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Optimal Control of a Queueing System with Two Interacting Service Stations and Three Classes of Impatient Tasks

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Abstract—**In this paper, the problem of task selection and service priority is studied for a queueing network with two interacting service stations and three classes of impatient tasks. By using stochastic dynamic programming, a functional equation for the optimal, state-dependent priority assignment policy is derived. Properties of the optimal cost-to-go functions and the optimal policy are established through inductive proofs. It is shown that the optimal policy is governed by two switching surfaces in the three-dimensional state space (one dimension for each task class). For the infinite time horizon case, the optimal policy is shown to be stationary. In this case, the optimal cost-to-go function and switching surfaces are obtained numerically by using the over-relaxed Gauss-Seidel method. Sensitivities of the optimal policy with respect to key system parameters are also investigated.**

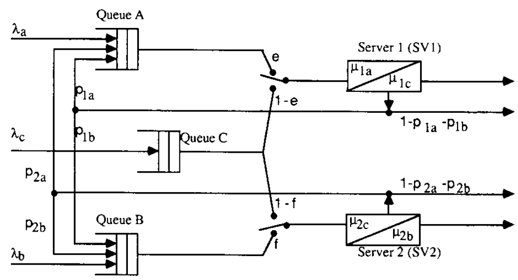

Fig. 1. The two-server queueing control system.

I. Introduction

OPTIMAL queueing control problems have recently been studied by many researchers. Harrison [1] and Pattipati and Kleinman [2] worked on cases with a single service station (server) and multiple classes of tasks (customers). The work of Pattipati and Kleinman was motivated by the attention allocation problem in a supervisory control environment. They started with a general queueing formulation and tried to incorporate balking and reneging into the model to handle random deadlines. Difficulty in solving it was discussed. The authors then reduced their scope to Markovian problems, and developed solution methodologies including a recursive algorithm and approximations under the heavy traffic environment.

Rosberg, Varaiya, and Walrand [3], Lin and Kumar [4], and Hajek [5] dealt with various cases involving two servers and a single class of tasks. Hajek presented a semi-Markov network with linear costs. The two servers have a quite general interaction structure between them. Decisions in the problem are task routing and service priorities. The author considered both finite horizon and long-run average cost cases, and showed the existence of switching curves for the optimal control policy.

The problem considered in this paper is an exponential queueing network as shown in Fig. 1. Three classes of impatient tasks (classes A, B, and C) arrive randomly and wait to be processed by two heterogeneous servers ($SV1$ and $SV2$). The structure of processing is that class A tasks have to be processed by $SV1$, and class B tasks by $SV2$. After processing, these tasks leave the system. Class C tasks, on the other hand, are the so-called *unknown tasks*. A class C task can turn out to be of class A, or of class B, or it can be a *neutral* task (a neutral task requires no further processing). Thus, a class C task has to go through a *preprocessing* or *identification* stage, which can be done by either $SV1$ or $SV2$. If it is *identified* to be of class A (B), it is routed back to be processed by $SV1$ ($SV2$). If it is identified to be a *neutral* task, it is sent out of the system. Tasks are impatient, and have class-dependent reneging rates. The two servers, on the other hand, are heterogeneous, and have class-dependent processing rates. The controls are task selection and service priority decisions e and f as shown in Fig. 1. We want to find the optimal control to minimize a linear objective function.

The problem was motivated by the Naval Battle Force/Battle Group operation, where several commanders with different levels of expertise and different areas of responsibility work together in processing hostile threats. One issue is how should these commanders divide and sequence the randomly arriving threats to optimize certain objective functions such as survivability, rewards, etc. (see Kleinman, Serfaty, and Luh [6]). Regarding Fig. 1, heterogeneous servers with class-dependent service rate represent commanders with different levels of expertise; the specific processing structure signifies different areas of responsibility; and reneging symbolizes the sudden penetration of threats. Problems of this nature also arise in production scheduling of manufacturing systems, air traffic control, etc.

Problems with both finite and infinite time horizons are considered. By using stochastic dynamic programming and inductive proofs, we show that the optimal policy is governed by two switching surfaces in the three-dimensional state space (one dimension for each task class). For the infinite horizon case, the optimal policy is shown to be stationary, and numerical studies are performed by using the over-relaxed Gauss-Seidel method. The optimal cost-to-go function and switching surfaces are obtained numerically and shown pictorially. Sensitivities of the optimal policy with respect to key system parameters are also investigated.

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The organization of this paper is as follows. The two-server queueing problem is formulated in Section II and from it a semi-Markov decision model is developed. By applying stochastic dynamic programming, optimality conditions for the control policy are obtained. In Section III we use the mathematical induction procedure to establish properties of the optimal cost-to-go functions and properties of the optimal policy. The effects of having Poisson reneging of tasks are also considered. Finally, Section IV discusses numerical testings and results.

II. Problem Formulation

Consider the queueing network shown in Fig. 1. Three classes of tasks (classes A, B, and C) arrive in Poisson streams with rates λ_a, λ_b, and λ_c, respectively. The two servers ($SV1$ and $SV2$) have exponentially distributed service rates. As mentioned, $SV1$ is capable of processing class A and class C tasks with mean rates μ_{1a} and μ_{1c}, and $SV2$ is capable of processing class B and class C tasks with mean rates μ_{2b} and μ_{2c}. Class A and class B tasks leave the system after being processed. A class C task, on the other hand, requires *identification* by either $SV1$ or $SV2$ to find out whether it belongs to class A or class B, or it is a neutral task requiring no further processing. If it turns out to be of class A (B), it is sent to queue A (B) to be processed by $SV1$ ($SV2$) as a regular class A (B) task. If it needs no further processing, it is sent out of the system. Let P_{1a} (P_{1b}) be the probability that a class C task is identified to be of class A (B) by $SV1$. Probabilities P_{2a} and P_{2b} are similarly defined for $SV2$. Although it is natural to think $P_{1a} = P_{2a}$ and $P_{1b} = P_{2b}$, this requirement is not enforced in the general derivation. We assume that

$$\mu_{1c} > \mu_{1a} \text{ and } \mu_{2c} > \mu_{2b},$$

i.e., identification is faster than processing.

