Power Portfolio Optimization in Deregulated Electricity Markets With Risk Management

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***Abstract*—In a deregulated electric power system, multiple markets of different time scales exist with various power supply instruments. A load serving entity (LSE) has multiple choices from these instruments to meet its load obligations. In view of the large amount of power involved, the complex market structure, the risks in such volatile markets, the stringent constraints to be satisfied, and the long time horizon, a power portfolio optimization problem is of critical importance for an LSE to serve its load, maximize its profit, and manage its risks. In this paper, a midterm power portfolio optimization problem with risk management is presented. Key instruments are considered, risk terms based on semi-variances of spot market transactions are introduced, and penalties on load obligation violations are added to the objective function to improve algorithm convergence and constraint satisfaction. To overcome the inseparability of the resulting problem, a surrogate optimization framework is developed, enabling a decomposition and coordination approach. Numerical testing results show that our method effectively provides decisions for various instruments to maximize profit and manage risks, and it is computationally efficient.**

***Index Terms*—Deregulated electricity market, power portfolio optimization, risk management.**

I. Introduction

IN a deregulated electric power system, multiple markets of different time scales exist, e.g., forward market, day-ahead market, and real-time market; each market has multiple power supply instruments. A load serving entity (LSE) has multiple choices from these instruments to meet its load obligations. For example, in the forward market, different contracts for purchasing or selling power at fixed prices are available months before the operating day to hedge against risks. In the day-ahead market, an LSE can submit demand bids and generation offers to purchase and sell power at the market-clearing price (MCP). Also, generation units can be self-scheduled to provide power for the operating day. In the real-time market, certain contracts, such as options, can be executed or not based on needs, and an LSE can directly purchase power from or sell power to the spot market at real-time market prices. In view of the large amount of power involved, the complex market structure, and the risks in such volatile markets, a power portfolio problem is of critical importance for an LSE to serve its load, maximize its profit, and manage its risks.

A midterm power portfolio optimization problem with risk management is presented in this paper. The time horizon ranges from a month to a year. The problem is difficult in view that the instruments of different markets have different time scales but are coupled through load obligation constraints. Also, the conventional risk term defined by portfolio variances [9] is not appropriate in the power market because an LSE is at risk only when purchasing power at prices higher than expected or when selling power at prices lower than expected. It is difficult to define a risk term that captures this characteristic and can be efficiently managed within an optimization framework. Finally, in view that some decision variables such as the amount of strips/options and pumping levels of pumped-storage units have discrete values and that there are tight limits on spot market purchases/sales to bound market risks, it is difficult to obtain fast convergence and good feasible solutions.

A literature review is presented in Section II, where it can be seen that a good method that efficiently manages risks within an optimization framework for a midterm large-scale portfolio optimization is lacking. The problem formulation is described in Section III. Risk terms based on semi-variances of spot market costs are introduced. To improve algorithm convergence and constraint satisfaction, quadratic penalty terms on the violation of load obligation constraints are added to the objective function. In view that the penalty term is not additive, the resulting problem is inseparable. To overcome this difficulty, a surrogate optimization framework is developed in Section IV. In the process, all decision variables associated with a particular instrument are pulled out to form an instrument subproblem, and decision variables for this instrument are optimized while keeping all other variables at their latest available values. Numerical testing results presented in Section V show that our method effectively provides near-optimal decisions to serve the load, maximize the profit, and manage risks, and the method is computationally efficient.

II. Literature Review

Studies on midterm power portfolio optimization and risk management can be categorized into three major approaches: portfolio evaluation based on existing financial models; Markowitz mean-variance based methods; and stochastic programming.

Existing financial models have been used in portfolio optimization to evaluate power supply instruments and portfolios. For example, an option-pricing model, which evaluates an option by creating a replicating portfolio whose costs equal those

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of the option, was used to evaluate individual instruments in [14]. The problem is then to maximize the overall profit, subject to operational and financial constraints. The problem thus formulated, however, is a large-scale, nonlinear, mixed-integer stochastic optimization problem and cannot be directly solved by using mathematical programming techniques. Heuristics were used to solve the problem. A value-at-risk model, which measures the minimum expected cost of a portfolio within a given confidence interval, was used to evaluate a power portfolio in [5]. The problem is then to minimize the cost of the portfolio subject to operational and financial constraints. The resulting problem is also a nonlinear, mixed-integer, stochastic optimization problem. Stochastic dynamic programming was used to solve a small portfolio problem. A similar approach was presented later in [11].

The Markowitz mean-variance model has been widely used in portfolio optimization [9]. The problem is to minimize the risk, which is defined as the variance of a portfolio, for a given return. Quadratic programming is used to solve the problem. The mean-variance model was extended to power portfolio optimization problems in [4], [16], and [17]. A power portfolio optimization problem considering forward contracts and generation units was presented in [17]. It is to minimize the cost variance of the portfolio subject to transaction cost limits, generation unit constraints, and financial constraints. Because of the on/off decision variables of generation units, the problem thus formulated is a mixed-integer programming problem. Heuristics were used to solve a small problem. The risk definition, however, is not appropriate in the power market as explained before.

A stochastic programming model was presented in [13]. The power supply instruments include forward contracts, generation units, and spot market transactions. The problem is to maximize the profit while serving the load. To solve the problem, a large number of scenarios were generated through statistical models. Scenario analysis based on a decomposition technique was then used to select portfolio positions that perform well. A similar approach was presented earlier in [12]. A shortcoming of this method is that it is highly dependent on the choice of scenarios.

From the above, it can be seen that a good method that efficiently manages risks within an optimization framework for a midterm large-scale portfolio problem is lacking.

