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Scheduling of Hydrothermal Power Systems

Houzhong Yan, Peter B. Luh, Xiaohong Guan
Department of Electrical and System Engineering
University of Connecticut
Storrs, CT 06269-3157, U.S.A.

Peter M. Rogan
Northeast Utilities Service
Berlin, CT 06037-1616, U.S.A.

ABSTRACT - This paper presents a method for scheduling hydrothermal power systems based on the Lagrangian relaxation technique. By using Lagrange multipliers to relax system-wide demand and reserve requirements, the problem is decomposed and converted into a two-level optimization problem. Given the sets of Lagrange multipliers, a hydro unit subproblem is solved by a merit order allocation method, and a thermal unit subproblem is solved by using dynamic programming without discretizing generation levels. A subgradient algorithm is used to update the Lagrange multipliers. Numerical results based on Northeast Utilities data show that this algorithm is efficient, and near-optimal solutions are obtained. Comparing with our previous work where thermal units were scheduled by using the Lagrangian relaxation technique and hydro units by heuristics, the new coordinated hydro and thermal scheduling generates lower total costs and requires less computation times.

1. Introduction

Hydrothermal scheduling of a power system is concerned with thermal unit commitment and dispatch, and the hourly generation of hydro units. The objective is to minimize the total operating cost of thermal units over a period of up to one week, subject to system-wide demand and reserve requirements and individual unit constraints. Because of potential cost savings, this class of mixed integer programming problems has been an active research subject for several decades. However, since the problem belongs to the class of NP-hard combinatorial problems, consistent generation of optimal schedules for problems of practical sizes has proven to be extremely difficult.

Recently, impressive results have been obtained by using the Lagrangian relaxation technique for generating near optimal solutions ([2, 3, 8, 9]). Lagrangian relaxation is a mathematical technique for solving constrained optimization problems ([6]). Its basic idea is to use Lagrange multipliers to relax system-wide demand and reserve requirements. The problem can then be decomposed into the scheduling of individual thermal units and the scheduling of individual watersheds. The multipliers are then adjusted iteratively at the high level, the so called "dual problem." The disadvantage of the approach is that the dual solution is generally infeasible, i.e., the once relaxed system-wide constraints are not satisfied. Some techniques, usually heuristics, are needed to modify the dual solution to obtain a good feasible schedule. Nevertheless, since the value of the dual function is a lower bound on the optimal cost, the quality of the feasible schedule can be quantitatively measured.

The relaxed subproblem for a hydro watershed usually considers hydraulic coupling among reservoirs, pond level, generation capacity and other individual unit constraints. The hydro subproblems in [9] are formulated as optimal control problems and solved by using the multiplier and Newton's methods, with the multipliers progressively tightened until all reservoir constraints are met. A network flow algorithm fits the structure of a watershed consisting of a number of cascading reservoirs, and is used to solve hydro subproblems in [3]. A feasible direction method is used in [8]. Starting with an initial feasible solution, increments to state and control variables are obtained along a feasible direction to minimize the cost of each hydro subproblem.

The hydro subproblems in this paper are simpler than those of [3, 8, 9]. According to the billing rules of New England Power Pool (NEPOOL), hydro units of the Northeast Utilities Service Company (NU) are divided into two categories: daily hydro units and weekly hydro units. A daily hydro unit is subject to water resource constraint for each day, and a weekly hydro unit water resource constraint for the entire scheduling horizon under consideration. For both types of units, limited water resources are converted into energy in MWhr, and their generations are subject to capacity constraints. Therefore, there is no pond level constraint, hydraulic coupling among reservoirs is ignored, and the efficiency of hydro units is predetermined by NEPOOL billing rules and is not a factor in NU scheduling. In the paper, this type of subproblems is efficiently solved by a merit order allocation method based on the multipliers associated with demand and reserve requirements.

The relaxed subproblem for a thermal unit usually considers minimum up/down times, generation capacity, ramp rate and other individual unit constraints, and is usually solved by using dynamic programming ([3, 4, 8, 9]). By using another set of Lagrange multipliers to relax ramp rate constraints, the method of [4] first calculates the optimal generation level for an "up" state at each hour. This can be easily done since there is no system dynamics, and the cost function is hourly additive and piecewise linear with only a few corner points. A thermal unit subproblem can then be efficiently solved by using the dynamic programming technique without discretizing generation levels.

The purpose of this research is to generate hourly schedules for NU, which has seven hydro units and about 70 thermal units. The maximum scheduling horizon is ten days (240 hours), with a seven day horizon as the usual cycle. The individual constraints of a hydro unit are formulated based on the billing practice of NEPOOL. The hydro subproblems can therefore be efficiently solved, and the hydro energy is allocated to cut the demand peaks and/or contribute to reserve when it is expensive. In an earlier phase of the research, hydro generations were obtained by using a heuristic method, and their contributions were deducted from the total system demand and reserve requirements. The thermal units were then scheduled by using the Lagrangian Relaxation technique as reported in [4]. This paper presents the results of the second phase research where both hydro and thermal units are considered in the relaxation process. It is observed that the combined relaxation of hydro and thermal units yields lower costs than the previous work, since hydro and thermal units are coordinated to meet the demand and reserve requirements more economically. To our surprise, the computational time is also reduced. The reason is that coordinated hydro and thermal scheduling smooths thermal demand and reserve requirements, and this causes the high level Lagrangian to converge more smoothly in the optimization process, and therefore reduces the total computation time. Numerical testing results based on NU data show that this method is efficient, and the near-optimal solutions are obtained.

