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Credibility in Stackelberg games *

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In a Stackelberg game, the leader can form incentives and raise threats upon followers to improve his own performance. The issue of credibility concerns whether the followers believe or not in the incentives and threats declared by the leader. In this paper, credibility is studied for two-person, single-stage games. A strategy is said to be fully credible if it satisfies the Principles of Optimality under the Stackelberg setting. The conditions for the existence of a fully credible Stackelberg strategy are explicitly stated and proved. It is shown that these conditions are fairly stringent, and are satisfied only by a very restricted class of games. When a fully credible strategy does not exist, several possible solutions are then discussed.

Keywords: Game theory, Stackelberg games, Incentives, Credibility, Principle of optimality.

1. Introduction

In a Stackelberg game, the leader is in a position to declare his strategy at the very beginning of the game to induce the behavior of other decision makers (followers). Recent studies have shown that this leader's ability to form incentives and raise threats can greatly improve the leader's cost function, provided that the followers are convinced that the leader is really committed to the incentives and threats he declared if those circumstances arise [1,2,5,11,13,14]. The credibility assumption that the followers believe in any incentive and threat announced by the leader was routinely made, and only recently has been examined by a few authors. In Ho and Olsder [6], the issues of information structures, incentives, threats and bluffing were studied for a three-level game. In Ho and Tolwinski [7], credibility was investigated in the context of a repetitive, two-level game. In Luh, Chang and Chang [9], the authors used the Inducible Region Concept to explain credibility in terms of the Principles of Optimality, and this point was then further elaborated by Ho [3].

We shall in this paper examine credibility for two-person, single-stage games along the line of [9] and [3]. We say that a strategy is fully credible if it satisfies the principles of optimality under the Stackelberg setting as defined in [9]. The conditions for the existence of a fully credible Stackelberg strategy are explicitly stated and proved. It is shown that these conditions are fairly stringent, and are satisfied only by a very restricted class of games. When a fully credible strategy does not exist, several possible solutions are then discussed.

2. Problem formulation and background

Consider a two-person, single-stage Stackelberg game. At the very beginning, the leader DM0 declares to the follower DM1 his strategy $\gamma_0(\cdot)$ (to be a function of DM1's control u_1). Knowing γ_0, DM1 selects a control $u_1 \in U_1$ to minimize his cost function $J_1(\gamma_0(u_1), u_1)$. Observing u_1, DM0 then calculates his control according to the declared strategy, i.e., $u_0 = \gamma_0(u_1) \in U_0$. DM0's major task is the design of his strategy γ_0, by taking into account DM1's rational reaction, to minimize his cost function $J_0(u_0, u_1)$. This problem is known to have the 'reversed' information structure since although DM0 announces γ_0 first, he acts after DM1 (Ho, Luh and Muralidharan [4]).

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Since DM0 can only select γ_0 (u_1 is determined by DM1 in minimizing J_1, and $u_0 = \gamma_0(u_1)$), not all the outcomes in the $U_0 * U_1$ space are realizable. The set of all realizable outcomes was called the 'Inducible Region' (IR) in Chang and Luh [2], and was delineated by Theorem 1 there. The best outcome DM0 can achieve, denoted as $u^r = (u_0^r, u_1^r)$, is the outcome in IR with the smallest J_0, i.e.,

$$u^r = \arg \min_{(u_0, u_1) \in \text{IR}} J_0(u_0, u_1)$$

(superscript r stands for reversed). For simplicity of discussion, we shall assume that u^r exists and is unique. Any γ_0 which passes through u^r and otherwise lies outside the region E defined by

$$E = \{(u_0, u_1) | (u_0, u_1) \in U_0 * U_1, \\ J_1(u_0, u_1) \le J_1(u^r)\}$$

is a 'Stackelberg strategy' for DM0 since the desired outcome u^r can be induced under the credibility assumption ([2,9], also see Figure 1). The idea is that DM1 should have higher J_1 (be punished) upon selecting any $u_1 \ne u_1^r$, so that u^r can be realized. For later use, we shall define DM0's 'Reaction Set' $R_0(u_1)$ as

$$R_0(u_1) = \{u_0' | u_0' \in U_0, \\ J_0(u_0', u_1) \le J_0(u_0, u_1) \; \forall u_0 \in U_0\}.$$

It is DM0's *best* control giving u_1. For simplicity of discussion, we shall assume that $R_0(u_1)$ is a singleton for every $u_1 \in U_1$. Also define

$$R = \{(R_0(u_1), u_1) | u_1 \in U_1\}$$

as the 'Reaction Curve'.