III. Problem Formulation

Consider a midterm power portfolio optimization problem with time horizon ranging from a month to a year. The time horizon consists of K discrete time intervals of equal duration Δt, with time index k ranging from 1 to K. For simplicity, physical constraints such as network limitations (congestion) are not considered. In view that decisions are made months before the operating day, it is assumed that the day-ahead market is merged with the real-time market. The power supply instruments include strips, call/put options, generation units, and spot market purchases/sales. In the following, uncertainties, individual instruments, load obligation constraints, risk terms, and the objective function are presented in detail.

A. Uncertainties

There are various uncertainties in a deregulated electricity market, such as spot market prices, load obligations, and strip/option prices. In view that spot market prices are much more volatile than other sources of uncertainties, they are modeled as random variables, while others are considered as deterministic. Assume that probability density functions (pdfs) of the spot market prices [S(k)] are given, e.g., lognormal distributions [5], [8], and are independent across hours in view of the long-term prediction. Based on these pdfs, a set of price levels are obtained for each hour after discretization.

B. Strips

A strip is a contract of purchasing or selling a fixed amount of power at a fixed price months before the operating day. For example, LSEs can purchase a December weekday on-peak strip at the beginning of the year. The price and amount of power are the same for every on-peak weekday hour in December.

To describe strips mathematically, let N_{strip} be the number of available strips, $c_{i,strip}$ ($/MW) the price of Strip i, and $P_{i,strip}$ (MW) the level of Strip i. For purchasing, $P_{i,strip}$ is positive; otherwise, $P_{i,strip}$ is negative. Usually, strips are purchased or sold at multiples of a discrete block

$$P_{i,strip} = \{0, \Delta P_{i,strip}, 2\Delta P_{i,strip}, \ldots, n_{i,strip}\Delta P_{i,strip}\} \quad (1)$$

where $\Delta P_{i,strip}$ (MW) is the size of a block, and $n_{i,strip}$ is the maximum number of blocks for Strip i. Binary indicators $\{\Psi_{i,strip}(k)\}$ are introduced to describe the strip type. For example, for a weekday on-peak strip, $\Psi_{i,strip}(k)$ is given as

$$\psi_{i,strip}(k) = \begin{cases} 1, & \text{if k is a weekday on peak hour} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Among the above, $P_{i,strip}$ is the decision variable, and other variables are assumed given.

The cost of Strip i is given as

$$J_{i,strip} = P_{i,strip} \sum_{k=1}^{K} \psi_{i,strip}(k) c_{i,strip}. \quad (3)$$

C. Options

Another contract in the forward market is an option, which is the right to purchase or sell a fixed amount of power at a fixed price months before the operating day. A certain amount of premium will be paid, and the decision to execute the option or not is made on the operating day/hour. There are two types of options: call options (to purchase) and put options (to sell).

Let N_{option} be the number of available options, $c_{i,prem}$ ($/MW) the premium, $c_{i,option}$ ($/MW) the price, and $P_{i,option}$ (MW) the level of Option i. For purchasing, $P_{i,option}$ is positive; otherwise, $P_{i,option}$ is negative. Similar to a strip, Option i is purchased or sold at multiples of a discrete block

$$P_{i,option} = \{0, \Delta P_{i,option}, 2\Delta P_{i,option}, \ldots n_{i,option}\Delta P_{i,option}\} \quad (4)$$

where $\Delta P_{i,option}$ (MW) is the size of a block, and $n_{i,option}$ is the maximum number of blocks. The option type is described by $\{\Psi_{i,option}(k)\}$ defined similar to (2), and option executions are described by binary decision variables $\{e_i(k)\}$. Among the above, the option amount $P_{i,option}$ and execution variables $\{e_i(k)\}$ are decision variables, and other variables are assumed given.

The cost of Option i is given as

$$J_{i,option} = P_{i,option}\sum_{k=1}^{K}\left[c_{i,prem}\psi_{i,option}(k) + e_i(k)\psi_{i,option}(k)c_{i,option}\right]. \quad (5)$$

D. Generation Units

LSEs may possess generation units, including thermal, pumped-storage, and hydro units, and these units can be self-scheduled to serve load obligations. The decision variables are the generation or pumping levels.

Let N_t be the number of thermal units, $P_{ti}(k)$ (MW) the generation level, $C_{ti}(P_{ti}(k))$ ($) the generation cost, and $S_i(k)$ ($) the start-up cost of Unit i. Detailed description of individual thermal units can be found in [7]. The costs of Unit i include the generation cost and start-up cost given as

$$J_{ti} = \sum_{k=1}^{K}\left[C_{ti}\left(P_{ti}(k)\right) + S_i(k)\right]. \quad (6)$$

Let N_p be the number of pumped-storage units and $P_{pi}(k)$ (MW) the generation/pumping level of Unit i at k. For generating, $P_{pi}(k)$ is positive; otherwise, $P_{pi}(k)$ is negative. For the units under consideration, generation levels are continuous, but pumping takes a few discreet levels. Detailed description of a pumped-storage system can be found in Ni and Luh [10], where the operation cost is assumed zero.

Let N_h be the number of hydro units and $P_{hi}(k)$ (MW) the generation level from Unit i at k. Detailed description of a hydro unit can be found in [7], where the operation cost is assumed zero.