The paper is organized as follows. The hydrothermal scheduling problem is formulated in Section 2. Solution methodology is presented in Section 3 including the Lagrangian relaxation framework, the merit order allocation technique for hydro subproblems, dynamic programming for thermal subproblems, subgradient method for updating multipliers, initialization of multipliers, and the heuristics to obtain a feasible schedule. Numerical testing results are presented in Section 4. Concluding remarks are then given in Section 5.

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2. Problem Formulation

Consider a power system with J hydro units and I thermal units. It is required to determine the startup, shutdown, and generation levels of all units over a specified time period T. The objective is to minimize the total generation cost subject to system demand and spinning reserve requirements, and other individual unit constraints. The time unit is one hour and the planning horizon may vary from one day to ten days. To formulate the problem mathematically, the following notation is first introduced:

$C_i(p_i^t(t))$: fuel cost of thermal unit i for generating power $p_i^t(t)$ at time t, a piecewise linear function of $p_i^t(t)$, in dollars;

D: number of days in the scheduling horizon, in days;
d: day index, d = 1, ..., D;

E_j^w: total energy available in the scheduling horizon for weekly hydro unit j, in MWhr;

$E_j^d(d)$: total energy available in day d for daily hydro unit j, in MWhr;

$E_j^h(t)$: energy used by hydro unit j at time t, in MWhr ($E_j^h(t) = p_j^h(t)*1$);

I: number of thermal units;
i: index of thermal units, i = 1, ..., I;
J: number of hydro units;
j: index of hydro units, j = 1, ..., J;

$P_d(t)$: system demand at time t, in MW;

$p_i^t(t)$: power generated by thermal unit i at time t, in MW;

$\overline{p}_i(t)$: maximum generation level of thermal unit i at time t, in MW;

$\underline{p}_i(t)$: minimum generation level of thermal unit i at time t, in MW;

$p_j^h(t)$: power generated by hydro unit j at time t, in MW;

$\overline{p}_j(t)$: maximum generation level of hydro unit j at time t, in MW;

$\underline{p}_j(t)$: minimum generation level of hydro unit j at time t, in MW;

$P_r(t)$: system spinning reserve requirement at time t, in MW;

$\overline{r}_i$: maximum spinning reserve contribution of thermal unit i, in MW;

$r_i^t(x_i(t),p_i^t(t))$: spinning reserve contribution of thermal unit i at time t, $r_i^t(.) = 0$ if the unit is down ($x_i(t) = 0$), and $r_i^t(.) = \min\{\overline{p}_i(t) - p_i^t(t), \overline{r}_i\}$ if the unit is up ($x_i(t) > 0$), in MW;

$r_j^h(p_j^h(t))$: spinning reserve contribution of hydro unit j at time t, $r_j^h(.) = \overline{p}_j(t)$ if the unit is down ($p_j^h(t) = 0$), and $r_j^h(.) = \overline{p}_j(t) - p_j^h(t)$ if the unit is up ($p_j^h(t) > 0$), in MW;

R_i: ramp rate of thermal unit i, in MW/hr;

$S_i(x_i(t), u_i(t))$: startup cost of thermal unit i, a linear function of time since last shut down, in dollars;

T: time horizon of scheduling, in hours;
t: time index, t = 1, ..., T;

$u_i(t)$: discrete decision variables at time t for the up or down of thermal unit i at time t+1, 1 denotes up and -1 down;

$x_i(t)$: state of thermal unit i, denoting the number of hours that unit i has been on (positive values) or off (negative values);

Δ_i: maximum allowable change in generation between two consecutive hours for thermal unit i ($\Delta_i = R_i*1$);

$\overline{\tau}_i$: minimum up-time of thermal unit i, in hours;

$\underline{\tau}_i$: minimum down-time of thermal unit i, in hours;

The problem is then formulated as the following mixed integer programming problem:

(P):

$$\min_{u_i(t),p_i^t(t),p_j^h(t)} J, \quad \text{with } J \equiv \sum_{t=1}^{T} \sum_{i=1}^{I} \Big[C_i(p_i^t(t)) + S_i(x_i(t), u_i(t))\Big], \qquad (2.1)$$

subject to

system-wide constraints including

-- system demand:

$$\sum_{i=1}^{I} p_i^t(t) + \sum_{j=1}^{J} p_j^h(t) = P_d(t), \qquad (2.2)$$

-- spinning reserve:

$$\sum_{i=1}^{I} r_i^t(x_i(t), p_i^t(t)) + \sum_{j=1}^{J} r_j^h(p_j^h(t)) \ge P_r(t), \qquad (2.3)$$

individual hydro unit constraints including

-- capacity:

$$\underline{p}_j(t) \le p_j^h(t) \le \overline{p}_j(t), \quad \text{if } p_j^h(t) > 0, \qquad (2.4)$$
$$p_j^h(t) = 0 \quad \text{otherwise;}$$