Let $x \equiv (x_a, x_b, x_c) \in Z_+^3$ be the state of the system, where x_a, x_b, and x_c denote, respectively, the number of class A, B, and C tasks in the system (Z_+ is the set of nonnegative integers). The controls are $u = (e, f)$, where e is the probability that $SV1$ selects the next task from queue A, and is a mapping from x into the set of real numbers in between and including 0 and 1 (i.e., e: $Z_+^3 \rightarrow [0, 1]$). The value $(1 - e)$ is therefore the probability of selecting a class C task. The mapping f: $Z_+^3 \rightarrow [0, 1]$ is similarly defined for $SV2$ (f is the probability of selecting the next task from queue B). Following the modeling procedure of Hajek [5] and Stidham and Prabhu [7], we shall in the sequel set up a semi-Markov decision model for this problem.

State transitions include arrivals, potential departures, and potential transfers between queues. For $i = a$, b, or c, let A_i and D_i denote, respectively, an arrival and a potential departure of a class i task, and R_{ca} (R_{cb}) a potential transfer of a class C task to class A (to class B). For example,

$$A_a x = (x_a + 1, x_b, x_c),\ D_a x = ((x_a - 1)^+, x_b, x_c),$$

$$R_{ca} x = (x_a + 1, x_b (x_c - 1)^+)$$

where $(x_i - 1)^+ \equiv x_i - 1$ if $x_i > 0$, and 0 otherwise.

The actual total event rate is

$$r' = \lambda_a + \lambda_b + \lambda_c + e\mu_{1a} + (1-e)\mu_{1c} + f\mu_{2b} + (1-f)\mu_{2c} \quad (2.1)$$

which is state dependent as e and f are functions of the state. To bypass the difficulty of state dependence, we utilize the *pseudo epoch* concept of Lippman [8] and define a constant total event rate r as follows:

$$r = \lambda_a + \lambda_b + \lambda_c + \mu_{1c} + \mu_{2c} > r'.$$

Note that r is greater than r', the actual total event rate. The difference $r - r'$ is the contribution of *pseudo events* artificially created to represent transitions from a state back to itself. For a given $u = (e, f)$, the transition probability function $P(\cdot|\cdot, u)$ is defined on $Z_+^3 * Z_+^3$ by

$$\begin{aligned} P(y|x, u) = r^{-1}\{&\lambda_a I(y = A_a x) + \lambda_b I(y = A_b x) + \lambda_c I(y = A_c x) \\ &+ e\mu_{1a} I(y = D_a x) + f\mu_{2b} I(y = D_b x) \\ &+ [(1-e)(1 - P_{1a} - P_{1b})\mu_{1c} \\ &\cdot (1-f)(1 - P_{2a} - P_{2b})\mu_{2c}] I(y = D_c x) \\ &+ [(1-e)P_{1a}\mu_{1c} + (1-f)P_{2a}\mu_{2c}] I(y = R_{ca} x) \\ &+ [(1-e)P_{1b}\mu_{1c} + (1-f)P_{2b}\mu_{2c}] I(y = R_{cb} x) \\ &+ \left(1 - \frac{r'}{r}\right) I(y = x) \end{aligned} \quad (2.2)$$

where I is the indicator function defined as

$$I(x, y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the last term in (2.2) is related to the *pseudo events.*

Since controls are state dependent and are made at instants of event epochs, we denote the sequence $(u_0, u_1, \cdots)$ as U, where $u_k = (e_k, f_k)$ is the control at the kth transition. Given U and an initial state i_0 in Z_+^3, a semi-Markov decision process $(x(t), t > 0)$ with jump rate r and imbedded transition probability P is defined as

$$x(t) = x_{n(t)}$$

where $X = (x_0, x_1, \cdots)$ is a sequence of random variables with $P(x_0 = i_0) = 1$ and

$$P(x_{k+1} = j | x_k = i_k, \cdots, x_0 = i_0) = P(j | i_k, u_k(i_0, \cdots, i_k)) \quad (2.3)$$

and $(n(t)$: $t > 0)$ is a rate r Poisson process independent of x.

The instantaneous cost is a linear function of the number of tasks in the system, i.e.,

$$C'x = c_a x_a + c_b x_b + c_c x_c$$

where c_i, $i = a$, b, and c are nonnegative weighting coefficients. Let τ_n be the random time at which nth event happens. Then the cost for a control policy U over the time interval $[0, \tau_n]$ is

$$E_x^U \left[\int_0^{\tau_n} e^{-\alpha t} C' x(t)\, dt \right]$$

where α is a nonnegative discount constant ($0 < \alpha < 1$), and E_x^U denotes the expectation with respect to $\{x(t)\}$. Following Rosberg, Varaiya, and Walrand [3], this cost can be rewritten as

$$(\alpha + r)^{-1} E_x^U \left[\sum_{k=0}^{n-1} \beta^k C' x_k \right] \quad (2.4)$$

where $\beta \equiv r/(\alpha + r) < 1$. In view of (2.3), (2.4) can be interpreted as the cost over n time stages for a discrete-time decision process with discount factor β. Ignoring the constant factor $(\alpha + r)^{-1}$, let us define the n-stage optimal cost-to-go for a given initial state x as

$$V_n^\beta(x) = \min_U E_x^U \left[\sum_{k=0}^{n-1} \beta^k C' x_k \right], \qquad n > \infty, \text{ and}$$

$$V_0^\beta(x) = 0 \qquad \text{by convention.}$$

It can be shown by following Hajek [5] and Schal [9] that

$$\lim_{n \rightarrow \infty} V_n^\beta(x) = V_\infty^\beta(x).$$

The dynamic programming equation leads to the following optimality conditions:

$$\begin{aligned} V^\beta_{n+1}(x)=C'x+\beta r^{-1}\min_{e_n,f_n}\{&\lambda_a V^\beta_n(A_a x)\\ &+\lambda_b V^\beta_n(A_b x)+\lambda_c V^\beta_n(A_c x)\\ &+e_nE_{1n}(x)+(1-e_n)E_{2n}(x)\\ &+f_nF_{1n}(x)+(1-f_n)F_{2n}(x)\}\\ \equiv TV^\beta_n(x) \qquad (2.5)\end{aligned}$$

where T denotes the dynamic programming operator, and

$$\begin{aligned} E_{1n}(x)&=\mu_{1a}V^\beta_n(D_a x)+(\mu_{1c}-\mu_{1a})V^\beta_n(x),\\ E_{2n}(x)&=\mu_{1c}[(1-P_{1a}-P_{1b})V^\beta_n(D_c x)\\ &\quad+P_{1a}V^\beta_n(R_{ca}x)+P_{1b}V^\beta_n(R_{cb}x)],\\ F_{1n}(x)&=\mu_{2a}V^\beta_n(D_b x)+(\mu_{2c}-\mu_{2b})V^\beta_n(x),\\ F_{2n}(x)&=\mu_{2c}[(1-P_{2a}-P_{2b})V^\beta_n(D_c x)\\ &\quad+P_{2a}V^\beta_n(R_{ca}x)+P_{2b}V^\beta_n(R_{cb}x)]. \qquad (2.6)\end{aligned}$$