E. Spot Market Purchases/Sales

In the real-time market, an LSE can directly purchase power from or sell power to the spot market based on the stochastic spot market prices. The decision variables are power purchases or sales $P_{spot}(k)$ (MW). When purchasing power from the market, $P_{spot}(k)$ is positive; when selling power to the market, $P_{spot}(k)$ is negative. The spot market purchase/sale cost is

$$J_{spot} = \sum_{k=1}^{K} P_{spot}(k)S(k). \quad (7)$$

To confine market risks, a strict limit may be imposed on the power purchased from or sold to the spot market

$$\underline{P}_{spot} \le P_{spot}(k) \le \overline{P}_{spot}, \quad \forall k. \quad (8)$$

F. Load Obligation Constraints

The power from the above instruments must equal the given load obligation at each time k

$$P^D(k) = \left\{\sum_{i=1}^{N_{strip}} P_{i,strip}\psi_{i,strip}(k) + \sum_{i=1}^{N_{option}} e_i(k)P_{i,option}\psi_{i,option}(k) + \sum_{i=1}^{N_p} P_{pi}(k) + \sum_{i=1}^{N_t} P_{ti}(k) + \sum_{i=1}^{N_h} P_{hi}(k) + P_{spot}(k)\right\}, \ \forall k. \quad (9)$$

In view of the stochastic spot market prices, constraints (9) have to be satisfied for various scenarios. The problem is therefore complicated because of the large number of scenarios. For simplicity, the load obligation constraints (9) are approximated in the expected sense

$$h(k) = P^D(k) - E\left\{\sum_{i=1}^{N_{strip}} P_{i,strip}\psi_{i,strip}(k) + \sum_{i=1}^{N_{option}} e_i(k)P_{i,option}\psi_{i,option}(k) + \sum_{i=1}^{N_p} P_{pi}(k) + \sum_{i=1}^{N_t} P_{ti}(k) + \sum_{i=1}^{N_h} P_{hi}(k) + P_{spot}(k)\right\} = 0, \ \forall k. \quad (10)$$

G. Risk Terms

In view that spot market prices are assumed random and others, e.g., load obligations and strip/option prices, are considered deterministic, market risks can be analyzed by spot market purchases/sales. However, the conventional risk term defined by variances of costs is not appropriate in the power market because an LSE is at risk only for purchasing power at prices higher than expected or for selling power at prices lower than expected. A risk term is thus introduced based on semi-variances of spot market transactions as follows:

$$r(k) = \sum_{j\in\Omega(k)} a(k,j)\left\{P_{spot}(k,j)\left[S(k,j) - E\left(S(k)\right)\right]\right\}^2 \times \Pr\left[S(k) = S(k,j)\right] \quad (11)$$

where S(k,j) is price level j at k, $\Omega(k)$ is the set of possible price levels at k, E(S(k)) is the mean spot market price at k, and

$$a(k,j) \in \begin{cases} \alpha(\$^{-1}), & \text{if } P_{Spot} > 0 \text{ and } S(k,j) > E\left(S(k)\right) \\ & \text{or } P_{Spot} < 0 \text{ and } S(k,j) < E\left(S(k)\right) \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

H. Objective Function

The profit of a portfolio equals its income from load sales minus costs for different instruments. In view that an LSE sells power to its customers at fixed prices, it is assumed that the income is given. The problem is then to minimize the total cost while serving the load obligations (10) and managing risks. In addition, in view of the discrete characteristic of certain decision variables such as strip/option levels and pumping for pumped-storage units, the strict limit on spot market purchases/sales, and a long time horizon, the speed of algorithm convergence is an issue. To overcome the difficulties, quadratic penalty terms on the violation of load obligation constraints, $\{w_c(k)h(k)^2\}$, are introduced to the objective function, where $w_c(k)$ ($/MW^2) is the weight for load obligation violation at k. The total cost is thus the expected costs from different instruments plus risk and penalty terms as follows:

$$\begin{aligned} J = E\Bigg\{ &\sum_{i=1}^{N_{strip}} \left[P_{i,strip} \sum_{k=1}^{K} \psi_{i,strip}(k) c_{i,strip} \right] + \sum_{i=1}^{N_{option}} P_{i,option} \\ &\times \sum_{k=1}^{K} [c_{i,prem}\psi_{i,option}(k) + e_i(k)\psi_{i,option}(k)c_{i,option}] \\ &+ \sum_{i=1}^{N_t}\sum_{k=1}^{K}[C_{ti}(P_{ti}(k)) + S_i(k)] + \sum_{k=1}^{K} P_{spot}(k)S(k) \Bigg\} \\ &+ \sum_{k=1}^{K} r(k) + \sum_{k=1}^{K} w_c(k)h(k)^2. \end{aligned} \tag{13}$$

The overall problem is then to minimize (13) subject to expected load obligation constraints (10) and individual instrument constraints.[1] The decision variables are strip/option levels, option executions, power levels for generation units, and spot market purchases/sales. Among them, strip/option levels must be decided now and are therefore deterministic, and other decision variables are stochastic in view of stochastic spot market prices.

The problem thus formulated, however, is not "separable" since quadratic penalty terms are not additive. This presents challenges to the standard Lagrangian relaxation method.

IV. Solution Methodology

To overcome the inseparability difficulty, surrogate Lagrangian relaxation [18] is used to solve the problem by relaxing the expected load obligation constraints (10) using multipliers $\{\lambda(k)\}$

$$\text{Min } L, \text{ with } L \equiv J + \sum_{k=1}^{K} \lambda(k)h(k) \tag{14}$$

subject to individual instrument constraints. The key idea is then to pull out all decision variables associated with a particular instrument to form an instrument subproblem, and decision variables for this instrument are optimized while keeping other variables at their latest available values. A two-level structure is therefore formed where the low level consists of solving individual instrument subproblems, and the high level is to update the multipliers.

[1]The above formulation is a snapshot of an ongoing midterm portfolio optimization process and is generally solved periodically, e.g., every month, using the latest available information. To indirectly consider future opportunities such as new strips and options, the load obligations are assumed declining toward the future.