-- weekly or daily energy constraint:

$$\sum_{t=1}^{T} E_j^h(t) = E_j^w \quad \text{(weekly energy);} \qquad (2.5)$$

$$\sum_{t=24(d-1)+1}^{24d} E_j^h(t) = E_j^d(d), \; d = 1, \ldots, D, \text{ (daily energy);} \quad (2.6)$$

implying that a hydro unit has a fix amount of energy for generation within a week or a day.

individual thermal unit constraints including

-- state transition:

$$x_i(t+1) = x_i(t) + u_i(t) \quad \text{if } x_i(t).u_i(t) > 0, \text{ and} \qquad (2.7)$$
$$x_i(t+1) = u_i(t) \quad \text{if } x_i(t).u_i(t) < 0, \qquad (2.8)$$

where $x_i(t)$ is the state and $u_i(t)$ the decision variable of thermal unit i at hour t. These two equations state that the number of hours being up or down accumulates if no start-up or shut-down occurs; otherwise, the number of hours being up or down equals 1;

-- capacity:

$$\underline{p}_i(t) \le p_i^t(t) \le \overline{p}_i(t) \quad \text{if } x_i(t) > 0, \qquad (2.9)$$
$$p_i^t(t) = 0 \quad \text{if } x_i(t) < 0. \qquad (2.10)$$

Some thermal units may also have one or more of the following constraints:

-- ramp rate:

$$[p_i^t(t-1) - \Delta_i] \le p_i^t(t) \le [p_i^t(t-1) + \Delta_i] \quad \text{if } x_i(t-1) > 1 \text{ and } x_i(t) > 1; \tag{2.11}$$

where $\Delta_i = R_i{*}1$ is the maximum allowable change in generation between two consecutive hours;

-- minimum up/down time:

$$u_i(t) = 1 \qquad \text{if } 1 \le x_i(t) < \bar{\tau}_i, \tag{2.12}$$

$$u_i(t) = -1 \qquad \text{if } \underline{\tau}_i \le x_i(t) \le -1, \tag{2.13}$$

stating that thermal unit i must be kept on if it is up for less than the minimum-up time, or be kept off if down for less than the minimum-down time;

-- minimum generation for the first and last hour (required by NEPOOL for steam units):

$$p_i^t(t) = \underline{p}_i(t), \quad \text{if } x_i(t-1) < 0 \text{ and } x_i(t) > 0, \text{ or if } x_i(t) > 0 \text{ and } x_i(t+1) < 0; \tag{2.14}$$

-- must-run or must-not-run:

$$x_i(t) > 0 \qquad \text{for } t_1 \le t \le t_2, \tag{2.15}$$

if thermal unit i is must-run for $t \in [t_1,t_2] \subset [1,T]$, and

$$x_i(t) < 0 \qquad \text{for } t_3 \le t \le t_4, \tag{2.16}$$

if thermal unit i is must-not-run for $t \in [t_3,t_4] \subset [1,T]$. Multiple must-run or must-not-run periods are also possible.

3. Solution Methodology

3.1 The Lagrangian Relaxation Framework

The basic idea of the Lagrangian relaxation technique is to relax "coupling" system-wide constraints on demand and spinning reserve (eqs. (2.2) and (2.3)) by using Lagrange multipliers. The method then decomposes the problem into the scheduling of individual units. Intuitively, the "hard" system constraints are converted into "soft" prices, with Lagrange multipliers acting as prices to regulate the coordination between thermal and hydro units, and the generation and reserve contribution of each unit. At the low level, a hydro subproblem is an optimal resource allocation problem subject to a total energy constraint, and are efficiently solved by using a merit order allocation method. A thermal unit subproblem is solved by following the method of [4]. After all the low-level subproblems are solved, the multipliers are adjusted at the high level so that system demand and reserve constraints are gradually satisfied over the iterations.

From the cost function (2.1) and constraints (2.2), (2.3), the Lagrangian is formulated as following:

$$L = \sum_{t=1}^{T} \Big\{ \sum_{i=1}^{I} [C_i(p_i^t(t)) + S_i(x_i(t), u_i(t))] + \lambda(t)[P_d(t) - \sum_{i=1}^{I} p_i^t(t) - \sum_{j=1}^{J} p_j^h(t)] + \mu(t)[P_r(t) - \sum_{i=1}^{I} r_i^t(x_i(t), p_i^t(t)) - \sum_{j=1}^{J} r_j^h(p_j^h(t))] \Big\}, \tag{3.1}$$

where $\lambda(t)$ and $\mu(t)$ are Lagrange multipliers associated with demand and spinning reserve requirements at time t, respectively. For notational convenience, define

$$\lambda \equiv [\lambda(1), \ldots, \lambda(T)]^T, \tag{3.2}$$

$$\mu \equiv [\mu(1), \ldots, \mu(T)]^T. \tag{3.3}$$

By using the duality theorem ([1, 6]) and exploiting the decomposable structure of (3.1), a two-level max-mini optimization problem can be formed. Given multipliers λ and μ, the low level consists of the following hydro and thermal subproblems:

