An Optimization-Based Approach for Facility Energy Management with Uncertainties

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Effective energy management for facilities is becoming increasingly important in view of rising energy costs, the government mandate on reduction of energy consumption, and human comfort requirements. This paper presents a daily energy management formulation and the corresponding solution methodology for HVAC systems. The problem is to minimize the energy and demand costs through control of HVAC units while satisfying human comfort, system dynamics, load limit constraints, and other requirements. The problem is difficult in view of the facts that the system is nonlinear, time-varying, building-dependent, and uncertain and that the direct control of a large number of HVAC components is difficult. In this paper, HVAC setpoints are control variables developed on top of a direct digital control (DDC) system. A method that combines Lagrangian relaxation, neural networks, stochastic dynamic programming, and heuristics is developed to predict system dynamics and uncontrollable load and to optimize the setpoints. Numerical testing and prototype implementation results show that our method can effectively reduce total costs, manage uncertainties, and shed the load; is computationally efficient; and is significantly better than existing methods.

INTRODUCTION

Effective energy management for facilities (e.g., hospitals, factories, malls, or schools) is becoming increasingly important in view of rising energy costs, the government mandate on reduction of energy consumption (Capehart et al. 2000), and human comfort requirements. The problem is to minimize the energy and demand costs through control of HVAC units while satisfying human comfort, system dynamics, load limit constraints, and other requirements. A major portion of energy consumption of a building comes from HVAC units. For example, it was reported that energy consumption of HVAC units in general accounts for 40% of total energy use for a building (Cheng et al. 1998) and in an extremely hot day even 65% (Hasnain et al. 1999). Improvement in the control of HVAC systems can result in significant savings (e.g., 25% energy use, see Zaheer-uddin and Zheng [2001]).

To control HVAC units, the traditional method is to decide the on/off or levels of HVAC components, such as chillers, fans, and dampers. This is low-level control. Since an HVAC system is composed of a large number of components, such low-level control may not be suitable for a large complex facility in view of the large number of control variables. The recent advancement of direct digital control (DDC) provides an architecture allowing high-level control through setting temperature or other setpoints while letting DDC implement the low-level

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control. For example, the night-setback strategy under DDC is to maintain room temperatures at two static setpoints during occupied time and unoccupied time.

Selecting setpoints for a DDC system to minimize the cost, however, is complex in view of the facts that the dynamics of an HVAC system are nonlinear, time-varying, and building-dependent, and a facility may consist of a large number of HVAC units. Furthermore, uncertainties generally exist, such as uncontrollable loads (referring to loads other than the HVAC loads under consideration), weather, and building occupancies. Simple DDC-based control such as night setback may let multiple HVAC units run at high power levels at the same time, causing high peak demands and high demand charges. Finally, given a tight load limit constraint or load curtailment demand, it is difficult to adjust the HVAC units to shed the load, minimize the total cost, and avoid new peaks during the system recovery period.

This paper presents a daily energy management formulation and the corresponding solution methodology to control HVAC equipment based on a DDC system in stochastic settings, with thermal loads and uncontrollable electricity loads as uncertain. In the following, a literature review is presented in the next section. The problem formulation is then established to minimize both energy and demand costs. Because the dynamics of HVAC systems are nonlinear, time-varying, and building-dependent, a neural network is used to predict the dynamics of HVAC systems. Neural networks are also used to predict the uncontrollable load. The solution methodology is presented next. Because the peak demand is not additive, the original problem is not separable. To overcome this difficulty, two new variables are introduced to transform the original problem to have a separable structure. Lagrangian relaxation (LR), a decomposition and coordination approach, is then used. Stochastic dynamic programming (SDP) is used to solve the HVAC unit subproblems, quadratic approximation is used to solve the monthly and 11-month subproblems, and heuristics are developed to obtain feasible solutions. The overall method is a semi-closed loop because the LR method is an open loop and the SDP method is a closed loop. Numerical testing and prototype implementation results presented demonstrate that the LR-based method can be embedded into a DDC system and effectively reduce the total costs, manage uncertainties, and shed the load while quickly restoring room temperatures without new peaks. The results also match our intuition that our method does not let multiple HVAC units run at high power levels at the same time during peak hours and pre-cools the rooms on a hot summer day to reduce the total cost while satisfying human comfort, load limit, and other constraints.

LITERATURE REVIEW

Both traditional control and DDC-based control were discussed in the literature. For example, an optimal on-off control of a fan and a heating coil was presented in House et al. (1991). The objective was to minimize the total energy cost plus a penalty term if control variables (HVAC unit heat input and airflow rate) are not equal to their required values. The system dynamics were described by two dynamic equations derived from energy conservation principles. The problem was discretized in time and then solved by using the sequential quadratic programming (SQP) method. Dynamic control of one HVAC unit was presented in Daryanian and Norford (1994) with hourly electricity usage as the control variable. A set of heuristics was developed to minimize electricity costs under real-time pricing based on system dynamics described by linear equations. Optimal multistage daily operations of HVAC were presented in Zaheer-uddin and Zheng (2001). The control variables are energy input, flow rate of hot water, and flow rate of air, and the problem was solved by using the singular perturbation method. The above results, however, provide strategies for one or a very limited number of HVAC units, and it is difficult to scale up the results for larger problems. Furthermore, from the view that real systems are building-dependent, nonlinear, time-varying, and uncertain, the practicality of these methods is questionable.

A DDC-based dynamic control was presented in Braun (1990). The problem was to decide the hourly room temperature setpoints to minimize the energy cost and the daily peak demand. These two objectives, however, were not handled in a unified framework, and two separate problems were formulated, one for the minimization of energy consumption and the other for the minimization of daily peak demand. Both problems were solved by using heuristics. A DDC-based control was presented in Pape et al. (1991). The method assumed that the total electricity cost could be adequately represented as a quadratic function of the ambient conditions and control variables (e.g., temperature and fan speed setpoints). The control strategies were then determined by solving a deterministic quadratic optimization problem. The coefficients of the quadratic function, however, must be determined empirically for different buildings and different operating modes. Also, the control strategies are static and may not be able to manage uncertainties. Another DDC-based control focusing on human comfort was presented in Hamdi and Lachiver (1998), where fuzzy logic was used to determine air velocity and air temperature setpoints. The method is compared with the conventional night-setback control via simulation, and results show that human comfort levels were significantly improved at almost the same energy consumption level.

From the literature review, it can be seen that a good method that can handle a large number of HVAC units, minimize both energy and demand costs, and manage uncertainties is urgently needed. Also missing is a method than can effectively shed the load while quickly restoring room temperatures without new peaks by conrolling HVAC units.

In our preliminary results presented in a conference paper (Luh et al. 2002), a model and the corresponding solution methodology to control HVAC equipment in a stochastic setting were presented. The traditional control based on the HVAC compressors on/off was used, and the system dynamics were described by linear equations. In practical applications, however, system dynamics are building-dependent, nonlinear, and time-varying. More realistic models and methods are needed.

PROBLEM FORMULATION

Assume that the overall system consists of I single-zone HVAC units, with unit index i ranging from 1 to I, and I rooms each controlled by an HVAC unit. Our method can be extended to multi-zone systems, as will be discussed later. The time horizon (e.g., 24 hours) is divided into K discrete time intervals of equal duration Δt (e.g., 15 minutes), with time index k ranging from 1 to K. The problem is to decide the room temperature setpoint scheduling to minimize the expected total energy and demand costs in a stochastic setting while satisfying system dynamics, temperature requirements on individual rooms and on heat exchangers, and load limit constraints. The solution obtained here is a room temperature setpoint scheduling policy: what is the room temperature setpoint for each time interval k based on realization of uncertainties. In our formulation, a practical method based on neural networks is used to predict the HVAC system dynamics. The minimization of expected total costs and the load shedding function are formulized in a single framework. In the following, constraints, uncertainties, and the objective function will be explained in detail.

Constraints

1. Dynamics of the system:

The schematic of a single-zone HVAC system is shown in Figure 1. The dynamics of the HVAC unit can be derived from the energy conservation principle as discussed below. A more practical method to describe the system dynamics will then be presented.