The variables $E_{1n}(x)$ represent the expected cost if $SV1$ selects the next task from queue A, and $E_{2n}(x)$ the cost if $SV1$ selects the next task from queue C. Variables $F_{1n}(x)$ and $F_{2n}(x)$ can be similarly interpreted. The optimal control $u^*_n = (e^*_n, f^*_n)$ for x with n steps to go is therefore determined by

$$e^*_n=\begin{cases}1 & \text{if } E_{1n}(x)\le E_{2n}(x),\\ 0 & \text{if } E_{1n}(x)>E_{2n}(x), \text{ and}\end{cases}$$

$$f^*_n=\begin{cases}1 & \text{if } F_{1n}(x)\le F_{2n}(x),\\ 0 & \text{if } F_{1n}(x)>F_{2n}(x).\end{cases} \qquad (2.7)$$

This is a bang-bang control.

III. Structure and Properties of the Optimal Control Policy

In this section, the inductive approach of Hajek [5] is used to show that the optimal control policy is of switching type. The derivations, however, are more complicated than that of [5] because of the existence of multiple classes of tasks and the specific processing structure considered. We also show that for the infinite horizon case the policy is stationary.

The core of the proofs is to establish by induction the following properties for the optimal cost-to-go function $V^\beta_n(\cdot)$.

$P1$: $V^\beta_n(x)$ is increasing in x_a, in x_b, and in x_c.

$P2$: $V^\beta_n(x+y)+V^\beta_n(x)$ is increasing in x_a, in x_b, and in x_c for each fixed y in Z^3_+.

$P3$: $V^\beta_n(x)-G_{in}(x)$ is increasing in x_a, in x_b, and in x_c, where

$$G_{in}(x)\equiv(1-P_{ia}-P_{ib})V^\beta_n(D_c x)+P_{ia}V^\beta_n(R_{ca}x)+P_{ib}V^\beta_n(R_{cb}x), \qquad i=1,2. \quad (3.1)$$

$P4$: $E_{1n}(x)-E_{2n}(x)$ and $F_{1n}(x)+F_{2n}(x)$ are increasing in x_c, and decreasing in x_a and in x_b, where $E_{1n}(x)$, $E_{2n}(x)$, $F_{1n}(x)$, and $F_{2n}(x)$ are defined in (2.6).

For $n=1$, $V^\beta_1(x)=C'x=c_ax_a+c_bx_b+c_cx_c$. It can be easily checked that P1–P4 are satisfied. Assume that V^β_n satisfies P1–P4 for all x in Z^3_+. After lengthy derivations, it can be shown that V^β_{n+1} also satisfies P1–P4 (the proofs are provided in the Appendix). Thus by mathematical induction, we conclude the following.

Theorem 1: $V^\beta_n(x)$ satisfies P1–P4 for all n in N and $x_i>0$, $i=a, b, c$.

From the above properties of the optimal cost-to-go function $V^\beta_n(x)$, we now show the existence of switching surfaces for e_n and f_n. First we define the two switching functions.

$$S_{1n}(x_b, x_c)=\min\{x_a: E_{1n}(x)-E_{2n}(x)\le 0\},$$

and

$$S_{2n}(x_a, x_c)=\min\{x_b: F_{1n}(x)-F_{2n}(x)\le 0\} \qquad (3.2)$$

and their associated regions

$$SR_{1n}\{x\in Z^3_+: x_a>S_{1n}(x_b, x_c)\},$$

and

$$SR_{2n}=\{x\in Z^3_+: x_b>S_{2n}(x_a, x_c)\}. \qquad (3.2)$$

Theorem 2: The switching functions S_{1n} and S_{2n} define two switching surfaces in the state-space x. When there are n stages to go, the optimal decision is given by

$$e^*_n=\begin{cases}1 & \text{iff } x\in SR_{1n},\\ 0 & \text{otherwise}\end{cases}$$

$$f^*_n=\begin{cases}1 & \text{iff } x\in SR_{2n},\\ 0 & \text{otherwise.}\end{cases} \qquad (3.4)$$

Proof: This assertion is a consequence of (2.7) and Theorem 1.

For the infinite horizon case ($n=\infty$), by following Lippman [10, Theorem 1] we have the following.

Theorem 3:

$$V^\beta(x)=\lim_{n\to\infty}\min_U\left[\sum_{k=0}^n \beta^k C'x_k\right] \qquad (3.5)$$

is achieved by a stationary policy.

Note that $0<\beta<1$ according to (2.4).

For a system with reneging tasks, the analysis is similar. Suppose that the reneging processes are Poisson with mean rates α_i, $i=a, b$, and c. With state x, the system reneging rate is $\alpha_ax_a+\alpha_bx_b+\alpha_cx_c$. Suppose that under normal operation, the system reneging rate is bounded above by a constant M. Then the total event rate can be redefined as

$$r=\lambda_a+\lambda_b+\lambda_c+\mu_{1c}+\mu_{2c}+M. \qquad (3.6)$$

The transition probability function (2.2) can be modified accordingly. Let the linear instantaneous reneging cost be

$$c_{ra}\alpha_ax_a+c_{rb}\alpha_bx_b+c_{rc}\alpha_cx_c. \qquad (3.7)$$

Then the dynamic programming equation becomes

$$\begin{aligned} V^\beta_{n+1}(x)=C'_rx+\beta r^{-1}\min_{e_n,f_n}\{&\lambda_aV^\beta_n(A_ax)+\lambda_bV^\beta_n(A_bx)\\ &+\lambda_cV^\beta_n(A_cx)+\alpha_ax_aV^\beta_n(D_ax)\\ &+\alpha_bx_bV^\beta_n(D_bx)+\alpha_cx_cV^\beta_n(D_cx)\\ &+(M-(\alpha_ax_a+\alpha_bx_b+\alpha_cx_c))V^\beta_n(x)\\ &+e_nE_{1n}(x)+(1-e_n)E_{2n}(x)\\ &+f_nF_{1n}(x)+(1-f_n)F_{2n}(x)\}\\ \equiv TV^\beta_n(x) \qquad (3.8)\end{aligned}$$

where

$$C'_rx\equiv(c_a+c_{ra}\alpha_a)x_a+(c_b+c_{rb}\alpha_b)x_b+(c_c+c_{rc}\alpha_c)x_c$$

and E_{1n}, E_{2n}, F_{1n}, F_{2n} remain the same as in (2.6). The dynamic programming equation is therefore similar to (2.5) for the case without reneging. Consequently, all the previous results apply.