(P-j), j = 1, ..., J (hydro subproblems):

$$\min_{p_j^h(t)} L_j^h, \text{ with } L_j^h \equiv \sum_{t=1}^{T} [-\lambda(t)p_j^h(t) - \mu(t)r_j^h(p_j^h(t))], \tag{3.4}$$

subject to constraints (2.4) - (2.6);

(P-i), i = 1, ..., I (thermal subproblems):

$$\min_{u_i(t), p_i^t(t)} L_i^t, \text{ with } L_i^t \equiv \sum_{t=1}^{I} \{[C_i(p_i^t(t)) + S_i(x_i(t), p_i^t(t))] - \lambda(t)p_i^t(t) - \mu(t)r_i^t(x_i(t), p_i^t(t))\}, \tag{3.5}$$

subject to constraints (2.7) - (2.16).

Let $L_j^{h*}(\lambda,\mu)$ and $L_i^{t*}(\lambda,\mu)$ denote the optimal Lagrangian for (P-j) and (P-i), respectively, with the given λ and μ. Then the high level dual problem is

$$\max_{\lambda,\mu} \Phi(\lambda,\mu), \text{ with } \Phi(\lambda,\mu) \equiv \sum_{i=1}^{I} L_i^{t*}(\lambda,\mu) + \sum_{j=1}^{J} L_j^{h*}(\lambda,\mu) + \sum_{t=1}^{T} [\lambda(t)P_d(t) + \mu(t)P_r(t)], \tag{3.6}$$

subject to

$$\mu(t) \ge 0, \quad t = 1, \ldots, T. \tag{3.7}$$

The above derivation presents the Lagrangian decomposition framework for solving the scheduling problem. There are several steps to obtain a near optimal solution: solving subproblems, solving the dual problem, and constructing a feasible solution. They are presented below.

3.2 Solving Hydro Subproblems

Given λ and μ, the hydro subproblem (P-j) is to determine the generations of hydro unit j so as to minimize the cost function (3.4). From (3.4), L_j^h depends on $r_j^h(p_j^h(t))$ and $p_j^h(t)$. According to the billing rules of NEPOOL, the reserve contribution $r_j^h(p_j^h(t))$ is a linear decreasing function of $p_j^h(t)$ as depicted in Fig. 1.

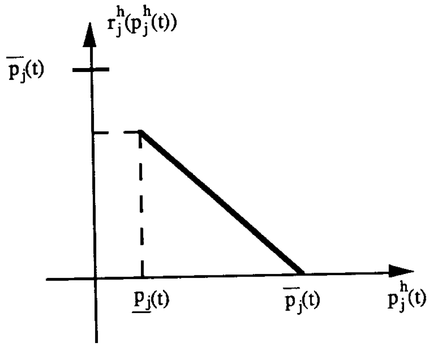

Fig. 1 Reserve contribution $r_j^h(p_j^h(t))$ of hydro unit j

The cost function L_j^h is therefore a linear function of $p_j^h(t)$. For weekly hydro unit j, (3.4) can be rewritten as

$$\min_{p_j^h(t)} L_j^h, \text{ with } L_j^h \equiv \sum_{t=1}^{T} -[\lambda(t)-\mu(t)]p_j^h(t) - \sum_{t=1}^{T} \mu(t)\overline{p}_j(t), \quad (3.8)$$

subject to weekly energy constraint (2.5).

In the absence of constraint (2.5), it is clear that the optimal solution at time t is:

$$p_j^{h*}(t) = \overline{p}_j(t), \quad \text{if } \lambda(t) - \mu(t) > 0, \text{ and} \quad (3.9)$$

$$p_j^{h*}(t) = 0, \quad \text{if } \lambda(t) - \mu(t) \leq 0. \quad (3.10)$$

To obtain the solution to (3.8) satisfying (2.5), the merit order allocation method is used. In the method, hours are arranged in the descending order of $(\lambda(t) - \mu(t))$. The total available energy E_j^w is then allocated to individual hours at the maximum generation level $\overline{p}_j(t)$ according to this ordering until all the energy is allocated. For the remaining hours, generations are zero. The solution generated by this method satisfies the first order necessary conditions for optimality, and is optimal for problem (P-j). The optimal Lagrangian $L_j^{h*}(\lambda,\mu)$ can thus be obtained for the given λ and μ, and capacity and energy constraints ((2.4) and (2.5), respectively) are satisfied. The subproblem for a daily hydro unit is solved in a similar manner for each day.

