249	285	80000
2	1565	9407	0	50	5000
3	1.6	205	0	45	50
4	89886	243120	0	217.5	100000
5	26	647	0	174.8	300
6	101	6563	0	133.6	3000
7	90	560	0	68	400
8	0	340	0	36.7	170
9	0	202	0	13.2	100
10	0	13430	0	31.7	700

Table 5: Test results for P_2

CORDDP			CORMIX			S_p
T(h)	T_s(s)	cost	NT'(h)	T_p(s)	cost	
48	239	$701,239	2×24	163	$701.235	1.47
72	305	$1,120,053	3×24	145	$1,120,051	2.10
168	1211	$2,611,352	7×24	389	$2,611,350	3.11

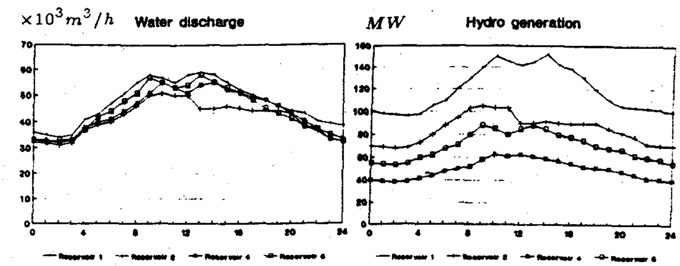

Fig.6. Test results for P_2

From Table 5, it can be seen that the CPU time is approximately linearly proportional to the scheduling horizon. With CORMIX, the CPU time is substantially reduced as the number of subproblems increases. The speedup is always less than the number of subproblems because of the uneven processing time for the subproblems and the time for (PH-H) updating.

For comparison, other nonlinear programming (NLP) techniques instead of EDDP were also used to solve (PH-n). One of them is the Davidon-Fletcher-Powell (DFP) method [10]. Although considered one of the best NLP techniques, DFP performed poorly compared with EDDP in solving (PH-n) with CPU time several times longer. This is because of the line search involved at each iteration and the heavy computional load in finding the gradient and Hessian information for (PH-H) [13].

5 Handling Unpredictable Changes in Natural Inflow

Up to now, the focus has been on the calculation of an optimal "nominal" schedule under a deterministic setting. In actual operation, however, there are many unexpected changes in natural inflow and in system demand. It is usually not clear what the impact of such a change is on total cost, and how the system should respond to it. It is impractical to regenerate the schedule whenever a change occurs. It would therefore be desirable to have a quick estimate about the impact of the change on total cost, and to have a systematic and effective way to handle the change without rescheduling from scratch every time.

In this section, a simple and effective way to handle changes in natural inflow is presented. The key idea is to use the optimal variational cost-to-go of EDDP to estimate the impact of a change on total cost, and to exploit properties of variational feedback controls to adjust water discharge. It is first assumed that change occurs only once in the scheduling horizon and the change is small so that the set of active constraints remain the same. Three cases are discussed, and for simplicity, subscript j is omitted.

Case 1. Suppose that the current time is t_1 within subproblem n and the change occurs at t_1. In this case, there is no need to redesign nominal states and controls for the future. From (5) one can see that the change on w_{nt_1} and δw_{nt_1} will be reflected through $x_{n(t_1+1)}$ in the form of $\delta x_{n(t_1+1)}$. The optimal variational cost-to-go $V_{nj(t_1+1)}(\nu_n, \delta x_{n(t_1+1)})$ provides a quick and accurate estimate about the impact of the change on total cost J. Furthermore, from the variational feedback control (18), we have

$$\delta u_{nt} = \alpha_{nt} + \beta_{nt}\delta x_{nt} + \gamma_{nt}\nu_n, \qquad (22)$$

with $\alpha_{nt} = 0$ as mentioned earlier. Thus if a component of $u_{n(t_1+1)}$ is on the bound and the direction of the variational component is to move against the bound, the component of $\delta u_{n(t_1+1)} = 0$. Otherwise $\delta u_{n(t_1+1)}$ is determined according to (22). From there, $\delta x_{n(t_1+2)}$, $\delta u_{n(t_1+2)}$,..., and so on can be similarly obtained at the on-line implementation phase. Consequently, no extra work is needed to redesign the schedule.

Case 2. Suppose that the current time is t_1 within subproblem n and the change occurs at $t_2 (> t_1)$ within the same subproblem. One example is the sudden intensification of a rainstorm in the forecast. In this case, it would be desirable to adjust water discharge from t_1 onwards, rather than wait till t_2 to react as discussed in Case 1. This look-ahead feature, however, can not be reflected by $\beta_{nt}\delta x_{nt}$ since $\delta x_{nt} = 0$. Examining the detailed steps of EDDP, it is found that the look-ahead feature can be obtained through the recalculation of α_{nt} for $t_1 \le t < t_2$. This calculation is done backwards in time from t_2 to t_1, and only involves matrix-vector manipulations, therefore computational requirements are small. After this, the revised optimal variational cost-to-go at t_1 provides an estimate about the impact of the change, and the process described in Case 1 can be used to calculate δu_{nt} and δx_{nt} for future stages during the on-line implementation phase.

Case 3. Suppose that the current time is t_1 within subproblem j, and the change occurs at t_2 of a future subproblem. For simplicity, assume that t_1 is in subproblem 1, and t_2 is in subproblem 2. The procedure described in Case 2 can be used to calculate α'_{2t}s for $t = t_2, t_2 - 1, ..., 1$ of subproblem 2. To propagate the effect backwards to subproblem 1, the initial state of subproblem 2, x_{21}, has to be adjusted by:

$$\min_{\delta x_{21}}[V_{21}(\nu_2, \delta x_{21}) + \nu_1 \delta x_{21}]. \qquad (23)$$

Once $\delta x_{21} = \delta x_{1(T+1)}$ is obtained, the effect can be propagated backwards within subproblem 1 following the procedure described in Case 2. The sum of revised V_{11} and V_{21} provides a quick and accurate estimate about the change in total cost because of δw_{2t_2}. The extension to the general case is obvious, and is omitted here.