IV. Numerical Results

In this section, we perform numerical studies for problems with infinite time horizon and finite queue sizes. Note that all previous results are developed assuming infinite queue sizes. For numerical consideration, however, we shall assume in this section that each queue has a finite size. Let N_i be the size of each queue, $i = A, B, C$. For the sake of simplicity, we further assume that $N_A = N_B = N_c \equiv N$. If N is large enough, the finite queue size model is a good approximation of the infinite queue size problem and vice versa, except for regions close to the queue capacity. With N finite, the stationary verison of the dynamic programming equation (2.5) can be written as

$$V^{\beta}(i, j, k) = C(i, j, k) + \beta r^{-1} PV(i, j, k), \qquad 0 \le i, j, k \le N \tag{4.1}$$

where $i = x_a$, $j = x_b$, $k = x_c$, $C(i, j, k)$ is the instantaneous cost at state (i, j, k), PV is the expected optimal cost-to-go after one state transition from (i, j, k) and is a function of V^{β}. The time index n has been dropped since the stationary case is considered.

An iterative algorithm based on the over-relaxed Gauss-Seidel method (Pizer [11]) is used to solve (4.1). The procedure is to iterate coordinate by coordinate to find the *fixed point* $V^{\beta *}(\cdot, \cdot, \cdot)$ of (4.1). With the optimal cost-to-go determined for each state, analytic results such as P1–P4 are checked and switching surfaces are obtained. Sensitivity studies on the optimal control policy with respect to key system parameters are also performed.

It should be mentioned that the major purposes of this numerical study are (1) to get a feel of the optimal cost-to-go function and the optimal control policy, and (2) to verify analytic findings such as P1–P4. Therefore, no effort was spent in developing new algorithms trying to apply or exploit these properties. Rather, the standard over-relaxed Gauss-Seidel method is adopted.

In the numerical study, $N = 15$, implying that the dimension of (4.1) is $16^3 = 4096$. The algorithm is implemented in Fortran on an IBM 3081. For all the cases tested, the algorithm converges, and the number of iterations varies from 6 to 90. The switching surfaces S_1 and S_2 are depicted, respectively, in Figs. 2 and 3 for a case with the following set of parameters:

$$c_a = c_b = c_c = 0.8, \; \lambda_a = \lambda_b = \lambda_c \equiv \lambda = 1,$$

$$\mu_{1a} = 4, \; \mu_{1c} = 6, \; \mu_{2b} = 5, \; \mu_{2c} = 7,$$

$$P_{1a} = P_{1b} = P_{2a} = P_{2c} \equiv P = 0.1, \text{ and } \beta = 0.94.$$

From the figures, we see that except for those states in the neighborhood of queue capacity, numerical results do verify the properties established in Section III.

It is also of interest to investigate sensitivities of the optimal policy with respect to key system parameters. In doing this, the above example is used as a base line for comparison. A few observations from the study are as follows.

1) The increases in class C feedback probabilities P_{ij}, $i = 1, 2$, $j = a, b$, indirectly increase arrivals of class A and class B tasks. The switching surface S_1 (S_2) therefore shifts in favor of processing class A (B) tasks. Fig. 4 illustrates the variations of S_1 with respect to P_{ij} ($\equiv P$ for all i, j in this case). The curves are obtained with $x_b = 3$. This shifting of S_1 (S_2) can also be seen analytically by comparing E_{1n} and E_{2n} [F_{1n} and F_{2n}, see (2.6)] and using the property that $V_n^{\beta}(x)$ is increasing in x_a and in x_b (property P1).

2) With discount factor β, the effective look ahead time is $(1 - \beta)^{-1}$ stages. When β is small, $V^{\beta}(x)$ is dominated by the linear stagewise cost, thus it is approximately linear in x. In this

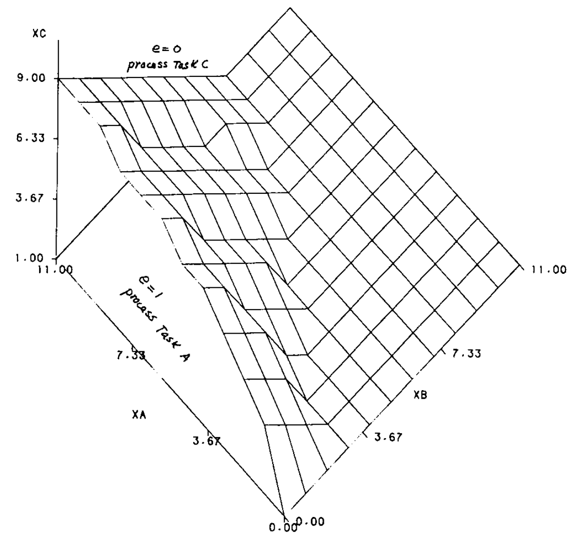

Fig. 2. Switching surface (E).

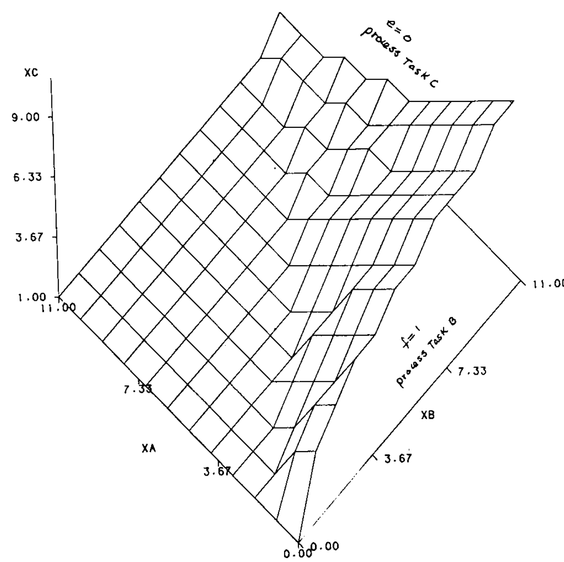

Fig. 3. Switching surface (F).