3.3 Solving Thermal Subproblems

The thermal subproblems are solved by following the method of [4]. A unit without the ramp rate constraint is presented first. For the cost function in (3.5) with λ and μ given, define the non-start-up cost as

$$f_i^t(p_i^t(t), x_i(t)) \equiv [C_i(p_i^t(t)) - \lambda(t)p_i^t(t) - \mu(t)r_i^t(x_i(t), p_i^t(t))]. \quad (3.11)$$

Then the cost function of (P-i) can be rewritten as

$$L_i^t \equiv \sum_{t=1}^{T} [f_i^t(p_i^t(t), x_i(t)) + S_i(x_i(t),u_i(t))]. \quad (3.12)$$

Note that L_i^t is hourly additive, there is no dynamics on generation levels, and the start-up cost $S_i(x_i(t), u_i(t))$ is independent of specific generation levels. The optimal generation level at time t for an up state $(x_i(t) > 0)$ can thus be obtained by minimizing $f_i^t(p_i^t(t), x_i(t))$ subject to the first and last hour generation constraint (2.14) when applicable. That is,

$$p_i^{t*}(t) = \arg\min f_i^t(p_i^t(t), x_i(t)), \quad (3.13)$$

if (2.14) is not active; otherwise, $p_i^{t*}(t) = \underline{p}_i(t)$. To solve (3.13), note that the fuel cost $C_i(p_i^t(t))$ and spinning reserve $r_i^t(x_i(t), p_i^t(t))$ are piecewise linear functions of $p_i^t(t)$, therefore $f_i^t(p_i^t(t), x_i(t))$ defined in (3.11) is also piecewise linear. The solution to (3.13) can thus be easily obtained by checking the vertices of the piecewise linear $f_i^t(p_i^t(t), x_i(t))$, and selecting the one that minimizes the function as shown in Fig. 2.

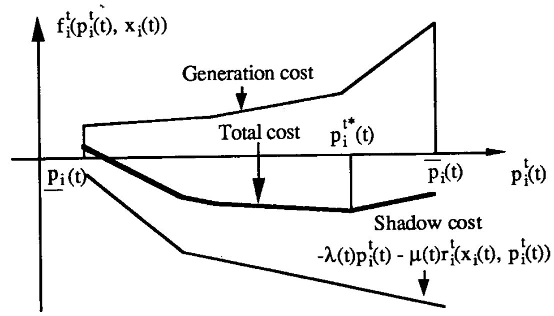

Fig. 2 Hourly cost function of thermal subproblem i

According to the billing rules of NEPOOL, the time varying start-up cost $S_i(x_i(t), u_i(t))$ is a linear function of time since last shut down. The number of down states $(x_i(t) < 0)$ needed to describe different start-up costs is therefore equal to the cold start-up time. Since a thermal unit can be kept on or shut down after it is up for the minimum up time, the required number of up states $(x_i(t) > 0)$ is the minimum up time plus one where the extra one is needed to describe the first and last hour minimum generation when (2.14) is active. Combining the above analysis for up and down states, a state transition diagram can then be constructed as in [4]. The optimal commitment and generation of thermal unit i can thus be obtained by using dynamic programming without discretizing generation levels.

For a thermal unit with ramp rate constraint (2.11), the generation levels of two consecutive hours are coupled. The approach taken here is to use additional sets of multipliers to relax the ramp up and ramp down constraints. An intermediate level is introduced to update these multipliers. The subproblem is then solved at the low level as if there were no ramp rate constraint ([4]).

3.4 Solving the Dual Problem

The high level dual problem is to update the multipliers λ and μ associated with demand and reserve requirements so as to maximize the dual function :

$$\max_{\lambda,\mu} \Phi(\lambda,\mu), \text{ with } \Phi(\lambda,\mu) \equiv \sum_{i=1}^{I} L_i^{t*}(\lambda,\mu) + \sum_{j=1}^{J} L_i^{h*}(\lambda,\mu) + \sum_{t=1}^{T} [\lambda(t)P_d(t) + \mu(t)P_r(t)]. \quad (3.14)$$

Since discrete decision variables are involved at the low level, the objective function $\Phi(\lambda,\mu)$ in (3.14) may not be differential at certain points. A subgradient algorithm is therefore used to update λ and μ as follows ([4, 5, 7, 10]):

$$\lambda^{l+1}(t) = \max[0, \lambda^{l}(t) + \alpha^{l} g_\lambda(t)], \quad (3.15)$$

$$\mu^{l+1}(t) = \max[0, \mu^{l}(t) + \alpha^{l} g_\mu(t)], \quad (3.16)$$

where l is the high level iteration index,

$$g_\lambda(t) = P_d(t) - \sum_{i=1}^{I} p_i^t(t) - \sum_{j=1}^{J} p_j^h(t) \quad (3.17)$$

is the subgradient of $\Phi(\lambda,\mu)$ with respect to $\lambda(t)$, and

$$g_\mu(t) = P_r(t) - \sum_{i=1}^{I} r_i^t(x_i(t), p_i^t(t)) - \sum_{j=1}^{J} r_j^h(p_j^h(t)) \quad (3.18)$$

is the subgradient of $\Phi(\lambda,\mu)$ with respect to $\mu(t)$.