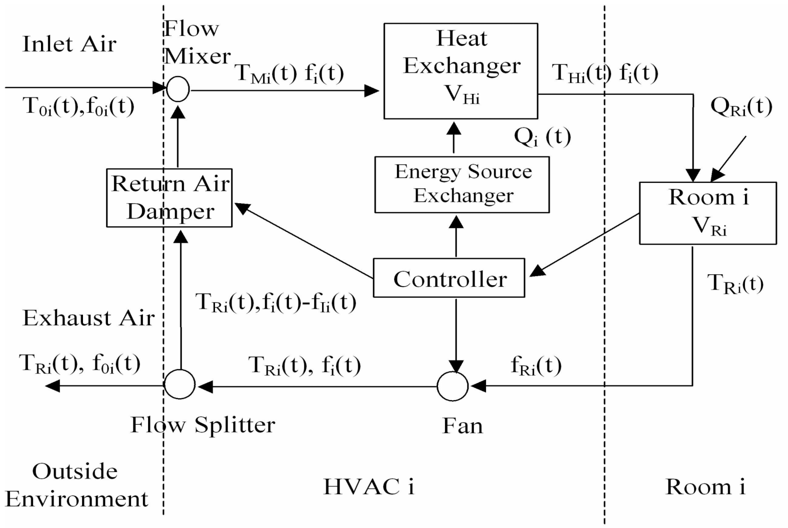

Figure 1. Schematic of the HVAC system.

Fresh inlet air enters System *i* with temperature $T_{0i}(t)$ and volumetric flow rate $f_{0i}(t)$. It is mixed with the recirculated air at the flow mixer. The mixed air with temperature $T_{Mi}(t)$ and flow rate $f_i(t)$ passes through the heat exchanger with effective volume V_{Hi}. The amount of heat $Q_i(t)$ (positive for heating and negative for cooling) is input to the heat exchanger, and the air is heated or cooled to temperature $T_{Hi}(t)$. Infiltration and exfiltration effects are neglected, so airflow rates after the heat exchanger and after the room to be mentioned later are equal to $f_i(t)$. A linear dynamic equation can be established based on the energy balance requirement as follows (House et al. 1991).

$$\dot{T}_{Hi}(t) = \frac{1}{V_{Hi}}\left[\alpha_{Hi}(T_{Mi}(t) - T_{Hi}(t))f_i(t) + \frac{Q_i(t)}{\rho c_p}\right] \tag{1}$$

In the above, ρ is the air density, C_p is the specific heat of air, and α_{Hi} is the heat transfer effective factor, which reflects thermal losses through insulation, etc.

After being heated or cooled in the heat exchanger, the discharged air with temperature $T_{Hi}(t)$ passes through Room *i* with effective volume V_{Ri} and thermal load $Q_{Ri}(t)$ to be discussed later. After mixing in the room, the air exits the room with temperature $T_{Ri}(t)$ and flow rate $f_i(t)$. Similar to the heat exchanger, a linear dynamic equation can be established as follows:

$$\dot{T}_{Ri}(t) = \frac{1}{V_{Ri}}\left[\alpha_{Ri}(T_{Hi}(t) - T_{Ri}(t))f_i(t) + \frac{Q_{Ri}(t)}{\rho c_p}\right] \tag{2}$$

where α_{Ri} is the heat transfer effective factor that reflects thermal losses through insulation, door openings, etc.

In the flow mixer, the returned air with temperature $T_{Ri}(t)$ and flow rate $f_i(t) - f_{0i}(t)$ is mixed with the inlet air. Assuming perfect mixing, an energy balance equation is obtained as follows:

$$T_{Mi}(t)f_i(t) = T_{0i}(t)f_{0i}(t) + T_{Ri}(t)[f_i(t) - f_{0i}(t)] \tag{3}$$

By replacing $T_{Mi}(t)$ in Equation 1 using Equation 3, two linear dynamic equations are obtained. The state variables are $T_{Hi}(t)$ and $T_{Ri}(t)$, and control variables are heat inputs $Q_i(t)$. The thermal load, the outside weather condition, and the airflow rate are assumed for simplicity as system parameters.

As mentioned earlier, the dynamics of HVAC systems in practice, however, are nonlinear, time-varying, and building-dependent. It is difficult to assume a set of equations and then tune the parameters for different buildings at different times. To overcome these difficulties, a neural network is used to predict the dynamics of HVAC systems, instead of using the linear system dynamics (Equations 1 to 3). In this paper, a multilayer perceptron (MLP) neural network trained by back propagation (BP), as presented in Zhang and Luh (2001), is used. Since our daily management model is developed on top of DDC in discrete time, the decision variables are the room temperature setpoints $T^s_{Ri}(k)$. The neural network predicts the heat exchanger temperature and the unit energy consumption for the Δt interval. Additional inputs for the network include the thermal load, the outside weather condition, and the room airflow rate as described earlier; day of week and time of day; and the room temperature and heat exchanger temperature at k. An MLP network with a single hidden layer is shown in Figure 2.

The neural network can be used for different rooms, different buildings, and different seasons by training based on historical data and then by updating based on the latest information. Also, the method can be used for HVAC systems with different structures, e.g., multi-zone HVAC systems, by assuming that the multi-zones have the same room temperature setpoints.

The room temperature changes from the previous setpoint to a new one and reaches equivalence somewhere during the Δt. Assume that the temperatures are measured at the ending instance of a Δt interval (as shown in Figure 3).

2. Room temperature comfort range:

The comfort range for room i can be different for occupied and unoccupied periods; for example:

$$T_{Ri}(k) \in \begin{cases} [20°\text{C}, 23°\text{C}] & \text{occupied,} \\ [19°\text{C}, 25°\text{C}] & \text{unoccupied.} \end{cases} \tag{4}$$

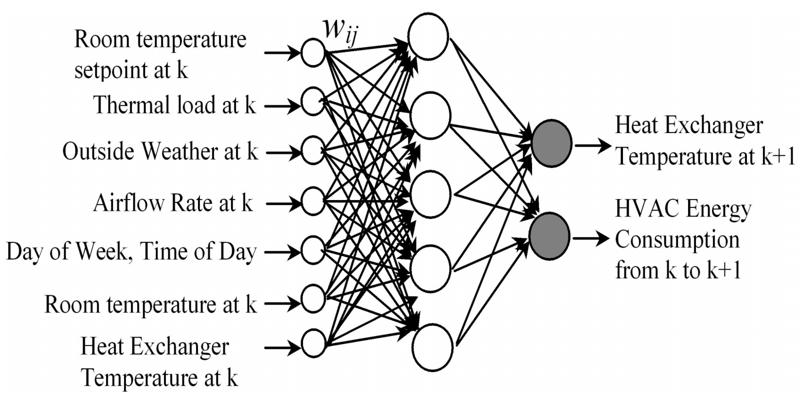

Figure 2. Schematic of a neural network for system dynamics prediction.

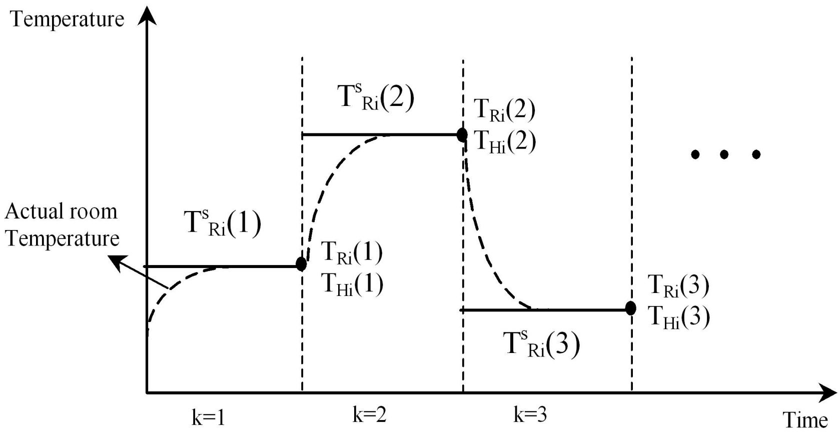

Figure 3. Timing of room temperature and setpoints.

Compared with the occupied period, the lower bound of $T_{Ri}(k)$ during the unoccupied period is usually lower to allow precooling, and the upper bound is higher to save electricity. Assume that the initial and terminal room temperatures are given.