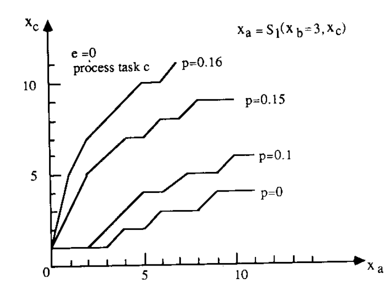

Fig. 4. Switching surface variations w.r.t. the feedback probability p.

degenerated case, the constants $c_a\mu_{1a}$ and $c_c\mu_{1c}$ reflect, respectively, the average gain per unit time for $SV1$ to process class A and class C tasks. $SV1$ would therefore keep on processing the class of tasks with the larger gain until that queue is depleted. This in turn implies that S_1 would either be the $x_a = 0$ plane or the $x_c = 1$ plane. A similar statement holds for S_2. When β is large, $V^\beta(x)$ becomes convex in each axis as stated by P1 and P2. The switching surfaces are no longer degenerate. Fig. 5 demonstrates the variations of S_1 for several values of β.

3) Following (2), increases in μ_{1a} or c_a (μ_{2b} or c_b) shift the control policy of $SV1$ ($SV2$) in favor of processing class A (B) tasks. Figs. 6 and 7 show variations of S_1 for several values of μ_{1a} and c_a, respectively. The shifting of S_1 (S_2) when μ_{1a} (μ_{2b}) increases can also be seen analytically by comparing E_{1n} and E_{2n} (F_{1n} and F_{2n}) and using the property that $V_n^\beta(x)$ is increasing in x_a and in x_b (property P1).

4) Variations of S_1 for several values of arrival rates are shown in Fig. 8 ($\lambda_i \equiv \lambda$ for $i = a, b, c$ in this case). The insensitivity of the control policy with respect to arrival rate variations for $\lambda > 3$ is caused by the finite queue size assumption.

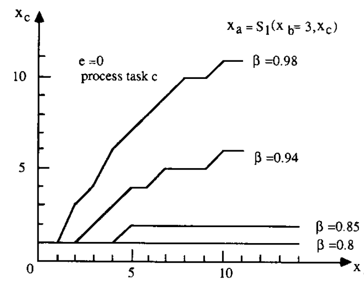

Fig. 5. Switching surface variations w.r.t. the discount factor β.

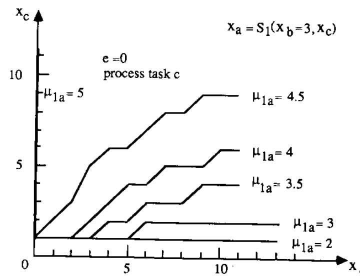

Fig. 6. Switching surface variations w.r.t. the service rate μ_{1a}.

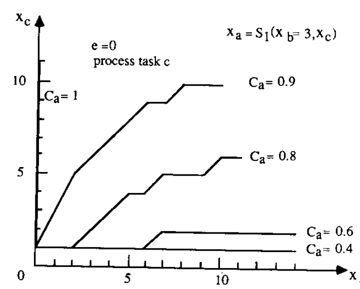

Fig. 7. Switching surface variations w.r.t. weighting coefficients C_a.

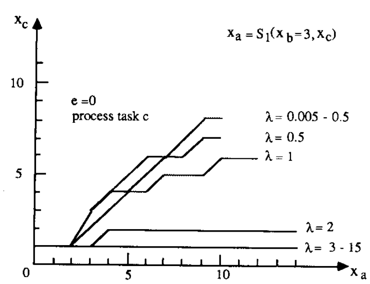

Fig. 8. Switching surface variations w.r.t. arrival rate λ.

V. Summary

In this paper, the problem of task selection and service priority is studied for a queueing network with two interacting servers and three classes of impatient tasks. By using stochastic dynamic programming, a functional equation for the optimal, state-dependent priority assignment policy is derived. Properties of the optimal cost-to-go functions and the optimal policy are established through inductive proofs. An iterative algorithm is used to compute the optimal stationary policy for problems with infinite horizon. Numerical results support our analytic findings and also provide further insights to the problem.

Appendix

Assume that V_n^β satisfies P1–P4 for $x \in Z_+^3$. To show that V_{n+1}^β satisfies P1–P4, we first establish a few properties for the components of $V_{n+1}^\beta(x)$. In doing this, we shall assume the following.

Assumption 1 (AS1): $\mu_{1c} > \mu_{1a}$ such that

$$\mu_{1c}(V_n^\beta(A_bx)-V_n^\beta(x))-\mu_{1a}(V_n^\beta(A_bx)-V_n^\beta(A_bD_ax))$$

is increasing in x_a, in x_b, and in x_c for all x.

This assumption assumes that the difference between the cost reduction from A_bx to x and the cost reduction from A_bx to A_bD_ax satisfies certain properties. They are assumed as an assumption rather than proved as a lemma because of the special processing capability considered in the paper ($DM1$ is not capable of processing class B tasks). It is easy to check that (AS1) is true for $n = 1, 2$ if $c_b\mu_{1c} - c_a\mu_{1a} > 0$.

Lemma 3.1:

1) $E_{jn}(A_ix) - E_{jn}(x)$, $i = a, b, c, j = 1, 2$,
2) $E_{1n}(A_ix) - E_{2n}(x)$, $i = a, b$, and
3) $E_{2n}(A_cx) - E_{1n}(x)$

are increasing in x_a, in x_b, and in x_c.

Proof:

1) It follows directly from P2.

2) For $i = a$,

$$\begin{aligned}E_{1n}(A_ax)-E_{2n}(x)&=\mu_{1a}V_n^\beta(x)+(\mu_{1c}-\mu_{1a})V_n^\beta(A_ax)\\&\quad-\mu_{1c}((1-P_{1a}-P_{1b})V_n^\beta(D_cx)\\&\quad+P_{1a}V_n^\beta(R_{ca}x)+P_{1b}V_n^\beta(R_{cb}x))\\&=\mu_{1c}(V_n^\beta(x)-G_{1n}(x))\\&\quad+(\mu_{1c}-\mu_{1a})(V_n^\beta(A_ax)-V_n^\beta(x)).\end{aligned}$$

From P2 and P3, the property follows.