A. Solving the Strip Subproblem

By pulling out all terms related to $P_{i,strip}$ from L in (14), the subproblem for Strip i is given as

$$\begin{aligned} &\min_{P_{i,strip}} L_{i,strip}, \text{ with} \\ &L_{i,strip} \equiv P_{i,strip} \sum_{k=1}^{K} \psi_{i,strip}(k)[c_{i,strip} - \lambda(k)] \\ &\qquad + \sum_{k=1}^{K} w_c(k)[P_{i,strip}\psi_{i,strip}(k) - h_{i,strip}(k)]^2 \end{aligned} \tag{15}$$

where $h_{i,strip}(k)$ is calculated based on (10) by dropping the term $P_{i,strip}\psi_{i,strip}(k)$ and using latest expected values for other variables, including other strip terms j ($j \neq i$), i.e.,

$$\begin{aligned} h_{i,strip}(k) \equiv P^D(k) - E\Bigg\{ &\sum_{j=1, j\neq i}^{N_{strip}} P_{j,strip}\psi_{j,strip}(k) \\ &+ \sum_{j=1}^{N_{option}} e_j(k) P_{j,option}\psi_{j,option}(k) \\ &+ \sum_{j=1}^{N_p} P_{pj}(k) + \sum_{j=1}^{N_t} P_{tj}(k) \\ &+ \sum_{j=1}^{N_h} P_{hj}(k) + P_{spot}(k) \Bigg\}. \end{aligned} \tag{16}$$

The problem is subject to discrete values of $P_{i,strip}$ in (1). There is no expectation in (15) because the decision variable $P_{i,strip}$ is deterministic as discussed before, and all other variables are also deterministic. For this quadratic optimization problem, the solution is obtained by setting the first-order derivative of (15) equal to zero and selecting the discrete value in (1) closest to the solution.

B. Solving the Option Subproblem

By collecting terms related to Option i from L in (14), the option subproblem is given as

$$\begin{aligned} &\min_{P_{i,option}, \{E[e_i(k)]\}} L_{i,option}, \text{ with} \\ &L_{i,option} \\ &\equiv P_{i,option} \sum_{k=1}^{K} \{c_{i,prem}\psi_{i,option}(k) + E[e_i(k)]\,\psi_{i,option}(k) \\ &\qquad \times [c_{i,option} - \lambda(k)]\} \\ &+ \sum_{k=1}^{K} w_c(k)[P_{i,option}E[e_i(k)]\,\psi_{i,option}(k) - h_{i,option}(k)]^2 \end{aligned} \tag{17}$$

where $h_{i,option}(k)$ is calculated based on (10) by dropping the term $P_{i,option}E[e_i(k)]\psi_{i,option}(k)$ and using latest expected values for other variables. The subproblem is subject to discrete values of $P_{i,option}$ in (4). The decision variables are the deterministic option level $P_{i,option}$ and stochastic execution variables $\{e_i(k)\}$. In view that stochastic spot market prices, which cause $\{e_i(k)\}$ to be stochastic, do not appear in this subproblem after decomposition, and all the parameters here are deterministic, the expected execution variables $\{E(e_i(k))\}$ are therefore considered as decision variables in (17).

There are only a few discreet option levels since they are defined to be the same for hours. Also, for a given option level, the subproblem can be decomposed into K independent small problems, one for each k, in view that binary execution variables are independent across hours. There are therefore only a limited number of solutions for the option subproblem, and the problem is solved by using exhaustive search.

C. Solving the Thermal Subproblem

By collecting terms related to $P_{ti}(k)$ from L in (14), the thermal unit subproblem is given as

$$\min_{\{E[P_{ti}(k)]\}} L_{ti}, \text{ with}$$

$$L_{ti} \equiv \sum_{k=1}^{K} \{E[C_{ti}(P_{ti}(k)) + S_i(k)] - \lambda(k)P_{ti}(k)\} + \sum_{k=1}^{K} w_c(k)\left[E[P_{ti}(k)] - h_{ti}(k)\right]^2 \quad (18)$$

where $h_{ti}(k)$ is calculated based on (10) by dropping the term $E(P_{ti}(k))$ and using latest expected values for other variables. The problem is subject to thermal unit constraints. Similar to what was discussed for the option subproblem, the decision variables are the expected generation levels. Dynamic programming is used to solve the subproblem, with time instances as stages and generator statuses as states [7].

D. Solving the Pumped-Storage Subproblem

By collecting terms related to $P_{pi}(k)$ from L in (14), the pumped-storage subproblem is given as

$$\min_{\{E[P_{pi}(k)]\}} L_{pi}, \text{ with}$$

$$L_{pi} \equiv -\sum_{k=1}^{K} E[P_{pi}(k)]\lambda(k) + \sum_{k=1}^{K} w_c(k)\left[E[P_{pi}(k) - h_{pi}(k)\right]^2 \quad (19)$$

where $h_{pi}(k)$ is calculated based on (10) by dropping the term $E(P_{pi}(k))$ and using latest expected values for other variables. The problem is subject to pumped-storage unit constraints. The decision variables are the expected generation or pumping levels. In view of discrete pumping levels together with a long time horizon, the traditional LR-based method (Ni and Luh [10]) is difficult to converge. In this paper, dynamic programming is developed with time instances as stages and pond levels as states. One limitation of the DP method is the curse of the dimensionality. Because of the small number of discrete pumping levels, the number of feasible states is limited, and the DP method is computationally efficient.

E. Solving the Hydro Subproblem

By collecting terms related to $P_{hi}(k)$ from L in (14), the hydro unit subproblem is given as

$$\min_{\{E[P_{hi}(k)]\}} L_{hi}, \text{ with}$$

$$L_{hi} \equiv -\sum_{k=1}^{K} E[P_{hi}(k)]\lambda(k) + \sum_{k=1}^{K} w_c(k)\left[E[P_{hi}(k)] - h_{hi}(k)\right]^2 \quad (20)$$

where $h_{hi}(k)$ is calculated based on (10) by dropping the term $E(P_{hi}(k))$ and using latest expected values for other variables. The problem is subject to hydro unit constraints. The decision variables are the expected generation levels. The problem is solved by using a merit order allocation method, and the details can be found in [7].