Table 6: Comparisons on total cost

	Estimate	Algorithm	Error%
Case 1	\$ 218,255	\$ 218,108	0.069
Case 2	\$ 218,309	\$ 218,174	0.064
Case 3	\$ 218,507	\$ 218,416	0.041

The above discussions can be easily extended to the situation where multiple small changes occur. If a change (or changes) is large, the above estimate of the impact on total cost may not be accurate, and the strategy described may also cause feasibility problems. In this case, it would be better to run the algorithm, using the existing solution, multipliers, and penalty coefficients as initial conditions. It generally takes only a few more iterations to get a satisfactory result. Whether to re-run the algorithm or not can be determined by generating future states and controls to the end of the scheduling horizon using the variational control strategy as described above. If there is no change in the set of active constraints and there is no feasibility problem, then there is no need to re-run the algorithm.

Example 5.1. Consider problem P_1 with two subproblems in (PH). It is assumed that the current time is t=1 in subproblem 1, and three cases are examined where changes are $w_{1,1}$, $w_{1,4}$, and $w_{2,3}$, respectively. For each case, w is changed from $[10\ 8\ 1\ 0]^T$ to $[14\ 14\ 2\ 0]^T$, and the technique described above is used to find variational changes in water discharge and reservoir storage, and also to estimate total cost. To evaluate the performance of the technique, the algorithm is also re-run to generate the true optimal solution. Comparison is given in Table 6. From Table 6, one can see that the estimated costs are very close to the recalculated ones. Comparing with the original cost of \$221,905, the optimal cost decreases as the natural inflows increase. With the same amount of increase in natural inflow, the optimal cost decreases less as the time of change moves toward the end of the scheduling horizon. The percentage error between the estimated and recalculated cost, on the other hand, decreases as the time of the change increases. These are expected because as the time of the change increases, the impact on total cost is reduced, the percentage error is thus reduced.

6 Conclusions

In this paper, a new decomposition and coordination approach to the short-term hydrothermal generation scheduling and predispatch problem is presented. Test results show that the algorithm is numerically stable. With the compatibility of EDDP and mixed coordination method, significant speedup is obtained under a simulated parallel processing environment. Furthermore, unpredictable changes in natural inflows are effectively handled. Future research work includes finding a more efficient nonlinear programming technique for solving (PH-n).

Appendix. Derivation of EDDP

By taking a second-order Taylor series approximation of (PH-n) backwards in time, the approximate quadratic programming problem in variational terms can be formulated as follows (subscript j is omitted for simplicity). At any t $(1 \le t \le T')$

$$V_{tn}(v_n, \delta x_{tn}) = \min_{\delta u_{tn}} QP[L'_{Htn} + V_{(t+1)n}(v_n, \delta x_{(t+1)n})] \qquad (24)$$

where V_{tn} is the variational cost-to-go function and

$$\begin{aligned} QP[\cdot] &\equiv \delta x_{tn}^T B_{1tn}\delta x_{tn} + \delta u_{tn}^T B_{2tn}\delta x_{tn} \\ &+ \delta u_{tn}^T B_{3tn}\delta u_{tn} + B_{4tn}\delta u_{tn} + B_{5tn}\delta x_{tn} \\ &+ v_n^T B_{6tn} v_n + v_n^T B_{7tn}\delta u_{tn} + v_n^T B_{8tn}\delta x_{tn}. \quad (25) \end{aligned}$$

Coefficients B_{1tn}, B_{2tn}, B_{3tn}, B_{4tn}, B_{5tn}, B_{6tn}, B_{7tn}, and B_{8tn} are:

$$B_{1tn} = \frac{1}{2}\nabla_{xx}L'_{Htn} + Q_{(t+1)n}, B_{2tn} = \nabla_{xu}L'_{Htn} - Q_{(t+1)n},$$

$$B_{3tn} = \frac{1}{2}\nabla_{uu}L'_{Htn} + Q_{(t+1)n}, B_{4tn}^T = \nabla_{u}L'_{Htn} - U_{(t+1)n},$$

$$B_{5tn} = \nabla_{x}L'_{Htn} + U_{(t+1)n}, B_{6n} = S_{(t+1)n},$$

$$B_{7tn} = -R_{(t+1)n}, B_{8tn} = R_{(t+1)n} \quad (26)$$

with $Q_{(T'+1)n} = S_{(T'+1)n} = U_{(T'+1)n} = 0$ and $R_{(T'+1)n} = I$. Solution to (24) is then

$$\delta u_{tn}^* = \alpha_{tn} + \beta_{tn}\delta x_{tn} + \gamma_{tn}\nu_n,$$

which is (18), where

$$\alpha_{tn} = -\frac{1}{2}B_{3tn}^{-1}B_{4tn}^T, \beta_{tn} = -\frac{1}{2}B_{3tn}^{-1}B_{2tn}^T, \gamma_{tn} = -\frac{1}{2}B_{3tn}^{-1}B_{7tn}^T. \quad (27)$$

The optimal variational cost-to-go function is then

$$\begin{aligned} V_{tn}(v_n, \delta x_{tn}) &= \delta x_{tn}^T Q_{tn}\delta x_{tn} + v_n^T R_{tn}\delta x_{tn} \\ &+ v_n^T S_{tn} v_n + U_{tn}^T \delta x_{tn} + Y_{tn}^T v_n + \Theta_{tn} \end{aligned}$$

which is (19).

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