For $i = b$,

$$\begin{aligned}E_{1n}(A_bx)-E_{2n}(x)&=\mu_{1c}(V_n^\beta(x)-G_{1n}(x))+\mu_{1c}(V_n^\beta(A_bx)\\&\quad-V_n^\beta(x))-\mu_{1a}(V_n^\beta(A_bx)-V_n^\beta(A_bD_ax)).\end{aligned}$$

The increasing in x_a, in x_b, and in x_c follows from P3 and (AS1).

3)
$$\begin{aligned}E_{2n}(A_cx)-E_{1n}(x)&=\mu_{1a}(V_n^\beta(x)-V_n^\beta(D_ax))\\&\quad+\mu_{1c}P_{1a}(V_n^\beta(A_ax)-V_n^\beta(x))\\&\quad+\mu_{1c}P_{1b}(V_n^\beta(A_bx)-V_n^\beta(x)).\end{aligned}$$

The property then follows from P2. Q.E.D.

Lemma 3.2:

$$I_{1n}(x)=E_{1n}(x)-((1-P_{1a}-P_{1b})E_{1n}(D_cx)+P_{1a}E_{1n}(R_{ca}x)+P_{1b}E_{1n}(R_{cb}x)),$$

and

$$I_{2n}(x)=E_{2n}(x)-((1-P_{1a}-P_{1b})E_{2n}(D_cx)+P_{1a}E_{2n}(R_{ca}x)+P_{1b}E_{2n}(R_{cb}x)) \quad (3.2)$$

are increasing in x_a, in x_b, and in x_c.

Proof: Let

$$H_n(x)=V_n^\beta(x)-G_{1n}(x)=V_n^\beta(x)-((1-P_{1a}-P_{1b})V_n^\beta(x)+P_{1a}V_n^\beta(R_{ca}x)+P_{1b}V_n^\beta(R_{cb}x)). \quad (3.3)$$

By substituting (2.6) into (3.2) and expressing it in terms of $H_n(x)$, we have

$$I_{1n}(x)=\mu_{1a}H_n(D_ax)+(\mu_{1c}-\mu_{1a})H_n(x),$$

$$I_{2n}(x)=\mu_{1c}((1-P_{1a}-P_{1b})H_n(D_cx)+P_{1a}H_n(R_{ca}x)+P_{1b}H_n(R_{cb}x)). \quad (3.4)$$

According to P3, $H_n(x)$ is increasing in x_a, in x_b, and in x_c, so are $I_{1n}(x)$ and $I_{2n}(x)$. Q.E.D.

Lemma 3.3:

$$J_{1n}(x)=\mu_{1a}E_{1n}(D_ax)+(\mu_{1c}-\mu_{1a})E_{1n}(x)-\mu_{1c}((1-P_{1a}-P_{1b})E_{1n}(D_cx)+P_{1a}E_{1n}(R_{ca}x)+P_{1b}E_{1n}(R_{cb}x)),$$

and

$$J_{2n}(x)=\mu_{1a}E_{2n}(D_ax)+(\mu_{1c}-\mu_{1a})E_{2n}(x)-\mu_{1c}((1-P_{1a}-P_{1b})E_{2n}(D_cx)+P_{1a}E_{2n}(R_{ca}x)+P_{1b}E_{2n}(R_{cb}x)) \quad (3.5)$$

are increasing in x_c, and decreasing in x_a and in x_b.

Proof: Substituting (2.6) into the above equations and regrouping, we get

$$J_{1n}(x)=\mu_{1a}L_n(D_ax)+(\mu_{1c}-\mu_{1a})L_n(x),$$

$$J_{2n}(x)=\mu_{1c}((1-P_{1a}-P_{1b})L_n(D_cx)+P_{1a}L_n(R_{ca}x)+P_{1b}L_n(R_{cb}x)) \quad (3.6)$$

where

$$L_n(x)=E_{1n}(x)-E_{2n}(x). \quad (3.7)$$

Since $L_n(x)$ is increasing in x_c and decreasing in x_a and x_b from P4, so are $J_{1n}(x)$ and $J_{2n}(x)$. Q.E.D.

The above three lemmas are developed around $E_{1n}(x)$ and $E_{2n}(x)$. Since $F_{1n}(x)$ and $F_{2n}(x)$ are symmetric to $E_{1n}(x)$ and $E_{2n}(x)$, a similar set of lemmas hold for $F_{1n}(x)$ and $F_{2n}(x)$. They are omitted here for conciseness.

The following lemma states a useful fact that P1-P4 hold for any nonnegative linear combinations of functions satisfying P1-P4.

Lemma 3.4: If h_1, h_2: $Z_+^3 \to R$ are two functions satisfying P1-P4, then $h = \alpha_1 h_1 + \alpha_2 h_2$, $\alpha_1, \alpha_2 \in R^+$, also satisfies properties P1-P4.

The proof is straightforward and is omitted.

With these background lemmas, we now proceed to prove that $V_{n+1}^\beta(x) = TV_n^\beta(x)$ satisfies properties P1-P4. From (2.5),

$$V_{n+1}(x)=C'x+\beta r^{-1}[\lambda_a V_n^\beta(A_ax)+\lambda_b V_n^\beta(A_bx)+\lambda c V_n^\beta(A_cx)+\beta r^{-1}[\min(E_{1n}(x), E_{2n}(x))+\min(F_{1n}(x), F_{2n}(x))].$$

If the last term of the above expression satisfies P1-P4, then by Lemma 3.4 $V_{n+1}^\beta(x)$ satisfies P1-P4 and the inductive proof is completed. Since $(E_{1n}(x), E_{2n}(x))$ and $(F_{1n}(x), F_{2n}(x))$ are symmetric, it suffices that we prove P1-P4 for

$$E(x)=\min(E_{1n}(x), E_{2n}(x)) \quad (3.8)$$

only.

Lemma 3.5: $E_{nS5}(x)$ satisfies P1-P4.

Sketch of the Proof: As mentioned, $L_n(x) = E_{1n}(x) - E_{2n}(x)$ [defined by (3.7)] is increasing in x_c and decreasing in x_a and in x_b because of P4. $E_n(x)$ can be rewritten as follows:

$$E_x\ \ (x)=\begin{cases} E_{2n}(x) & \text{if } L(x)>0 \\ E_{1n}(x) & \text{if } L(x)\leqslant 0. \end{cases}$$

Proof for P1: $E_n(A_ix) = \min(E_{1n}(A_ix), E_{2n}(A_ix)) \geqslant \min(E_{1n}(x), E_{2n}(x)) = E_n(x)$, for $i = a, b, c$ from Lemma 3.1.