3. **Heat exchanger temperature range:**
The discharged air temperature at the heat exchanger i should be within a certain range; for example:

$$T_{Hi}(k) \in [10°\text{C}, 30°\text{C}] \quad \forall k \tag{5}$$

Assume that the initial and terminal heat exchanger temperatures are given.

4. **Load limit constraint:**
Sometimes, there is a load limit for the entire time horizon K, or there is a load shedding demand for a certain period. In these cases, the expected average Δt load at k, which is the sum of I HVAC units' load $d_{Hi}(k)$ and the uncontrollable load $d_{uc}(k)$, should be less than a given load limit $d_L(k)$,

$$E\left[\left[\sum_{i=1}^{I} d_{Hi}(k|T_k, f_k) + d_{uc}(k)\right] - d_L(k)\right] \le 0 \quad \forall k\ . \tag{6}$$

If the limit is large enough, or there is no load shedding demand, this constraint can be neglected.

Uncertainties

1. **Uncontrollable load:**
The uncontrollable load $d_{uc}(k)$ is uncertain and must be predicted in advance in our daily scheduling problem. An MLP neural network trained by BP, similar to the one shown in Figure 2, is used. The training was based on outside weather conditions (e.g., outside temperature, humidity, solar radiation, etc., available from the Internet; time, day, season; historical uncontrollable load; and building occupancies.

2. Thermal load:

Assume that the thermal load of room *i*, $\{Q_{Ri}(k)\}$, consists of a deterministic part, such as the energy from lighting, and a stochastic part, contributed for example by uncertain room occupancies. To simplify the derivation, it is assumed that the deterministic part can be calculated by the heat generated by lighting, equipment, and other loads, and the room occupancy can be approximated according to room event schedules. The thermal load is described by a single-state Markov chain with the following one-step transition matrix:

$$P\{Q_{Ri}(k) = x_{bi}(k) \mid Q_{Ri}(k-1) = x_{ai}(k-1)\} = \pi_{abi},\ i = 1, \ldots I;\ a, b = 1, \ldots, p_i, \tag{7}$$

where p_i is the number of thermal load levels for room *i*, and $x_{ai}(k-1)$ and $x_{bi}(k)$ are possible thermal loads at $k-1$ and k, respectively. If the room occupancy data are difficult to obtain, the thermal load can be simplified to have only the deterministic part, which is not uncertain. As shown is Equation 2 and Figure 2, the thermal load affects the system dynamics, as will be discussed in the next section.

The Objective Function

Assume that a monthly electricity cost includes energy cost, monthly demand cost, and 11-month demand cost, as commonly seen for industrial or commercial facilities, e.g., Connecticut Light and Power Rate 56-58. For generality, a time-of-day energy price structure is assumed, i.e., the price at time *k* is given by $c(k)$, $k = 1, 2, \ldots K$. The flat rate and the on/off peak-based pricing are therefore special cases. The energy cost at *k* is the product of energy price $c(k)$ and energy consumption, which is the sum of I HVAC units' consumptions $\Sigma_i d_{Hi}(k)\Delta t$ plus the uncontrollable consumption $d_{uc}(k)\Delta t$. The total energy cost is then the summation of the above over *k*.

The current month peak demand is the maximum demand of the current month. Since it is difficult to consider the demand over a month for our daily energy management problem, the monthly peak demand is approximated as the maximum of the current month peak demand $\bar{d}_{cm}$ obtained from previous days within the month and the peak over the *K* time intervals under consideration. At the first day of the month, the peak demand of the same month of the previous year is used, and the value can be scaled back by a certain percentage to encourage conservation. For our daily scheduling problem, the monthly peak demand price C_{cm}' is scaled to daily price C_{cm}, and the approximate daily charge is C_{cm} multiplied by the peak demand.

The 11-month peak charge is similarly defined by scaling the 11-month peak demand price C_{11m}' to daily price C_{11m}, and by approximating the 11-month peak demand as the maximum of the previous 11 months peak demand $\bar{d}_{11m}$ and the maximum demand over the *K* time intervals. If the daily peak demand exceeds $\bar{d}_{11m}$, the effect will last for the next 11 months.

The expected total cost is therefore the expected energy costs and expected demand costs as follows:

$$\begin{aligned} J \equiv E\Bigg\{ &\sum_{k=1}^{K} c(k)\Delta t \left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) \right] + C_{cm}\max\left(\bar{d}_{cm}, \left(\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) \right)_{k=1,2,\ldots,K} \right) \\ &+ C_{11m}\max\left(\bar{d}_{11m}, \left(\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) \right)_{k=1,2,\ldots,K} \right) \Bigg\} \end{aligned} \tag{8}$$

The overall problem is to decide the room temperature setpoints $T_{Ri}^{s}(k)$ to minimize Equation 8 subject to system dynamics, temperature requirements (Equations 4 and 5), load limit constraints (Equation 6), uncertain uncontrollable load, and uncertain thermal load. In the formulation, the constraints are for individual HVAC units or rooms, and the energy costs are the sum of energy costs of individual HVAC units. The peak demand, however, is not the sum of individual HVAC units' peak demands. The problem formulated is therefore not "separable." In addition, in the view that the problem is similar to the unit commitment problem, it is believed to be NP-hard.

SOLUTION METHODOLOGY

Our idea to solve the above problem is to apply Lagrangian relaxation (LR) to obtain a near-optimal solution. LR is a decomposition and coordination approach applicable to "separable" problems. The problem thus formulated, however, is not separable. To overcome this difficulty, new variables y_{cm} and y_{11m} are introduced that are, respectively, defined as the expected current month peak demand in Equation 9 and the expected current 11-month peak demand in Equation 10.

$$y_{cm} \equiv \max\left(\bar{d}_{cm}, E\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right]_{k=1,2,\ldots,K}\right) \tag{9}$$

$$y_{11m} \equiv \max\left(\bar{d}_{11m}, E\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right]_{k=1,2,\ldots,K}\right) \tag{10}$$

The original problem is then transformed to have a "separable" structure,

$$\min_{T^s_{Ri}(k)} J, \text{ with } J \equiv E\left\{\sum_{k=1}^{K} c(k)\Delta t\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right] + C_{cm}y_{cm} + C_{11m}y_{11m}\right\}, \tag{11}$$

subject to system dynamics, temperature requirements (Equations 4 and 5), load limit constraints (Equation 6), and the following new constraints enforcing Equations 9 and 10.

Monthly Peak Demand Constraint

$$E\left[\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right] - y_{cm}\right] \le 0 \quad \forall k \tag{12}$$

$$\bar{d}_{cm} - y_{cm} \le 0 \tag{13}$$

11-Monthly Peak Demand Constraint

$$E\left[\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right] - y_{11m}\right] \le 0 \quad \forall k \tag{14}$$

$$\bar{d}_{11m} - y_{11m} \le 0 \tag{15}$$

For this separable problem, Lagrangian relaxation can be effectively applied with the Lagrangian given by

$$\underset{T^s_{Ri}(k)}{\text{Min}}\ L,\ \text{with}\ L \equiv E\left\{\sum_{k=1}^{K} c(k)\Delta t\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k)\right] + C_{cm}y_{cm} + C_{11m}y_{11m}\right\}$$
$$+ \sum_{k=1}^{K} \lambda(k)E\left\{\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) - y_{cm}\right]\right\} + \sum_{k=1}^{K} \mu(k)E\left\{\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) - y_{11m}\right]\right\} \quad (16)$$
$$+ \sum_{k=1}^{K} \nu(k)E\left\{\left[\sum_{i=1}^{I} d_{Hi}(k) + d_{uc}(k) - d_L(k)\right]\right\},$$

where λ(k), μ(k), and ν(k) are, respectively, the multipliers relaxing the current month, 11-month peak demands, and load limit constraints (Equations 12, 14, and 16). The multipliers are non-negative and are shadow prices for the violation of the constraints. After regrouping relevant terms, a two-level structure can be formed. The low level consists of solving HVAC unit subproblems, the current month peak charge subproblem and the 11-month peak charge subproblem to be explained next. The high-level problem is to update the multipliers.