F. Solving the Spot Market Subproblem

By collecting terms related to $P_{spot}(k)$ from L in (14), the spot market subproblem is given as

$$\min_{\{P_{spot}(k)\}} L_{spot}, \text{ with}$$

$$L_{spot} \equiv \sum_{k=1}^{K} E\Big\{P_{spot}(k)\left[(S(k) - \lambda(k,d)\right] + \sum_{j\in\Omega} a(k,j)\left\{P_{spot}(k,j)\left[S(k,j) - E(S(k))\right]\right\}^2 \times \Pr[S(k) = S(k,j)] + w_c(k)\left[P_{spot}(k) - h_{spot}(k)\right]^2\Big\} \quad (21)$$

where $h_{spot}(k)$ is calculated based on (10) by dropping the term $P_{spot}(k)$ and using latest expected values of other variables. The problem is subject to spot market constraints (8), and the decision variables are the stochastic power purchases/sales $\{P_{spot}(k)\}$. In view that L_{spot} is a sum of costs over all the hours and the spot market prices $\{S(k)\}$ are assumed independent across hours, the subproblem can be decomposed to K small problems, one for each k. Solutions are obtained by solving a quadratic optimization problem at each time k for each price level. The outputs are a set of spot market purchase/sale policies based on stochastic spot market prices.

G. Solving the High-Level Surrogate Dual Problem

The high-level problem is to find a set of optimal multipliers to maximize the surrogate dual function, i.e.,

$$\max_{\{\lambda(k)\}} \tilde{q}, \text{ with}$$

$$\tilde{q}(\lambda(k)) \equiv \sum_{i=1}^{N_{strip}} L^*_{i,strip} + \sum_{i=1}^{N_{option}} L^*_{i,option} + \sum_{i=1}^{N_t} L^*_{ti} + \sum_{i=1}^{N_p} L^*_{pi} + \sum_{i=1}^{N_h} L^*_{hi} + L^*_{spot} + \sum_{k=1}^{K} \lambda(k)P^D(k) + \sum_{k=1}^{K} w_c(k)h(k)^2 \quad (22)$$

where $L^*_{i,strip}$ is the optimal $L_{i,strip}$ in (15) by dropping the penalty terms and similar for others. To solve the problem, the surrogate subgradient method [18] is used as presented below.

The surrogate subgradient component with respect to $\lambda(k)$ is obtained from (10) as

$$\tilde{g}_\lambda(k) = h(k) = P^D(k) - E\left\{\sum_{i=1}^{N_{strip}} P_{i,strip}\psi_{i,strip}(k) + \sum_{i=1}^{N_{option}} e_i(k)P_{i,option}\psi_{i,option}(k) + \sum_{i=1}^{N_p} P_{pi}(k) + \sum_{i=1}^{N_t} P_{ti}(k) + \sum_{i=1}^{N_h} P_{hi}(k) + P_{spot}(k)\right\}. \quad (23)$$

The multipliers are updated according to

$$\lambda(k)^{n+1} = \lambda(k)^n + s^n\tilde{g}\left(\lambda(k)^n\right). \quad (24)$$

In the above, n is the iteration index, s^n is the step size at the nth iteration given as

$$0 < s^n < (L^* - \tilde{L}^n)/\left\|g^n_\lambda\right\|^2 \quad (25)$$

where L^* is the optimal dual value, and $\tilde{L}^n$ is the surrogate dual value obtained at the nth iteration. Since L^* is not known, it is estimated as

$$L^* \cong (1 + \beta(n))\,\hat{L}^n \quad (26)$$

where $\hat{L}^n$ is the best current surrogate dual, and $\beta(n)$ is a positive number.

Given $\{\lambda^{n+1}\}$, one subproblem is selected and solved. To guarantee algorithm convergence, the following surrogate optimization condition is checked after solving each subproblem:

$$L(\lambda^{n+1}, x^{n+1}) < L(\lambda^{n+1}, x^n) \quad (27)$$

where x refers to the set of decision variables. If such an x^{n+1} cannot be obtained, set $x^{n+1} = x^n$, and the next subproblem is solved.

H. Checking Stopping Criteria

The iterative optimization process is terminated if the number of iterations is greater than a preset number or if the level of constraint violation, i.e., the L2 norm of the subgradient vectors (23), is less than a specified small number.

I. Obtaining a Feasible Solution

The solutions for subproblems, when put together, however, are generally infeasible, i.e., the expected load obligation constraints (10) are not satisfied for some hours. To obtain feasible solutions, heuristics have been developed stepping through k from 1 to K. Based on the flexibilities of decisions, the method first adjusts the power levels of generation units in the sequence of thermal, hydro, and pumped-storage units within their capacities. If a feasible solution is obtained, the method stops. If not, option executions for the infeasible hours are checked and adjusted. Because of the discrete characteristic of option values, the adjustment of options alone may not be able to meet the load obligations. The adjustment of generation units is needed. If a feasible solution is obtained, the method stops. Otherwise, the spot market purchases/sales are adjusted within spot market transaction limitations (8).

J. Summary of the Algorithm

The overall algorithm is summarized as follows.

Step 1) [Initialize.] Initialize the multipliers $\{\lambda(k), k = 1 \text{ to } K\}$.

Step 2) [Solve one subproblem.] In the sequence of strip, call, thermal, hydro, pumped-storage, and spot market subproblems, one is selected and solved using (15)–(21).

Step 3) [Update the multipliers.] Update the multipliers using (23)–(27).

Step 4) [Check stopping criteria.] If stopping criteria have not been satisfied, go to Step 2 and solve the next subproblem. Otherwise, go to Step 5.