Proof for P2:

We need to show that

2-a) $E_n(A_a^2x)-E_n(A_ax) \geqslant E_n(A_ax)-E_n(x)$;

2-b) $E_n(A_aA_bx)-E_n(A_bx) \geqslant E_n(A_ax)-E_n(x)$;

2-c) $E_n(A_aA_cx)-E_n(A_cx) \geqslant E_n(A_ax)-E_n(x)$;

2-d) $E_n(A_bA_cx)-E_n(A_cx) \geqslant E_n(A_bx)-E_n(x)$;

2-e) $E_n(A_b^2x)-E_n(A_bx) \geqslant E_n(A_bx)-E_n(x)$;

2-f) $E_n(A_c^2x)-E_n(A_cx) \geqslant E_n(A_cx)-E_n(x)$.

The proofs for the six inequalities are similar. For each one we examine all the possible cases of E and utilize Lemma 3.1 extensively. The procedure is very cumbersome. We therefore only include the proofs of 2-a) and 2-b) to illustrate key ideas.

Proof of 2-a) of Lemma 3.5: Since L_n is decreasing in x_a, we have $L_n(x) \geqslant L_n(A_ax) \geqslant L_n(A_a^2x)$. There are four cases for $(E_n(x), E_n(A_ax), E_n(A_a^2x))$:

c_1) $(E_{1n}(x), E_{1n}(A_ax), E_{1n}(A_a^2x))$ if $0 \geqslant L_n(x)$;

c_2) $(E_{2n}(x), E_{1n}(A_ax), E_{1n}(A_a^2x))$ if $L_n(x) \geqslant 0 \geqslant L_n(A_ax)$;

c_3) $(E_{2n}(x), E_{2n}(A_ax), E_{1n}(A_a^2x))$ if $L_n(x) \geqslant L_n(A_ax) \geqslant 0 \geqslant L_n(A_a^2x)$;

c_4) $(E_{2n}(x), E_{2n}(A_ax), E_{2n}(A_a^2x))$ if $L_n(x) \geqslant L_n(A_ax) \geqslant L_n(A_a^2x) \geqslant 0$.

For case c_1), $E_n(A_a^2x) - E_n(A_ax) = E_{1n}(A_a^2x) - E_{1n}(A_ax)$

$$\leqslant E_n(A_ax)-E_n(x)=E_{1n}(A_ax)-E_{1n}(x)$$

from Lemma 3.1.

For case c_3),

$$E_{1n}(A_a^2x)-E_{2n}(A_ax) \geqslant E_{1n}(A_ax)-E_{2n}(x)$$

by Lemma 3.1, and

$$E_{1n}(A_ax) \geqslant E_{2n}(A_ax) \text{ since } L_n(A_ax) \geqslant 0.$$

It follows that

$$E_{1n}(A_a^2x)-E_{2n}(A_ax) \geqslant E_{1n}(A_ax)-E_{2n}(x) \geqslant E_{2n}(A_ax)-E_{2n}(x).$$

Therefore,

$$E_{2n}(A_a^2x)-E_n(A_ax) \geqslant E_n(A_ax)-E_n(x).$$

Proofs for c_2) and c_4) are similar and therefore skipped.

Proof of 2-b) of Lemma 3.5: L_n is decreasing in x_a and in x_b, so

$$L_n(A_aA_bx) \leqslant L_n(A_ax) \leqslant L_n(x), \quad \text{and } L_n(A_aA_bx) \leqslant L_n(A_bx) \leqslant L_n(x).$$

There are six cases of $(E_n(x), E_n(A_ax), E_n(A_bx), E_n(A_aA_bx))$:

c_1) $(E_{2n}(x), E_{2n}(A_ax), E_{2n}(A_bx), E_{2n}(A_aA_bx))$ if $0 \leqslant L_n(A_aA_bx)$;

c_2) $(E_{1n}(x), E_{1n}(A_ax), E_{1n}(A_bx), E_{1n}(A_aA_bx))$ if $L_n(X) \leqslant 0$;

c_3) $(E_{2n}(x), E_{2n}(A_ax), E_{2n}(A_bx), E_{1n}(A_aA_bx))$ if $L_n(A_aA_bx) \leqslant 0 \leqslant L_n(A_ax)$ and $L_n(A_aA_bx) \leqslant 0 \leqslant L_n(A_bx)$;

c_4) $(E_{2n}(x), E_{2n}(A_ax), E_{1n}(A_bx), E_{1n}(A_aA_bx))$ if $L_n(A_aA_bx) \leqslant 0 \leqslant L_n(A_ax)$ and $L_n(A_bx) \leqslant 0 \leqslant L_n(x)$;

c_5) $(E_{2n}(x), E_{1n}(A_ax), E_{2n}(A_bx), E_{1n}(A_aA_bx))$ if $L_n(A_ax) \leqslant 0 \leqslant L_n(x)$ and $L_n(A_aA_bx) \leqslant 0 \leqslant L_n(A_bx)$;

c_6) $(E_{2n}(x), E_{1n}(A_ax), E_{1n}(A_bx), E_{1n}(A_aA_bx))$ if $L_n(A_ax) \leqslant 0 \leqslant L_n(x)$ and $L_n(A_bx) \leqslant 0 \leqslant L_n(x)$.

By applying Lemma 3.1 to the above cases, it can be shown that 2-b) holds for all of them.

Proof for P3: Define

$$G_n'(x)=E_n(x)-((1-P_{1a}-P_{1b})E_n(D_cx)+P_{1a}E_n(R_{ca}x)+P_{1b}E_n(R_{cb}x)). \quad (3.9)$$

To investigate the properties of $G_n'(x)$, we again have to go through all the possible cases of $(E_n(x), E_n(D_cx), E_n(R_{ca}x), E_n(R_{cb}x))$. From the fact that L_n is increasing in x_c and decreasing in x_a and in x_b, we have

$$L_n(R_{ca}x) \leqslant L_n(D_cx) \leqslant L_n(x) \text{ and } L_n(R_{cb}x) \leqslant L_n(D_cx). \quad (3.10)$$

According to (3.10), there are six possible cases for $(E_n(x), E_n(D_cx), E_n(R_{ca}x), E_n(R_{cb}x))$:

c_1) $(E_{2n}(x), E_{2n}(D_cx), E_{2n}(R_{ca}x), E_{2n}(R_{cb}x))$ if $0 \leqslant L_n(R_{ca}x)$ and $0 \leqslant L_n(R_{cb}x)$.