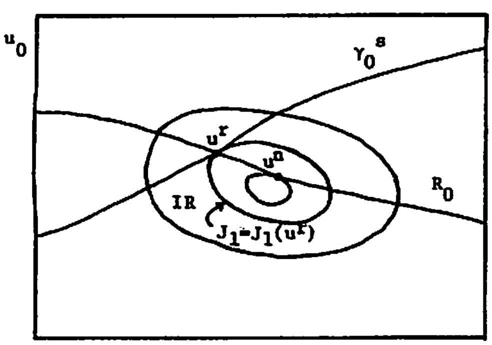

Fig. 1. A Stackelberg strategy γ_0^s.

3. Credibility and the principles of optimality

It was discussed in [9] that there are two kinds of principles of optimality for Stackelberg games: POP-I along the optimal path, and POP-II off the optimal path. They are concerned with the issue whether the leader would be tempted to deviate from his declared strategy, given desired or undesired follower's behavior. Under our formulation, if after DM1's rational selection of $u_1 = u_1^r$, there is no incentive for DM0 to deviate from his original desired outcome u^r by selecting some $u_0 \ne u_0^r$, then the desired outcome u^r is said to satisfy POP-I. Note that there are often situations where DM0 is allured to break his promise even though DM1 did exactly what he was supposed to do. On the other hand, if after DM1's irrational selection of $u_1 = u_1' \ne u_1^r$, there is no incentive for DM0 to deviate from his previously declared strategy γ_0^s by setting $u_0 = u_0'' \ne \gamma_0^s(u_1')$, then the Stackelberg strategy γ_0^s is said to satisfy POP-II. Consequently if a γ_0^s satisfies both POP-I and POP-II, there will be no incentive for DM0 to break away from it whether DM1 behaves rationally or not. It is therefore a 'fully credible' Stackelberg strategy. Note that these principles of optimality correspond to, in a loose sense, the concept of perfect Equilibrium and Sequential Equilibrium and Sequential Equilibrium for Nash games as discussed in [12,10,8]. We now have the following propositions on the principles of optimality.

Proposition 1. *If a Stackelberg strategy* γ_0^s *satisfies both* POP-I *and* POP-II, *then* $\gamma_0^s(u_1) = R_0(u_1)$ *for all* $u_1 \in U_1$.

Proof. Suppose that γ_0^s satisfies both POP-I and POP-II but not identically equals R_0. Firstly along the optimal path (u_0^r, u_1^r), if

$$\gamma_0^s(u_1^r) = u_0^r \ne R_0(u_1^r),$$

then u_0^r is not DM0's best control giving u_1^r, and POP-I is violated. Secondly off the optimal path, i.e., for some $u_1' \ne u_1^r$, if

$$\gamma_0^s(u_1') = u_0' \ne R_0(u_1'),$$

then u_0' is not DM0's best control giving u_1', and POP-II is violated. The proposition is thus established by contradiction.

Proposition 2. *If $R_0(u_1)$ can be used as DM0's Stackelberg strategy to induce u^r, then it is a Stackelberg strategy satisfying both* POP-I *and* POP-II.

This proposition is correct as can be easily seen from the definition of the reaction set.

4. A related game with normal information structure

Consider the problem formulated in Section 2 with the exception that we now let DM1 be the leader. In this new problem, DM1 announces his strategy γ_1 (a constant strategy) first, and he also acts first ($u_1 = \gamma_1$). Knowing γ_1 (or u_1), DM0 then selects the best u_0 to minimize J_0. This problem is known to have 'normal' information structure since DM1 announces his strategy first and also acts first [4]. Clearly for any u_1 announced by DM1, DM0 would select $u_0 = R_0(u_1)$. The solution to this problem, $u^n = (u_0^n, u_1^n)$, is given by

$$u_1^n = \arg\min_{u_1} J_1(R_0(u_1), u_1),$$

$$u_0^n = R_0(u_1^n)$$

(superscript n stands for normal). For simplicity, we shall assume that u^n exists and is unique. This result, together with Propositions 1 and 2, yields the following theorem.