Step 5) [Generate feasible solutions.] Use heuristics to obtain feasible solutions if the supbroblem solutions obtained are infeasible.

K. Monte Carlo Simulations and Feasible Solutions

The outputs of the algorithm are deterministic strip/option values, expected option executions, expected generation levels, and spot market purchase/sale policies. The outputs, however, need to be verified because our problem formulation is a simplified one with expected load obligation constraints. Monte Carlo simulation (for a single snapshot of an ongoing midterm portfolio optimization process) was therefore used to check the total cost of each simulation run and the sample mean/variance and to compare the sample mean with the expected costs from the algorithm.

For each Monte Carlo simulation run, the spot market price for each hour is generated based on its distribution, and the spot market purchases/sales are determined based on the policies from the spot market subproblem. Because the solutions for subproblems are generally infeasible, e.g., the load obligation constraints (9) are not satisfied for some hours, heuristics similar to those in Section I were used to obtain feasible solutions for each simulation run.

After M simulation runs, the sample mean J_m provides an estimate of the expected cost J

$$J \approx \bar{J} = \frac{1}{M}\sum_{m=1}^{M} J_m. \quad (28)$$

To measure the optimality of the algorithm, the relative duality gap is estimated by

$$(\bar{J} - \tilde{q})/\tilde{q} \quad (29)$$

where $\tilde{q}$ is the surrogate dual cost in (22).

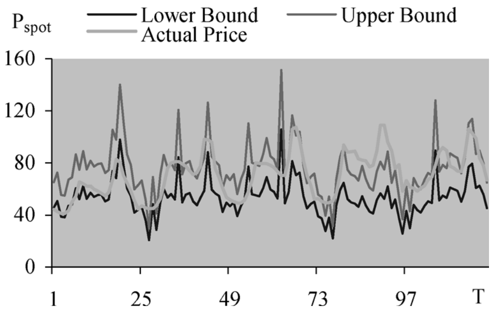

Fig. 1. Predicted spot market price confidence region and actual price.

The sample standard deviation from Monte Carlo runs is given by

$$\sigma_{\bar{J}} = \sqrt{\frac{1}{M^2}\sum_{m=1}^{M}(J_m - \bar{J})^2}. \tag{30}$$

By using (28) and (30), different methods can be compared based on the confidence region for a given probability of error.

V. NUMERICAL TESTING RESULTS

The algorithm was implemented using C++ and ran on a Pentium 4 2.5-GHz PC with 512 MB memory. In the following, hourly resolution is used. Lognormal distributions were used to describe the random spot market prices. The pdf is

$$p(x) = \frac{1}{xs\sqrt{2\pi}} e^{-(\ln x - d)^2/2s^2} \tag{31}$$

where d and s^2 are the mean and variance of the normally distributed $ln(x)$, respectively. In the testings, the standard deviation for the logarithm of a spot market price is set to be 10% of logarithm of the corresponding hourly mean price.

To examine how the lognormal distribution characterize the spot market prices, the 95% confidence intervals of the predicted spot market prices for New England Connecticut zone from January 6–10 and the actual spot market prices are depicted in Fig. 1. It shows that 86% of actual spot market prices are within the predicted confidence region. Therefore, lognormal distributions can be used to approximate the spot market prices.

Three examples are presented. In Example 1, a simple problem is tested to demonstrate how our risk management scheme affects the cost variance and decisions. In Example 2, a realistic yearly portfolio problem is tested. Algorithm performance and decisions on different instruments are analyzed. In Example 3, portfolio problems with different numbers of instruments and time horizons are tested to examine the algorithm scalability. The complete input data and results for the three examples are available at http://www.engr.uconn.edu/msl/test_data/JunX/Portfolio.txt.

A. Example 1

A simple portfolio problem of one day duration is tested. Available instruments include two strips, two hourly call options, and spot market purchases/sales as presented in Table I. The load obligations are shown in Fig. 2, and the mean spot market prices are shown in Fig. 3. It can be seen that both figures have peaks at around 6 P.M.

TABLE I
INSTRUMENT PARAMETERS

	Type	Price	Limits
Strip 1	On-peak (8 am – 11 pm)	$74.80/MWH	600 MW
Strip 2	Off-peak (Other hours)	$64.20/MWH	400 MW
Call 1	On-peak	$99.00/MWH, premium: $16.02/MWH	200 MW
Call 2	Off-peak	$70.00/MWH, premium: $14.20/MWH	100 MW
Spot market		See Figure 3	[-500, 500] MW

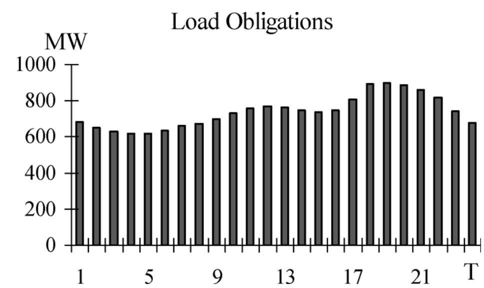

Fig. 2. Load obligations for Example 1.

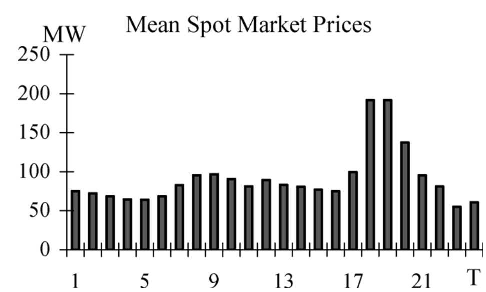

Fig. 3. Mean spot market prices for Example 1.