The adaptive step sizing method of [4] is used to obtain the step size α at iteration l. It is given by

$$\alpha = \gamma \frac{\bar{L} - L}{[g_\lambda^T, g_\mu^T][g_\lambda^T, g_\mu^T]^T},$$

where $\bar{L}$ is an estimate of the optimal value of L with $\bar{L} \geq L$, g_λ and g_μ are stack vectors of $g_\lambda(t)$ and $g_\mu(t)$ like λ and μ, and γ is a scaling constant. The high level iteration terminates when the dual cost L cannot be further improved, or a limit on the number of high level iterations has been reached.

3.5 Initializing the Multipliers

Good initialization of multipliers can significantly reduce the number of high level iterations. In our algorithm, the multipliers associated with system demand are initialized to the system marginal costs based on the priority-list commitment and dispatch. In the method, thermal units are first committed at each hour in the ascending order of their full load average rates until the committed capacity exceeds the sum of the demand and reserve requirements. The hydro energy is then allocated to hours by using the merit order allocation method based on marginal costs of committed thermal units. The multipliers $\lambda(t)$ are initialized to be the system marginal cost, i.e., the rate of the last thermal block dispatched at hour t ([4]). All other multipliers are initialized to zero since the reserve and ramp rate constraints are difficult to be considered in initialization.

3.6 Obtaining Feasible Solutions

The dual solution is generally infeasible ([3]), i.e., the once relaxed demand and reserve requirements are generally not satisfied. A heuristic method is developed to generate a good feasible solution based on dual results. Note that dual solutions of hydro subproblems satisfy both capacity and energy constraints in view of how the subproblems are solved. Hydro generations and reserve contributions are therefore fixed. In the absence of ramp rate constraint (2.11) and minimum generation for the first and last hour (2.14), a feasible solution can be obtained by adjusting generation levels of the committed thermal units if the following two inequalities are satisfied ([4]):

$$\sum_{j=1}^{J} p_j^h(t) + \sum_{i \in E_1} \bar{p}_i(t) \geq P_d(t) + P_r(t), \quad (3.19)$$

$$\sum_{j=1}^{J} r_j^h(t) + \sum_{i \in E_1} \bar{r}_i(t) \geq P_r(t), \quad (3.20)$$

where E_1 denotes the set of all committed thermal units at hour t. Equation (3.19) requires that the sum of hydro generations and capacities of committed thermal units exceeds the sum of system demand and reserve requirements. Equation (3.20) then guarantees that hydro and committed thermal units can provide the required reserve. For units with ramp rate constraints (2.11) or at the first or last hour generation with (2.14) active, their generation levels cannot be adjusted or are difficult to adjust. The above conditions should be appropriately modified as in [4].

If (3.19) and (3.20) are not satisfied at hour t, more thermal units will be committed at that hour. Two feasible solutions are generated, and the better one is selected. The first solution is obtained by committing turbine units without minimum up/down time constraints, and this is good for isolated infeasible hours. The second solution is generated by adjusting the commitment of steam units based on unit full load average rates subject to minimum up/down time constraints. A feasible solution is usually obtained by starting up some units earlier or shutting down some units later than scheduled in the dual solution. A steam unit may also be started up to cover a number of consecutive infeasible hours. Economic dispatch is then carried out dispatching all committed thermal units to satisfy the system demand and reserve requirements.

After the feasible solution is obtained, a few more high level iterations (the so called "heuristic iterations") are carried out to obtain additional feasible solutions, and the best feasible solution is selected. The final feasible cost and the maximum dual function value are used to calculate the dual gap, a measure of the quality of the feasible schedule.

3.7 Summary of the Algorithm

The algorithm is summarized as follows:

1 [Initialize.] Initialize system demand multipliers λ according to priority-list commitment and dispatch. Initialize all other multipliers to zero.
2 [Solve subproblems.] Solve individual unit subproblems for the given λ and μ. For a hydro unit, go to 2a; for a thermal unit, go to 2b. If all the subproblems have been solved, go to 3.
   2a. Solve the hydro subproblem according to subsection 3.2. Go to 2.
   2b. Solve the thermal subproblem according to subsection 3.3. Go to 2.
3 [Update multipliers.] Update λ and μ according to (3.15) and (3.16).
4 [Check convergence.] If the stopping criteria for the high level problem (3.6) has not been satisfied, go to 2.
5 [Generate feasible solutions.] If a feasible solution can be obtained without changing commitment, go to 5a. Otherwise go to 5b.
   5a. Generate a feasible solution by economic dispatch. Go to 6.
   5b. Obtain two feasible solutions by using the methods discussed in subsection 3.6.
6 [Select the best feasible solution.] Select the best feasible solution obtained. If the desired number of heuristic iterations is reached, stop.
7 [Perform Heuristic Iterations.] Fellow 3 to update λ and μ, and follow 2 to solve low level subproblems. Go to 5.