Theorem 1. *The reversed game has a Stackelberg strategy γ_0^s satisfying both* POP-I *and* POP-II *if and only if $u^r = u^n$.*

Proof. Suppose that the reversed game has a Stackelberg strategy satisfying both POP-I and POP-II, with u^r as its outcome. From Proposition 1, $\gamma_0^s = R_0$. However, if DM0 declares $R_0(u_1)$ as his strategy, then DM1 would select u_1^n as his control. This would then yield u^n as the outcome of the game. Consequently under the uniqueness assumptions of u^r and u^n, we have $u^r = u^n$.

Now suppose that $u^r = u^n$. As we just discussed, if DM0 declares R_0 as his strategy, DM1 would select u_1^n ($= u_1^r$), and this would then yield u^n ($= u^r$) as the outcome of the game. Consequently $R_0(u_1)$ can be used as DM0's Stackelberg strategy to induce u^r. From Proposition 2, we then conclude that it satisfies both POP-I and POP-II. The proof is therefore complete.

Theorem 1 says that a fully credible strategy has the property that it induces a solution u^r which would have been picked by DM1 if he were given the choice to be the leader to announce a constant strategy γ_1 instead. In other words, *a reversed game has a fully credible strategy if and only if the resulting outcome remains unchanged under a swapping of the leadership.* This is a fairly stringent requirement, and is satisfied only by a very restricted class of games. For illustration purpose, we shall consider an example with quadratic cost functions, and show that the coefficients of the cost functions must satisfy certain conditions in order to have $u^r = u^n$.

Example. Suppose

$$J_0(u_0, u_1) = 0.5u_0^2 - 0.5u_0u_1 + a_1u_1^2 + a_2u_1,$$

$$J_1(u_0, u_1) = (u_0 - 0.5)^2 + 4(u_1 - 0.5)^2,$$

$$u_0 \in U_0 = [0, 1], \quad u_1 \in U_1 = [0, 1].$$

According to Theorem 1 of [2], the boundary of IR is delineated by the $J_1 = \frac{1}{4}$ constant contour. It is also easy to see that

$$R_0(u_1) = 0.5u_1 \quad \text{and} \quad (u_0^n, u_1^n) = (9/34, 9/17).$$

On the other hand,

$$(u_0^r, u_1^r) = \arg\min_{(u_0, u_1) \in \mathrm{IR}} J_0.$$

It can be shown that we need to have

$$a_2 = -18a_1/17 + 9/68$$

in order to achieve $u^r = u^n$.

5. Possible solutions

If a game does not have a fully credible Stackelberg strategy, what could the leader do? One way is to concentrate on optimality and forget about credibility, i.e., select any γ_0^s which induces u^r, as suggested by most existing results. Such a strategy can achieve DM0's minimum cost if DM1 behaves rationally. It could, however, create a disaster if DM1 deviates from being rational for some reason.

Another way is to emphasize credibility at the sacrifice of optimality. From above discussion, it is clear that DM0 should in this case select $R_0(u_1)$

as his strategy, and try to induce u^n. Since there is no incentive for DM0 to deviate from $R_0(u_1)$ for any $u_1 \in U_1$, such a strategy is credible. The cost is apparently not minimum (if $u^r \neq u^n$).

The third way is the so called 'Optimal Stackelberg Strategy' γ_0^* as suggested in [9]. The essential idea is to punish DM1 in the way that hurts DM0 the least upon any DM1's deviation. Such a strategy maintains optimality, and is 'partially credible'.

6. Summary

In this paper, the conditions for a single-stage Stackelberg game to have a fully credible Stackelberg strategy are stated and proved. It is then shown that a fully credible Stackelberg strategy exists if and only if the resulting outcome remains unchanged under a swapping of the leadership. When a fully credible Stackelberg strategy does not exist, several possible solutions are discussed.

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