To demonstrate how our risk management scheme affects the cost variance and decisions, three cases are tested. In Case 1, risk management is not considered, i.e., $r(k) = 0$ in (11); in Case 2, risk is defined by variance as opposed to semi-variance of spot market transactions; and in Case 3, risk is defined by semi-variance of spot market transactions as presented in our method. Results obtained after 50 Monte Carlo runs are summarized in Table II.

The low duality gaps for the three cases show that near-optimal solutions are obtained. By comparing the expected total costs after our algorithm and the sample mean of total cost after 50 runs for all three cases, it can be seen that the simulation results match the expected results from our algorithm. By comparing Cases 1 and 3, it can be seen that our semi-variance-based risk management scheme significantly reduces the expected total cost by 33% and the sample standard deviation by 47%. The reason is that more power is purchased from spot

TABLE II
SIMULATION RESULTS (AFTER 50 MONTE CARLO RUNS)

	Case 1: Without risk terms	Case 2: Variance based risk terms	Case 3: Semi-variance based risk terms
Expected total cost after algorithm ($\$10^3$)	2098.34	1734.56	1635.53
Sample mean of total cost ($\$10^3$)	2122.52	1675.97	1595.30
Sample mean compared to Case 3	33% more	5% more	
Sample Std. Dev. ($\$10^3$)	14.56	9.68	9.89
Sample Std. Dev. compared to Case 3	47% more	2% less	
Duality gap	0.2%	0.2%	0.1%

TABLE III
DECISIONS FOR STRIPS AND CALLS OF ONE SCENARIO

	Case 1	Case 2	Case 3
Strip 1	600 MW	600 MW	600 MW
Strip 2	400 MW	400 MW	400 MW
Call 1	140 MW; executed at 5-9 pm	200 MW; executed at 5-10 pm	180 MW; executed at 5-10 pm
Call 2	100 MW; executed at all off-peak hours	100 MW; executed at all off-peak hours	100 MW; executed at all off-peak hours

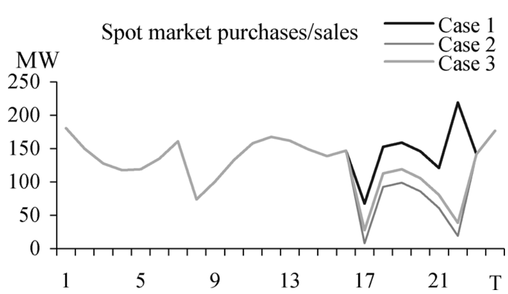

Fig. 4. Spot market purchases/sales of one scenario.

market at stochastic spot market prices in Case 1 by not considering the risks, resulting in higher total costs and larger variances. By comparing Cases 2 and 3, it can be seen that our semi-variance-based risk management scheme reduces 5% expected cost with 2% higher sample standard deviation. The reason is that less power at low prices is purchased from spot market in Case 2 by overestimating the risks, resulting in higher call costs but smaller variances.

To illustrate in detail how our risk management scheme achieves the savings, the decisions of three cases are compared for a particular scenario. The values of strips and calls are depicted in Table III, and spot market purchases/sales are shown in Fig. 4. It can be seen that on/off-peak strips are purchased at their capacities for all three cases because of the low prices. Comparing Cases 1 and 3, more power is purchased from the on-peak call, and less power is purchased from the spot market during 5–10 P.M. for Case 3 because of high spot

TABLE IV
MAJOR PARAMETERS

Strips	6 strips each month
Options	6 on-peak call/put options each month
Thermal units	3 units, capacities: 147, 192, and 19 MW
Pumped-storage units	1 unit, generation level: [60, 540] MW, pumping level: {125, 250, 375, 500} MW
Hydro units	1 unit with fixed scheduling
Spot market	Limitation: [-300, 300] MW

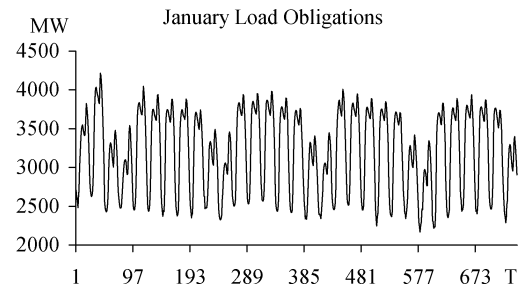

Fig. 5. Load obligation for Example 2.

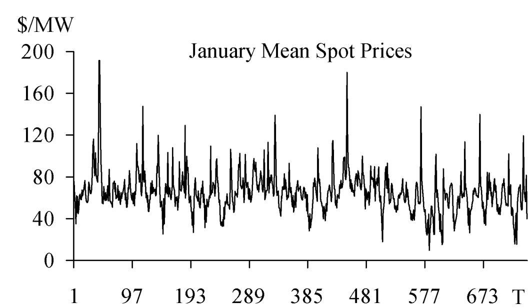

Fig. 6. Mean spot market prices for Example 2.

market price uncertainties around 6 P.M. To purchase less from the spot market and use other available instruments such as calls effectively reduces risks. Comparing Cases 2 and 3, spot market purchases/sales are further reduced for Case 2 because the variance-based method overestimates risks by weighting both sides of semi-variances of spot market transactions. In view that the expensive Call 1 is purchased more to reduce stochastic spot market purchases/sales, the total cost for Case 2 is higher than that for Case 3, although the sample standard variance is reduced as shown previously in Table II.

B. Example 2

A yearly portfolio problem is tested for an LSE in New England for 2004. The input data include available instruments, load obligations, and predicted spot market prices at the beginning of 2004. Key parameters of major instruments are presented in Table IV.

For illustration purpose, the load obligations and mean spot market prices of January 2004 are depicted in Figs. 5 and 6, respectively. It can be seen from the figures that spot market prices are much more volatile as compared to load obligations. The average on-peak spot market price for the month is \$70.7/MW.