4. Numerical Results

The algorithm was implemented in FORTRAN on a SUN Sparc Station 2. All the billing rules of NEPOOL are complied, and many practical considerations are included. Numerical results presented here are based on four NU data sets: second week in August, 1989; fourth week in February, 1990; first week in March, 1990; and third week in April, 1990. These data sets cover weeks in various seasons and are randomly selected from NU billing data files. The contributions of pumped storage units, together with power provided by co-generators and non-dispatchable contracts, were deducted from system demand and reserve requirements. A summary of major system parameters is given in Table 1. There are seven hydro units with capacity about 240 MW. These units account for 5% of total capacity, generate 7% of the peak load, and can provide 100% of reserve requirements according to

Table 1 and Figure 1. Hydro units thus play an important role in smoothing the peak load and also contributing to reserve requirements.

Numerical results for the four data sets are summarized in Table 2. Two methods are used to test each data set. Method 1 is the approach presented in this paper. In method 2, hydro units are scheduled by using heuristics currently employed by NU, and the unmet demand and reserve are provided by thermal units based on the Lagrangian relaxation technique of [4].

It can be seen from Table 2 that computation times and duality gaps of Method 1 are consistent across the four data sets. The CPU times are about four to five minutes on a Sparc Station 2, and the algorithm is efficient to be used on the daily basis. The quality of a solution is measured by the duality gap, defined as the relative difference between the final feasible cost and the maximum value of the dual function (a lower bound to the optimal cost). It can be seen that the duality gaps are below 0.3%. The duality gaps of other data sets tested not reported here are in the similar range.

Comparing to our previous results of Method 2, the cost is reduced by using the current approach for all data sets as expected. Combined hydro and thermal scheduling within the Lagrangian relaxation framework is better than heuristic hydro scheduling in conjunction with the Lagrangian relaxation of thermal units. The numbers of high level iterations for Method 1 are also lower than those of Method 2, and the CPU times are substantially reduced. The reason is that coordinated hydro and thermal scheduling smooths thermal demand and reserve requirements, and this reduces the number of start-up and shut-down of thermal units. This in turn causes the high level Lagrangian to converge more smoothly in the optimization process, and reduces the number of high level iterations and the number of line searches within each high level iteration. CPU times of Method 1 are therefore substantially lower than those of Method 2. Since duality gaps are related to the start-up and shut-down of thermal units ([4]), their reduction is also expected.

It is observed from our testing that hydro solutions may oscillate among hours with close values of $(\lambda(t) - \mu(t))$. This, however, is not serious since hydro units only generate a small percentage of power in the NU system, and the system demand usually changes significantly hour by hour. In comparison with a peak shaving technique, a better coordination among hydro and thermal units can be obtained, and the reserve contribution of hydro units can be appropriately considered by using our method as evidenced by the presence of the reserve multiplier $\mu(t)$ in the hydro cost function (3.8).

5. Concluding Remarks

An algorithm has been presented to solve hydrothermal scheduling problems based on the Lagrangian relaxation technique. By using Lagrange multipliers to relax complicating demand and reserve requirements, the method decomposes the problem into the scheduling of individual units. Good coordination of hydro and thermal units is achieved through Lagrange multipliers acting as prices to regulate the generation and reserve contribution of individual units. For hydro subproblems, the merit order allocation technique is proved to effective for our problem formulation. For thermal subproblems, dynamic programming without discretizing generation levels is an efficient approach. Numerical results for Northeast Utilities data show that this algorithm is efficient, and near-optimal solutions are obtained. Comparing to our previous work where hydro units were scheduled by using heuristics and thermal units by the Lagrangian relaxation technique, the costs generated by the new approach are lowered and computation times are substantially reduced. This algorithm has been embedded in the daily scheduling package of NU, and is being tested and to be used on the daily basis.

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Houzhong Yan was born in Yangzhou, Jiangsu Province, P. R. China on July 9, 1958. He received his B.S. degree in Mathematics and M.S. degree in Computer Sciences from East China Institute of Technology, Nanjing, P.R. China in 1982 and 1986, respectively.

From 1986,1989, he was a lecturer at the Department of Computer Sciences, Southeast University, Nanjing, P. R. China, working on Numerical Analysis, Computational Geometry, Computer Graphics. Currently he is a Ph.D. candidate in the Department of Electrical and Systems Engineering, University of Connecticut, Storrs, CT.

Peter B. Luh (S'77-M'80-SM'91) was born in Taipei, Taiwan, Republic of China on Dec. 21, 1950. He received his B.S. degree in Electrical Engineering from National Taiwan University, Taipei, Taiwan, Republic of China, in 1973, the M.S. degree in Aeronautics and Astronautics Engineering from MIT, Cambridge, Massachusetts, in 1977, and the Ph.D. degree in Applied Mathematics from Harvard University, Cambridge, Massachusetts, in 1980.

Since 1980, he has been with the University of Connecticut, and currently is a Professor in the Department of Electrical and Systems Engineering. His major research interests include schedule generation and reconfiguration for manufacturing systems, power systems, distributed decisionmaking, and game theory. He has been a Principal Investigator and Consultant to many industry and government-funded projects in the above areas, and has published about 100 papers.