Solving the HVAC Unit Subproblems

After collecting terms related to HVAC unit *i* from Equation 16, the following HVAC subproblem is formed:

$$\underset{T^s_{Ri}(k)}{\text{Min}}\ L_{hi},\ \text{with}\ L_{hi}(\lambda,\mu,\nu) \equiv E\left\{\sum_{k=1}^{K} [c(k)\Delta t + \lambda(k) + \mu(k) + \nu(k)]d_{Hi}(k)\right\} \quad (17)$$

This is subject to system dynamics as represented by the neural network for the unit and temperature range constraints (Equations 4 and 5). Let L_{hi}^* denote the optimal cost of HVAC unit *i*. The problem is to determine the room temperature setpoints for time $k = 1, 2, ..., K$ to minimize unit *i*'s energy costs and the unit's cost on monthly and 11-month peak demand charges. If the demand at *k* exceeds the monthly, 11-month peak demand, or load limit, λ(*k*), μ(*k*), or ν(*k*) will be positive to penalize energy consumption; otherwise, λ(*k*), μ(*k*), or ν(*k*) will be zero at convergence.

This multi-stage stochastic optimization problem is solved by using stochastic dynamic programming (SDP) as shown in Figure 4, with time instances as stages. At stage *k*, a state x_{jk} is characterized by the room temperature, heat exchanger temperature, and thermal load (described by a single-state Markov chain [Equation 7]). By considering all possible room temperature setpoints over their desired ranges and a few discretized thermal loads, a state transition diagram can be obtained. To determine the states and to obtain the transition costs, the neural network of Figure 2 is used to predict HVAC unit *i*'s heat exchanger temperature and energy consumption.

To obtain a best room temperature setpoint scheduling policy based on the uncertain thermal loads to minimize the expected cost, backward SDP is applied. At the terminal stage $K + 1$, the room temperature and heat exchanger temperature are given, and a few discretized thermal loads and their probabilities are given. The terminal cost at stage *K*, $L_{hi}(x_{jK})$, can be determined based on HVAC system dynamics predicted by NN. To move backward, the cumulative cost $L^*_{hi}(x_{j,K\text{-}1})$ at stage *K*-1 is calculated as

$$L^*_{hi}(x_{j,K-1}) = \underset{T^s_{Ri}(K-1)}{\min} [E([c(K-1)\Delta t + \lambda(K-1) + \mu(K-1) + \nu(K-1)]d_{Hi}(x_{j,K-1}) + L_{hi}(x_{jK}))] . \quad (18)$$

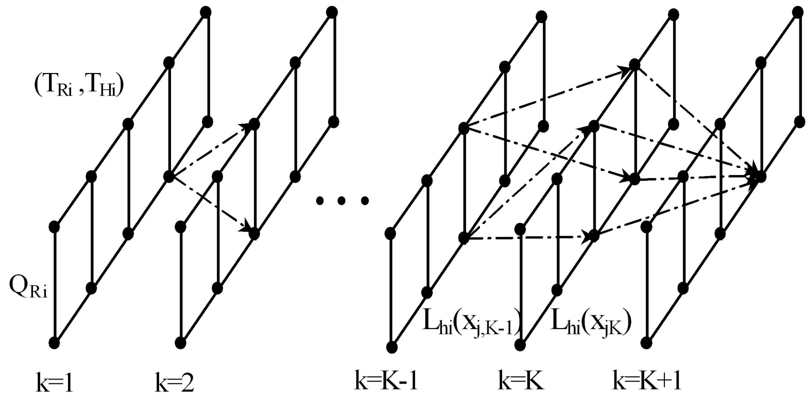

Figure 4. The structure of stochastic dynamic programming.

The method then moves backward, with optimal cumulative cost at stage k as

$$L^*_{hi}(x_{jk}) = \min_{T^s_{Ri}(k)} \left\lfloor E([c(k)\Delta t + \lambda(k) + \mu(k) + v(k)]d_{Hi}(x_{jk}) + L^*_{hi}(x_{j,k+1})) \right\rfloor . \quad (19)$$

At stage 1, the minimum cost related to HVAC unit i, $L_{hi}{}^*$, is equal to the optimal cumulative cost $L^*{}_{hi}(x_{j1})$ for the given initial state. The room temperature setpoint scheduling policy is also obtained. According to the policy, the optimal room temperature setpoints can be obtained by tracing forward the stages based on the realizations of thermal loads. The SDP method is closed loop, and the overall method is semi-closed loop since the LR method is open loop.

As a special case, if the thermal load is treated as deterministic as discussed before, the DP state will be the combination of the room temperature and heat exchanger temperature. The DP method will be modified accordingly.

Solving the Current Month Peak Charge Subproblem

By collecting terms related to y_{cm}, the current month peak charge subproblem is formed as

$$\operatorname*{Min}_{y_{cm}} L_{ycm}\text{, with } L_{ycm}(\lambda(k)) = \left[C_{cm} - \sum_{k=1}^{K} \lambda(k)\right] y_{cm}\text{,} \quad (20)$$

subject to Equation 13. Let $L_{ycm}{}^*$ denote the optimal current month peak charge. The solution for this linear optimization problem, however, may oscillate. For example, if $C_{cm} - \Sigma\lambda(k)$ varies around zero over the iterations, the optimal solution will oscillate between $\bar{d}_{cm}$ and $\propto$. Our idea to overcome such solution oscillation is to approximate the linear function by a strictly convex function, e.g., a quadratic function (Guan et al. 1995):

$$C_{cm}y_{cm} = ay^2_{cm} + by_{cm} + c \quad (21)$$

Based on testing experience, parameter b is selected as $0.8C_{cm}$, and a and c are decided so that Equation 21 is satisfied when y_{cm} equals $\bar{d}_{cm}$ and $\alpha\bar{d}_{cm}$, where $1 < \alpha < 2$.

Solving the 11-Month Peak Charge Subproblem

The 11-month peak charge subproblem is similar to the current month peak charge subproblem. The optimization problem is to

$$\underset{y_{11m}}{\text{Min}}\ L_{y11m}\text{, with } L_{y11m}(\mu(k)) = [C_{11m} - \mu(k)]y_{11m}, \tag{22}$$

subject to Equation 15. Let L_{y11m}^* denote the optimal 11-month peak charge. The problem can be solved by using a method similar to the above.

Solving the High-Level Dual Problem

The high-level dual problem is given by

$$\max_{\lambda(k)\geq 0, \mu(k)\geq 0, v(k)\geq 0} q\text{, with } q(\lambda(k),\mu(k),v(k)) \equiv$$
$$\sum_{i=1}^{I} L_{hi}^* + L_{ycm}^* + L_{y11m}^* + \sum_{k=1}^{K} [c(k)\Delta t + \lambda(k) + \mu(k) + v(k)]E[d_{uc}(k)] - \sum_{k=1}^{K} v(k)d_L(k)\ . \tag{23}$$

Because the decision variables $\{T_{Ri}^s(k)\}$ are discretized over their desirable ranges, the dual function q in Equation 23 is nondifferentiable. The subgradient method (Bertsekas 1999) is thus used to solve the above problem, where $\lambda(k)$, $\mu(k)$, and $\nu(k)$ are updated by using the following formula:

$$\lambda(k)^{n+1} = \max[0, \lambda(k)^n + \alpha^n g(\lambda(k)^n)] \tag{24}$$

$$\mu(k)^{n+1} = \max[0, \mu(k)^n + \alpha^n g(\mu(k)^n)] \tag{25}$$

$$v(k)^{n+1} = \max[0, v(k)^n + \alpha^n g(v(k)^n)] \tag{26}$$

In the above, n is the high-level iteration index, α^n is the step sizes at the nth iteration, and $g(\lambda(k)^n)$, $g(\mu(k)^n)$, and $g(\nu(k)^n)$ are the subgradient components of the dual function with respect to λ, μ, and ν, respectively, and are:

$$g(\lambda(k)^n) = E\left\{\left[\sum_{i=1}^{I} d_{Hi}^*(k)^n + d_{uc}(k) - y_{cm}^{*\,n}\right]\right\}, \tag{27}$$

$$g(\mu(k)^n) = E\left\{\left[\sum_{i=1}^{I} d_{Hi}^*(k)^n + d_{uc}(k) - y_{11m}^{*\,n}\right]\right\}\text{, and} \tag{28}$$

$$g(v(k)^n) = E\left\{\left[\sum_{i=1}^{I} d_{Hi}^*(k)^n + d_{uc}(k) - d_L(k)\right]\right\}. \tag{29}$$

The step size α^n is given by

$$\alpha^n = \beta^n \times \frac{L^U - L^n}{g(\lambda^n,\mu^n,v^n)^T g(\lambda^n,\mu^n,v^n)}, \tag{30}$$

where L^U is an upper bound of the optimal value of Equation 23, L^n is the value of the dual function at the nth iteration, and $0 < \beta^n < 2$. The convergence of this subgradient method depends on the selection of parameters L^U and β^n. It has been shown that adaptive adjusting of parameters L^U and β^n will speed up the convergence. In our implementation, parameters are reduced if the value of L^n remains approximately the same over several iterations (Chen et al. 1998).