c_2) $(E_{2n}(x), E_{1n}(D_cx), E_{1n}(R_{ca}x), E_{1n}(R_{cb}x))$ if $L_n(D_cx) \leqslant 0 \leqslant L_n(x)$.

c_3) $(E_{2n}(x), E_{2n}(D_cx), E_{1n}(R_{ca}x), E_{1n}(R_{cb}x))$ if $L_n(R_{ca}x) \leqslant 0 \leqslant L_n(D_cx)$ and $L_n(R_{cb}x) \leqslant 0 \leqslant L_n(D_cx)$.

c_4) $(E_{2n}(x), E_{2n}(D_cx), E_{2n}(R_{ca}x), E_{1n}(R_{cb}x))$ if $0 \leqslant L_n(R_{ca}x)$ and $L_n(R_{cb}x) \leqslant 0 \leqslant L_n(D_cx)$.

c_5) $(E_{2n}(x), E_{2n}(D_cx), E_{1n}(R_{ca}x), E_{2n}(R_{cb}x))$ if $L_n(R_{ca}x) \leqslant 0 \leqslant L_n(D_cx)$ and $0 \leqslant L_n(R_{cb}x)$.

c_6) $(E_{1n}(x), E_{1n}(D_cx), E_{1n}(R_{ca}x), E_{1n}(R_{cb}x))$ if $L_n(x) \leqslant 0$.

Lemma 3.2 and properties of E_{1n} and E_{2n} are used in showing $G_n'(x)$ to be increasing in x_a, in x_b, and in x_c. For example, consider case c_5)

$$G_n'(x)=E_{2n}(x)-((1-P_{1a}-P_{1b})E_{2n}(D_cx)+P_{1a}E_{1n}(R_{ca}x)+P_{1b}E_{2n}(R_{cb}x))=I_{2n}(x)-P_{1a}(E_{1n}(R_{ca}x)-E_{2n}(R_{cb}x)).$$

Substituting (3.4) and (2.6) into the above equation and regrouping, we obtain

$$\begin{aligned} G_n'(x)&=\mu_{1c}((1-P_{1a}-P_{1b})H_n(D_cx)+P_{1a}H_n(R_{ca}x)\\ &\quad+P_{1b}H_n(R_{cb}x))-P_{1a}\mu_{1a}(V_n^{\beta}(R_{ca}D_ax)\\ &\quad-V_n^{\beta}(R_{ca}x))-P_{1a}\mu_{1c}H_n(R_{ca}x)\\ &=\mu_{1c}((1-p_{1a}-P_{1b})H_n(D_cx)+P_{1b}H_n(R_{cb}x))\\ &\quad+P_{1a}\mu_{1a}(V_n^{\beta}(R_{ca}x)-V_n^{\beta}(R_{ca}D_ax)), \end{aligned}$$

where $H_n(x)$ is defined by (3.3). Since $H_n(x)$ is increasing in x_a, in x_b, and in x_c according to P3, and $V_n^{\beta}(R_{ca}x) - V_n^{\beta}(R_{ca}D_ax)$ is increasing in x_a, in x_b, and in x_c by P2, so is $G_n(x)$. The remaining five cases can be proved similarly.

Proof for P4: Define for $E_n(x)$

$$E_{1n}'(x)=\mu_{1a}E_n(D_ax)+(\mu_{1c}-\mu_{1a})E_n(x)$$

and

$$E_{2n}'(x)=\mu_{1c}((1-P_{1a}-P_{1b})E_n(D_cx)+P_{1a}E_n(R_{ca}x)+P_{1b}E_n(R_{cb}x) \quad (3.11)$$

as we did for V_n^{β} in (2.6). From P4, two sets of inequalities

$$L_n(R_{ca}x) \leqslant L_n(D_cx) \leqslant L_n(x) \leqslant L_n(D_ax)$$

and

$$L_n(R_{cb}x) \leqslant L_n(D_cx) \leqslant L_n(x)$$

can be derived as for (3.10). These inequalities, together with the positions that zero can be placed, yield seven possible cases for $(E_n(x), E_n(D_ax), E_n(D_cx), E_n(R_{ca}x), E_n(R_{cb}x))$.

We shall consider the case where $L_n(x) \leqslant 0 \leqslant L_n(D_ax)$ to demonstrate key ideas. In this case,

$$(E_n(x), E_n(D_ax), E_n(D_cx), E_n(R_{ca}x), E_n(R_{cb}x)) =((E_{1n}(x), E_{2n}(D_ax), E_{1n}(D_cx), E_{1n}(R_{ca}x), E_{1n}(R_{cb}x)).$$

Similar to the proof of Lemma 3.3, we obtain

$$\begin{aligned} E_{1n}'(x)-E_{2n}'(x)&=\mu_{1a}E_{2n}(D_ax)+(\mu_{1c}-\mu_{1a})E_{1n}(x)\\ &\quad-\mu_{1c}((1-P_{1a}-P_{1b})E_{1n}(D_cx)\\ &\quad+P_{1a}E_{1n}(R_{ca}x)+P_{1b}E_{1n}(R_{cb}x))\\ &=-\mu_{1a}(E_{1n}(D_ax)-E_{2n}(D_ax))+J_{1n}(x)\\ &=-\mu_{1a}L_n(D_ax)+\mu_{1a}L_n(D_ax)+(\mu_{1c}-\mu_{1a})L_n(x)\\ &=(\mu_{1c}-\mu_{1a})L_n(x). \end{aligned}$$

Since $\mu_{1c} - \mu_{1a} > 0$ and $L_n(x)$, is increasing x_c and decreasing in x_a and in x_b, $E'_{1n}(X) + E'_{2n}(X)$ satisfies P4. Consequently $E_n(x)$ satisfies P4. Q.E.D.

Thus, by mathematical induction we conclude the following.

Theorem 1: $V_n^\beta(x)$ satisfies P1–P4 for $\forall n \in N$, $x_i > 0$, $i = a$, b, c, and $\beta < 1$.

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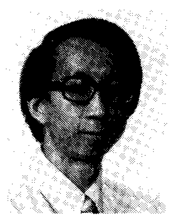

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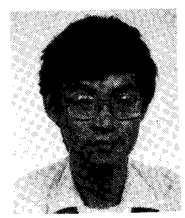

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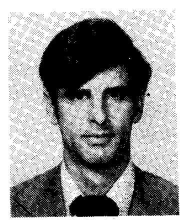

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