The CPU time for this yearly problem is 302.3 s after 50 iterations. Simulation results of 50 Monte Carlo runs show that there are on average six hours of violations of constraint (9), and all of

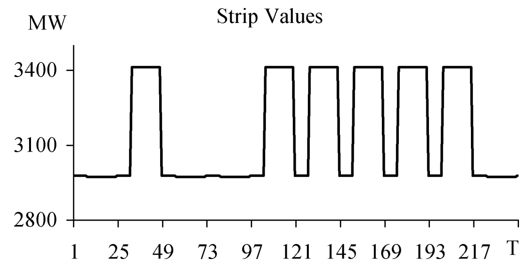

Fig. 7. Decisions for strips of one scenario.

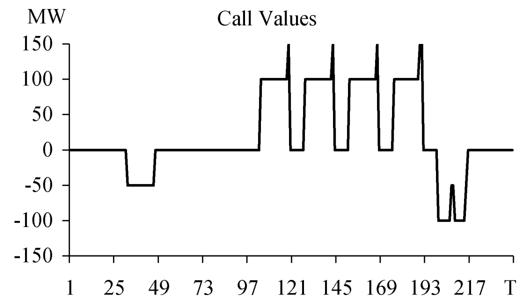

Fig. 8. Decisions for options of one scenario.

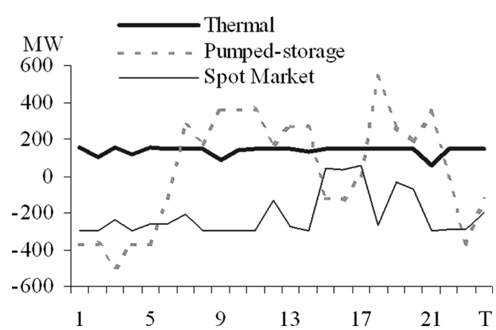

Fig. 9. Power from generation units and spot market.

them are caused by discrete values of certain decision variables. The relative duality gap (29) is 0.9%, showing that near-optimal solutions are obtained.

To examine various decisions, the results of strips and options for January 1–10 are displayed in Figs. 7 and 8, respectively, for one scenario. It can be seen that strips are the major sources to serve the load and hedge against risks (the minor strip value differences in hours 1–25 and 49–97 are due to on/off peaks in weekends/holidays). It can also be seen that call options (to purchase) are usually executed during high spot market price days/hours (e.g., January 5 on-peak hours from 104 to 119 with average spot market price $83.7/MW), and put options (to sell) are usually executed during low spot market price hours (e.g., January 9 on-peak hours from 200 to 215 with average spot market price $61.5/MW).

The sum of three thermal unit generations, the pumped-storage unit decisions, and spot market purchases/sales for January 5 (a weekday with high spot market prices) are depicted in Fig. 9. It can be seen that the units are scheduled to generate at high levels during most on-peak hours with high spot market prices (except for Hours 15–17, where spot market prices are low). During off-peak hours with low spot market prices, the pumped-storage units are scheduled to pump at high levels

TABLE V
TWO SETS OF INSTRUMENTS

Set 1	Set 2
6 strips/month	12 strips/month
6 options/month	12 options/month
1 pumped-storage unit	2 pumped-storage units
3 thermal units	6 thermal units
1 hydro unit	1 hydro unit
Spot market purchases/sales	Spot market purchases/sales

TABLE VI
COMPUTATION TIME AND PERFORMANCE OF SET 1

Time horizon	1-month	6-month	1-year
Computation time	23.3s	155.5s	302.3s
Constraint violations	1	4	9
Duality gap	0.7%	0.8%	0.7%

TABLE VII
COMPUTATION TIME AND PERFORMANCE OF SET 2

Time horizon	1-month	6-month	1-year
Computation time	41.3s	286.1s	577.4s
Constraint violations	0	3	9
Duality gap	0.6%	0.7%	0.8%

(except for Hour 7, where the load obligation is high). It can also be seen that the power is sold to the market during most hours in view that spot market prices are high for the day, and the pumped-storage unit is well scheduled.

C. Example 3

To examine the scalability of the SLR-based method, portfolio problems with different numbers of instruments and different time horizons are tested. Two sets of instruments as summarized in Table V are used, where Set 1 contains the same set of instruments as those in Example 2, and Set 2 consists of instruments of Set 1 plus the duplication of all the instruments except the hydro unit and the spot market. The load obligations for Example 2 are used for Set 1 and are increased by 95% for Set 2. Spot market prices of Example 2 are used for both sets. Three time horizons, 1-month, 6-month, and 1-year, are tested. For easy comparison, the iteration number is set as 50 for all the testings.

The simulation results of 50 Monte Carlo runs for different time horizons are summarized in Table VI for Set 1 and in Table VII for Set 2. It can be seen from the tables that computation time grows linearly with respect to the number of instruments and time horizons. For all the cases, the numbers of constraint violations are small, and by checking the results, the violations are all caused by discrete values of decision variables. Also, the small duality gaps (below 1% for all the cases) demonstrate that near-optimal solutions are obtained.

VI. CONCLUSION

This paper presents a midterm power portfolio optimization model and the corresponding methodology to serve the load, maximize the profit, and manage risks. Risk terms based on semi-variances of spot market transactions are introduced, and penalties on load obligation violations are added to the objective function to improve algorithm convergence and constraint satisfaction. A decomposition and coordination methodology based on surrogate optimization framework is then developed to solve the problem. Numerical testing results based on a load serving entity in New England show that our method provides near-optimal solutions with quantified quality for a large complex portfolio problem with different instruments to maximize the profit and manage risks, and it is computationally efficient.

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Frederick B. White, photograph and biography not available at time of publication.

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Krishnan Kasiviswanathan, photograph and biography not available at time of publication.