Obtaining Feasible Solutions

The solutions to subproblems, when put together, generally are infeasible, i.e., load limit constraint (Equation 6), monthly peak constraint (Equation 12), and 11-month peak constraint (Equation 14) are not satisfied. To obtain a good feasible solution, three heuristics have been developed. The first method is to adjust room temperature setpoint scheduling policies to satisfy Equations 6, 12, and 14. If no feasible solution can be found because of low monthly and/or 11-month peaks, Heuristics 2 increases the monthly and 11-month peaks while satisfying Equation 6. If this can be done, a feasible solution is still obtained from the original problem's point of view. If no feasible solution can be found because of a low load limit, Heuristics 3 sacrifices comfort by increasing the upper bound of the comfort constraint (Equation 4) to satisfy Equation 6. The details are presented below.

- *Heuristics 1.* This method is to check the total load from time $k = 1$ and work forward in time. If the total load at time k exceeds the load limit or monthly or 11-month peaks, the method backtracks to the previous stage $k - 1$ and pre-cools the rooms. For simplicity, there is no backtracking to $k - 2$. In the process, the temperature setpoint of a randomly selected room at time $k - 1$ is reduced by a fixed value, e.g., 0.2°C, so as to reduce the load in time k. The adjusted energy consumption can be estimated based on the HVAC system dynamic neural network. If the total load at time $k - 1$ exceeds the load limit or peaks, the setpoint will not be adjusted but kept at its original value. The method then checks if the total load is below the limit. If the load is below the limit, the method stops; Otherwise, the method continues for setpoints of another randomly selected room until the method stops or all the rooms are selected for adjustment.
- *Heuristics 2.* If no feasible solution can be found by Heuristics 1 because of low monthly or 11-month peaks, the second method increases y_{cm} and y_{11m} to the daily peak. This method therefore changes Equations 12 and 14, which are in the modified problem formulation. From the original problem's point of view, however, Equations 4 to 6 are satisfied.
- *Heuristics 3*. If no feasible solution can be found after Heuristics 1 because of low load limit constraints, the third method determines a proper room temperature comfort range. The method increases the upper bound in Equation 4, and restarts the algorithm. This method saves money and reduces peaks at the cost of human comfort.

After obtaining the feasible solution, the duality gap, i.e., the difference between the feasible cost and dual cost q in Equation 23, is obtained. Usually the duality gap is not zero, and a small duality gap shows that a near-optimal solution is obtained.

Summary of the Algorithm

The overall algorithm is summarized as follows:

- Step 1: [Initialize.] Initialize all the multipliers $\lambda(k)$, $\mu(k)$, and $\nu(k)$ to zero.
- Step 2: [Solve the HVAC unit subproblems.] Use SDP to solve the HVAC unit subproblems in Equation 17.
- Step 3: [Solve the monthly and 11-month subproblems.] Use quadratic approximation to solve the monthly and 11-month demand subproblems in Equations 20 and 22.
- Step 4: [Update the multipliers.] Update the multipliers using Equations 24 to 30.
- Step 5: [Check stopping criteria.] If stopping criteria have not been satisfied, go to Step 2.

Table 1. Rate Structure

Energy Charge:	
On-peak (Weekdays 7 a.m.-11 p.m.)	\$0.0653/kWH
Off-peak (All other hours)	\$0.0436/kWH
Demand Charge:	
Monthly charge:	\$5.09/kW
11-Month charge:	\$3.56/kW

Otherwise, go to Step 6.

- Step 6: [Generate feasible solutions.] Use heuristics to obtain feasible solutions if the sub-problem solutions obtained are infeasible.

NUMERICAL TESTING AND ACTUAL CONTROL RESULTS

The algorithm was implemented in C++ and run on a P4 2.5GHz PC with 512MB memory. The 24-hour period is divided into 96 intervals of 15 minutes each, and control variables are room temperature setpoints for each of the time intervals. A commonly used rate* for industrial facilities is used, including energy prices based on on/off peaks and monthly/11-month demand prices as summarized in Table 1. A problem with hourly variable rate will have the same computational complexity as compared to a problem with on/off peak-based pricing because they are all based on the same price structure of $c(k)$, $k = 1, 2, \ldots K$ in the algorithm.

Four examples are considered. In Example 1, a simple system with two single-zone HVAC units is examined. Our LR-based method is tested and compared with a night-setback control, which maintains a static setpoint (22°C) during occupied time (9 a.m.-6 p.m.) and keeps room temperature below 25°C during unoccupied time. In Example 2, the load shedding function is tested, and room temperature recovery is analyzed. In Example 3, a system with different numbers of single-zone HVAC units and thermal load levels is used to examine the computational requirements and quality of the LR-based method. In Example 4, our LR-based method is implemented for a building. Results from two groups of HVAC units running under the LR-based method and under a night-setback method are compared and analyzed.

Example 1

A system with two single-zone HVAC units and two thermal load levels at each stage is considered. The key parameters are presented in Tables 2 and 3.

To train the HVAC system dynamic neural network, three months' data were generated according to Equations 1 to 3. Table 4 shows the prediction results of Unit 1. To train the uncontrollable load neural network, three-month historical load data were collected from a building. The prediction results were scaled so that the uncontrollable load is about 40% of the total load at noon, as shown in Figure 6. For both neural networks, the results can be improved given more training data.

Results obtained after 50 Monte Carlo runs by using the LR method and the night-setback control are shown in Table 5. The results show that the LR-based method effectively manages the uncertainties to avoid a new peak (with expected daily peak 10.29 kW) and to reduce the expected total cost (about 12.5% of total costs saved compared with the night-setback method).

To show how the LR method works, the HVAC load of one scenario using the LR-based method is depicted in Figure 7. It can be seen that the LR-based method does not let the two HVAC units run at high power levels at the same time during peak hours, so that the daily peak load does not exceed current month and 11-month peaks of 10.3 kW. The figure also shows

*Connecticut Light & Power Rate 57

Table 2. Major Parameters of the System

HVAC Unit	1	2
Initial temperature	23°C	23°C
Terminal room temperature	23°C	23°C
Room volume	220 m^3	190 m^3
Outside temperature	In Figure 5, a hot summer day	
Expected uncontrollable load	Predicted by NN in Figure 6	
Current month peak demand	10.3 kW	
11-month peak demand	10.3 kW	
Load Limit	10.3 kW all the time	
Room temperature range: Occupied time: Unoccupied time:	 20-22°C 19-25°C	

Table 3. Uncertain Thermal Load

Possible thermal load	1		2	
Deterministic part	On/off peak: 100 W/50 W			
Stochastic part	100 W		300 W	
Initial probability	1		0	
Transition matrix	p_{11}	p_{21}	p_{12}	p_{22}
Stage 1 to 2	0.9	0.9	0.1	0.1
...	0.9	0.9	0.1	0.1
Stage 96 to 97	0.9	0.9	0.1	0.1

Table 4. NN Prediction for HVAC Dynamics (Unit 1)

	Max Value	**Mean Error**
Heat exchanger temperature	35.3°C	0.41°C
HVAC load	3.5 kW	0.22 kW

Table 5. Results of Example 1 (50 Monte Carlo Runs)

	LR Method	**Night-Setback Method**
Daily peak (kW)	10.29	10.8
New month/11-month peak (kW)	10.3	10.8
Total electricity cost ($)	11.2	12.6
Total cost compared to LR method		12.5% more

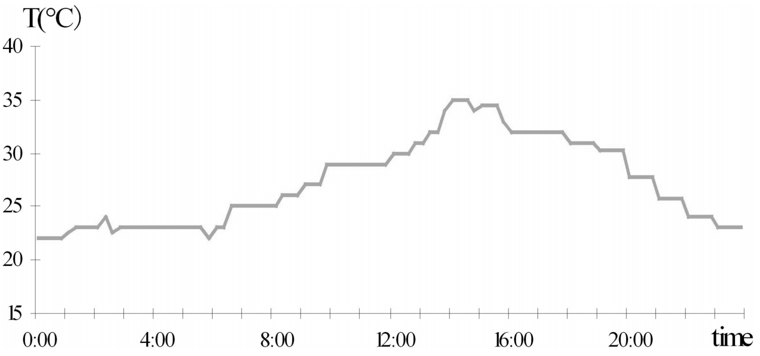

Figure 5. Outside temperature.

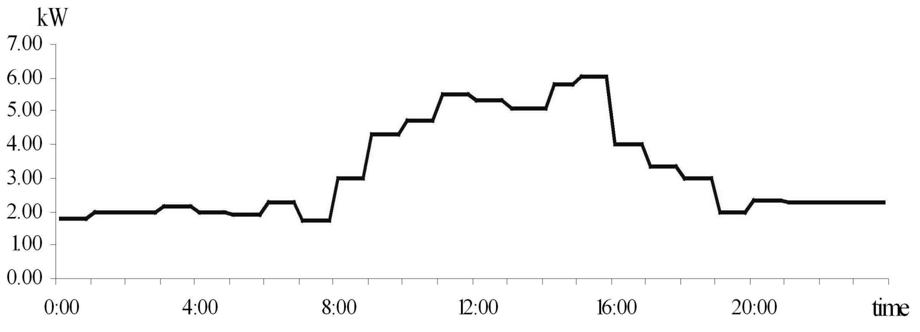

Figure 6. Uncontrollable electricity load (mean error: 0.07 kW).

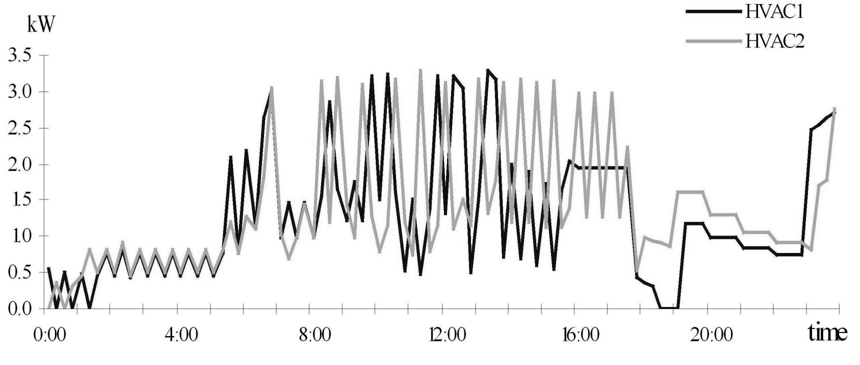

Figure 7. HVAC load by using the LR method.

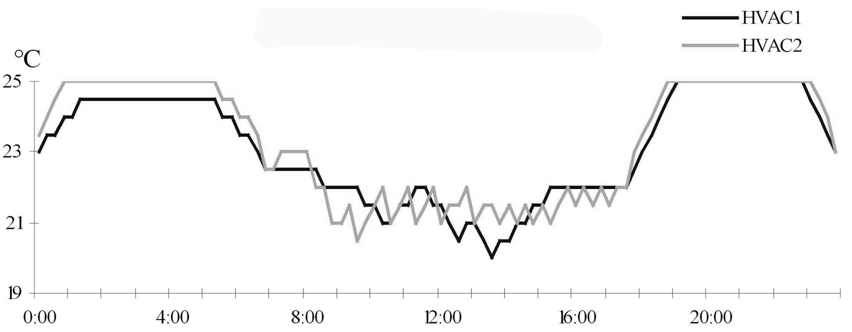

Figure 8. Room temperature setpoints by using the LR method.

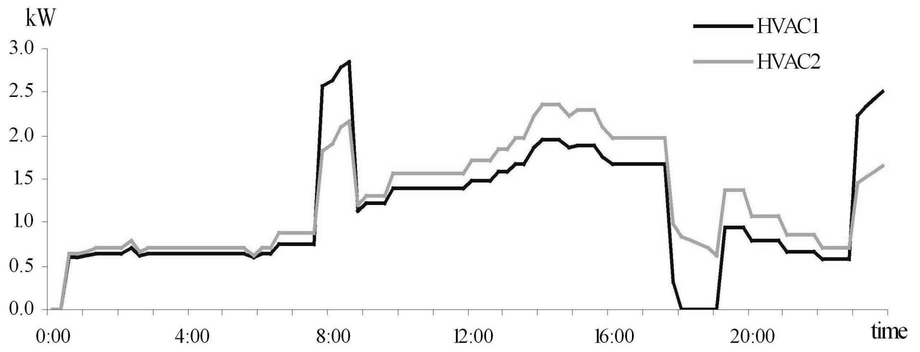

Figure 9. HVAC load by using the night-setback method.

pre-cooling (which began at about 5 a.m.) for both HVAC units. Precooling effectively reduces the total cost because of the low electricity price at off-peak hours. The corresponding room temperature setpoints are shown in Figure 8.

To compare the LR-based method with other methods, the HVAC load for the same scenario by using the night-setback control is depicted in Figure 9. It can be seen that both HVAC units run at high power levels during peak hours, causing the daily peak demand to exceed the current month and 11-month peaks. The figure shows no pre-cooling for either unit.

Example 2

The load shedding function is tested in this example. All the parameters are the same as those in Example 1, except that the total load between 12 p.m. to 4 p.m. is reduced to be less than or equal to 8.4 kW. After 50 Monte Carlo runs using the LR-based method, the average daily peak is 10.0 kW and average peak during the load shedding period is 8.29 kW. The results show that the load shedding requirements are met.

To show how the load shedding function operates, the total load, room temperature setpoints, and HVAC units' load of one scenario are depicted in Figures 10, 11, and 12, respectively. It can been seen from Figure 10 that the total loads from 12 p.m. to 4 p.m. are shedded below 8.4 kW,

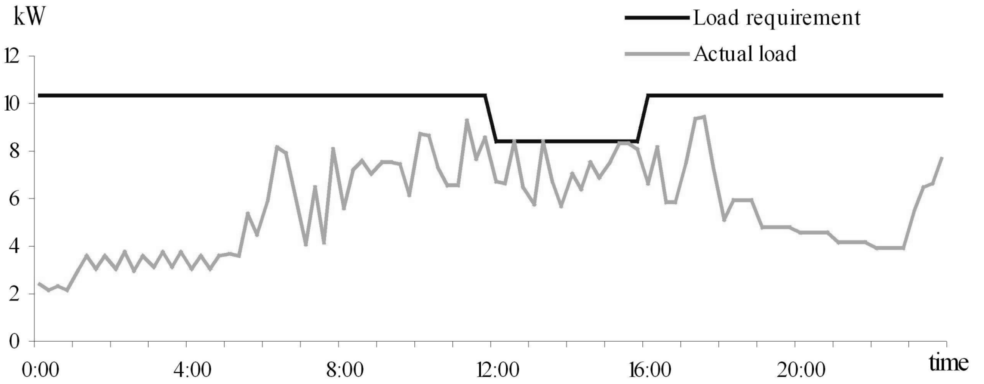

Figure 10. Total load of one scenario.

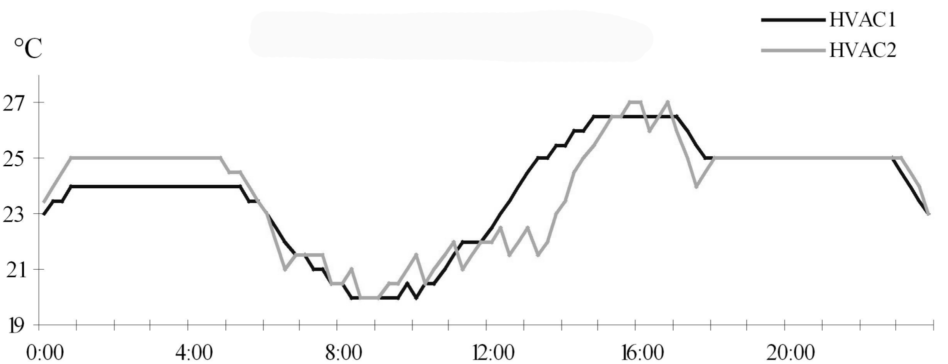

Figure 11. Room temperature setpoints.

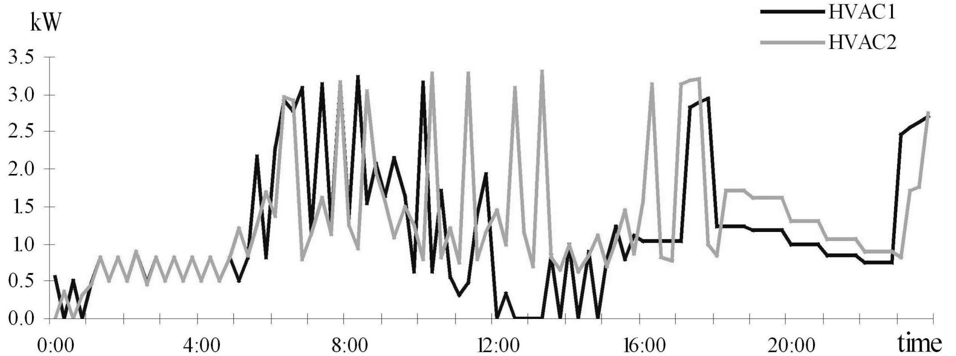

Figure 12. HVAC units' loads.

and the daily peak load does not exceed the monthly and 11-month peak loads of 10.3 kW. It can also been seen from Figure 11 that the room temperature comfort range during the load shedding period is raised to 27°C as decided by Heuristics 3 because of the limits on electricity consumption. The HVAC units' loads shown in Figure 12 show that both units are running at low power levels from 12 p.m. to 4 p.m.

After the load shedding period terminates at 4 p.m., the room temperatures should be gradually restored to their normal ranges. To turn on both the units at highest power level at 4 p.m., however, will create a new peak (>10.3 kW). To quickly restore the room temperatures while avoiding new peaks, Heuristics 3 decides the temperature setpoints one hour at a time. It can be seen from Figures 10 and 11 that the room temperatures are not fully restored until 6 p.m. with a small peak (9.5 kW) around 5:30 p.m. Figure 12 shows that two units are running at a high power level at different times from 4 p.m. to 6 p.m. to avoid a new peak.

In the above, the room temperatures are not fully restored until 6 p.m. If, however, the temperatures are required to be restored immediately at 4 p.m. with no new peaks, the uncontrollable load must be reduced during the load shedding period so that HVAC units can use more electricity and maintain a lower room temperature. If 40% uncontrollable loads are reduced during 12 p.m. to 4 p.m., the total load, room temperature setpoints, and HVAC units' load of one scenario are depicted in Figures 13, 14, and 15, respectively. It can be seen that the room temperatures are maintained at low levels (<24°C) during the load shedding period and are restored at 4 p.m. without introducing new peaks. It can also been seen that the two units are running at high power levels at different times so as to reduce the total load below the limit.

Example 3

To test the computational efficiency and quality of the LR method, a system with different numbers of single-zone HVAC units and thermal load levels was tested. The two units in Example 1 are used, and are duplicated when more units are needed. For simplicity, the load limit is assumed large enough so Equation 6 can be neglected. All other key parameters are the same as in Example 1. The algorithm is stopped if the iteration number is more than 100 or the relative duality gap is less than 0.5%. The results are summarized in Table 6 for 2 HVAC units and Table 7 for 20 HVAC units.

Table 6. Computation Time and Quality (2 HVAC Units)

Thermal Load Levels	Dual Cost ($/Day)	Primal Cost ($/Day)	Duality Gap	Computation Time (seconds)
1	11.28	11.40	1.10%	1.05
2	11.06	11.20	1.25%	1.82
3	11.08	11.17	0.80%	1.97

Table 7. Computation Time and Quality (20 HVAC Units)

Thermal Load Levels	Dual Cost ($/Day)	Primal Cost ($/Day)	Duality Gap	Computation Time (seconds)
1	57.31	51.37	0.40%	4.77
2	58.77	59.13	0.60%	9.25
3	52.01	52.49	0.90%	14.05

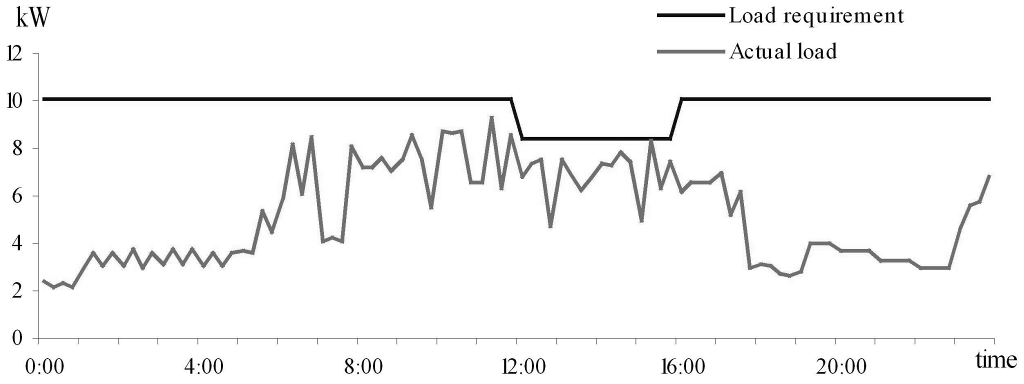

Figure 13. Total load with 40% uncontrollable load reduces.

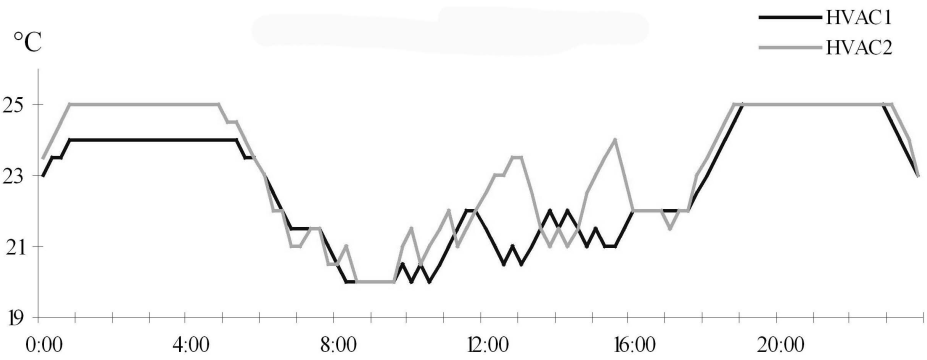

Figure 14. Room temperature setpoints with 40% uncontrollable load reduces.

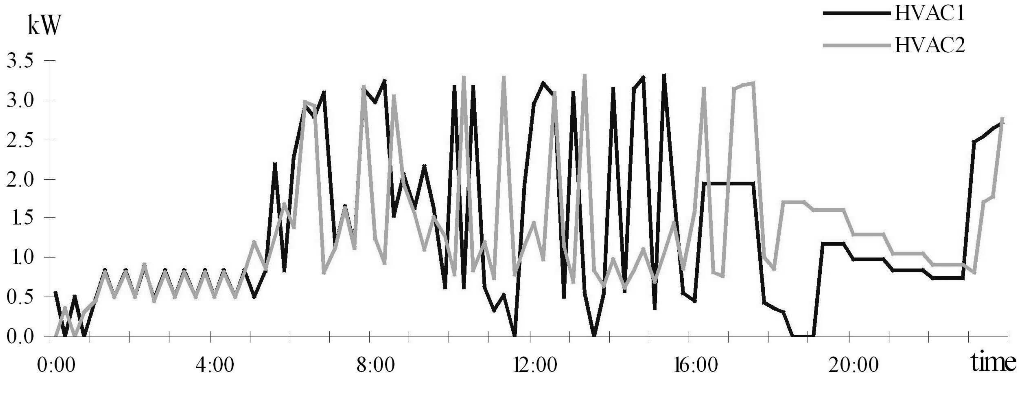

Figure 15. HVAC units' loads with 40% uncontrollable load reduces.

It can be seen from the tables that computation time grows linearly with respect to the number of units and thermal load levels. Furthermore, the duality gaps of all cases are below 1.2%, demonstrating consistent convergence of the algorithm and that near-optimal solutions are obtained. The LR-based method can therefore be extended to systems with large numbers of HVAC units and thermal load levels.

Example 4

The method was implemented for an industrial building. There are about 60 controllable single-zone HVAC units under original DDC controls. The DDC system uses a night-setback control, starting precooling at about 6 a.m., maintaining room temperature at 23.5°C from 7 a.m. to 6 p.m., and keeping room temperatures below 25°C for other hours. To compare the results using our LR-based method with those using the original DDC method, two groups of HVAC units were selected. Group 1 uses the LR method, and Group 2 uses the original DDC control. To achieve similar human comfort, the room temperature comfort range of the LR method is set as 20°C, 23.5°C from 9 a.m. to 6 p.m. and 20°C, 25°C during other hours. Each group has nine HVAC units, which accounts for about 5% of total load of the building, and the two groups have the same types of units, approximately equal cooling areas, and normal room occupancies. Because room occupancy data are difficult to obtain, the thermal load is simplified to have only the deterministic part. Because the nine units' load accounts for a small percentage of the total load, the load limit is given for the nine units of Group 1 instead of for the total load of the building. To test the load shedding function, the load limit is set at 34 kW for Group 1 during the whole day based on the normal peak of the nine units.

Many difficulties were encountered during the implementation, e.g., difficulties in learning and developing our software on top of the original DDC system, the computer having multiple users who may accidentally terminate our software, limited usable dynamic data exchange (DDE) links, causing the original DDC system to become stuck, and limited number of hot days in New England when the system is functioning. Consequently, about five weeks' valid historical data were collected, mostly from September 2002, for neural network training. In the following, results are presented for one relatively cool day and one relatively hot day with outside temperature profiles depicted in Figure 16. For both days, the monthly peak (50 kW) and 11-month peak (50 kW) are much larger than the daily peak.

The prediction results for the system dynamic neural network (energy consumption of Unit 1) and uncontrollable load neural network are summarized in Table 8. It shows that prediction results are much better for the cool day. This is because most historical data we collected are for cool days. The results should be improved when more data are available.

To compare the results of the two groups, the nine HVAC units' total loads for both groups are shown in Figure 17 for the hot day. The room temperatures of three units under our control are shown in Figure 18. It can be seen that the room temperatures are within the required comfort range during all hours. The figures show that the LR method does not let multiple HVAC units run at high power levels at the same time during peak hours so that the daily peak demand does not exceed load limit (34 kW), and there are precoolings in early morning.

To show the load shedding effects and savings of the LR method for the hot and cool days, the daily peak and energy costs are presented in Table 9. The results show that for both days, the daily peak loads of the nine units under our control are below the load limit of 34 kW. Comparing peak loads of the two groups, it shows that with similar human comfort, the LR method causes 24.2% lower peak in the hot day and 20.9% lower peak in the cool day. The energy cost savings of the LR method is not significant in this testing because we missed hot days. The demand costs are the same for both groups since the historical monthly and 11-month peaks are very high. Considering that demand costs usually account for 40% to 50% of total costs, the LR method may achieve 10% to 20% total savings when the HVAC's total load accounts for 40% to 50% of the total load in hot days without sacrificing human comfort.

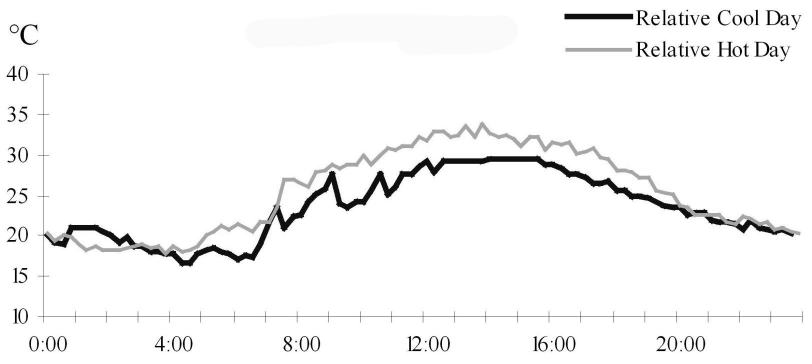

Figure 16. Outside temperatures.

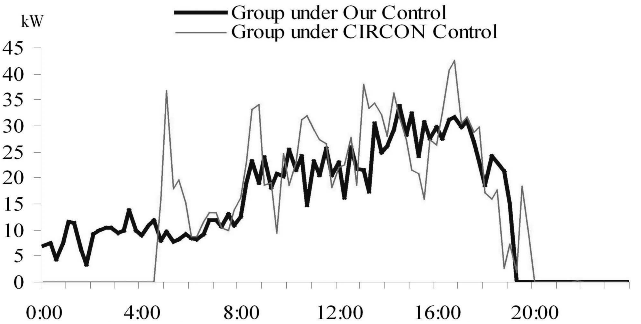

Figure 17. HVAC loads for the two groups.

Table 8. Neural Network Prediction Results

	Uncontrollable Load (Max Load/Mean Error)	**HVAC Load of Unit 1 (Max Load/Mean Error)**
Cool Day	877 kW/70 kW	5.7 kW/0.8 kW
Hot Day	1020 kW/110 kW	6.8 kW/1.4 kW

Table 9. Daily Peak and Energy Costs

	Daily Peak in Group 1	**Daily Peak in Group 2**	**Peak Load Shedding**	**Energy Cost in Group 1**	**Energy Cost in Group 2**
Cool Day	30.6 kW	38.7 kW	20.9%	$72.9	$73.1
Hot Day	33.8 kW	44.6 kW	24.2%	$84.6	$86.3

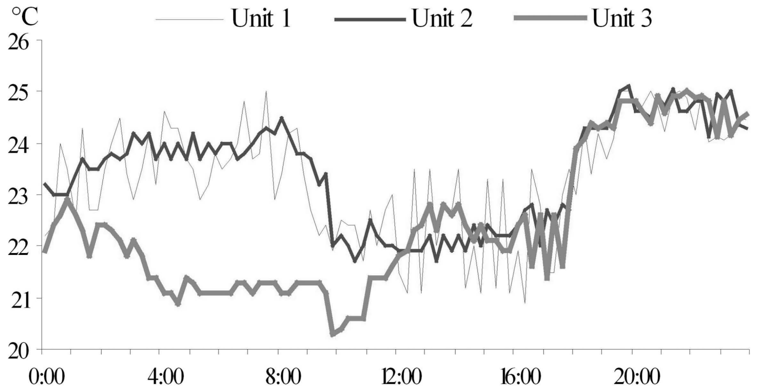

Figure 18. Room temperature setpoints for three HVAC units under the LR-based control.

CONCLUSIONS

This paper presents an optimization-based methodology to control HVAC units in stochastic settings. A DDC-based dynamic building control that decides HVAC setpoints for each time interval is used. Because the dynamics of HVAC systems are nonlinear, time-varying, and building-dependent, neural networks are used to predict the dynamics of HVAC systems. Lagrangian relaxation, a decomposition and coordinated approach, is used to obtain near-optimal solutions with quantified quality. Numerical testing and prototype implementation results show that the LR-based method can effectively manage uncertainties to minimize the expected total costs and shed the load while quickly restoring room temperatures without new peaks. Results also show consistent convergence of our algorithm and that near-optimal solutions are obtained with large numbers of HVAC units and uncertainty levels. The LR-based method can therefore be extended to large systems. The method was tested and implemented for single-zone HVAC systems, but it can be extended to multi-zone systems by assuming that the multi-zones have the same room temperature setpoints.

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