Dr. Luh is an Associate Editor of the *IEEE Transactions on Automatic Control*, Technical Editor of the *IEEE Transactions on Robotics and Automation*, won the Best Paper Award of the 1987 Joint Command and Control Research Symposium, and has served on Program Committees and Operating Committees of several major national, international, and inter-society conferences. His is a member of Sigma Xi and is listed in *Who's Who in Engineering, Who's Who in the East,* and *Who's Who in American Education.*

Xiaohong Guan (S'89) was born in Sichuan, P. R. China on Nov. 3, 1955. He received his B.S. and M.S. degree in Control Engineering from Tsinghua University, Beijing, P. R. China in 1982 and 1985, respectively.

From 1985 to 1988, he was with the Institute of Systems Engineering, Xian Jiaotong University, Xian, P. R. China as a lecturer, working on hierarchical and decentralized control of large scale systems, multiprocessor based and distributed computer control systems and application oriented control languages. Currently he is a Ph.D. candidate in the Department of Electrical and Systems Engineering, University of Connecticut, Storrs, CT.

Mr. Guan is a member of Eta Kappa Nu.

Peter M. Rogan was born in New Britain, CT on April 22, 1961. He graduated Magna Cum Laude from the Loomis-Chaffee School in 1979 and received his A.S. degree in Electrical Engineering from Hartford State Technical College in 1982.

Since 1982, he has been with Northeast Utilities Service Company, Berlin, CT, and currently is an Associate Analyst in the Power Supply Analysis and Power Contracts Department.

Table 1. Summary of System Characteristics

System characteristics	Total number				Total capacity or requirement (MW)			
	Aug w2 89	Feb w4 90	Mar w1 90	Apr w3 90	Aug w2 89	Feb w4 90	Mar w1 90	Apr w3 90
Hydro units	7	7	7	7	233	247	246	258
Steam units	24	32	34	31	1684	1804	1874	1776
Turbine units	27	23	24	24	257	293	244	238
Nuclear units	7	7	7	7	2471	1245	2112	2102
Dispatchable contracts	8	8	8	6	350	689	453	292
All units	74	77	80	75	4995	4278	4929	4666
Peak load					3531	3875	3981	3149
Minimum load					2265	2506	2570	2097
Maximum reserve					183	181	181	133

Table 2. Summary of Testing Results

	Data sets	Number of high level iterations	CPU time (sec)	Max. dual cost ($)	Best feasible cost ($)	Duality gap (%)
method 1	Aug w2, 89	39	215.92	4,540,893.97	4,554,061.69	0.29
	Feb w4, 90	32	157.88	5,683,409.39	5,695,651.40	0.20
	Mar w1, 90	31	206.07	7,063,741.06	7,074,114.14	0.15
	Apr w3, 90	31	267.38	5,854,873.80	5,861,493.22	0.11
method 2	Aug w2, 89	37	282.24	4,550,967.85	4,565,506.38	0.32
	Feb w4, 90	66	423.94	5,700,291.95	5,711,858.93	0.20
	Mar w1, 90	59	342.64	7,080,303..62	7,099,967.58	0.27
	Apr w3, 90	34	343.51	5,857,696.82	5,868,396.14	0.18

Method 1: The approach presented in this paper.

Method 2: Hydro units are scheduled by using heuristics and thermal units by using the Lagrangian relaxation technique

DISCUSSION

MILAN V. RAKIC (Institute Mihajlo Pupin) and RADMILA M. RAKIC (Energoprojekt), Belgrade, Yugoslavia: The authors should be congratulated for their excellent paper in the field which has been very intensively researched. Special value to the paper gives the fact that is applied to rather large, real power system. Anyway, some comments could be made and some questions could be put since in this field there are approaches which are even more general.

1. Power exchange with other utilities is not included in the model.

2. Since this is the optimization problem, any model simplification results in worsening of results. Instead of taking billing rules of NEPOOL, authors should have modeled the river basin precisely.

The hydro subproblem, as it is formulated in the paper, is a linear programming problem with upper and lower constraints and nonlinear production characteristics of hydro power plants are not taken into account. As it is formulated, the subproblem solution will have results on upper and lower bounds and oscillations in the process solution may appear, as it was mentioned in the paper.

3. Start-up cost function of thermal generating units is assumed as linear function of time since the last shut down. The result is that the units which have been out of operation for long time (for example close to the end of the optimization period) can not be started up due to high start up costs.

4. The method of pumped storage power plants scheduling was not presented in the paper. Are pumped storage hydro power plants scheduled in advance?

H. YAN, P. B. LUH, X. GUAN, (Department of Electrical & Systems Engineering, University of Connecticut, Storrs, CT 06269-3157), and P. ROGAN (Northeast Utilities Service Company, Berlin, CT 06037-1616): We would like to thank the discussers for their comments and questions on this paper. The comments and questions will be answered as follows:

1. Power exchange with other utilities was not considered in this paper. The method for integrated consideration of scheduling and exchange is currently under investigation.

2. New England Power Pool (NEPOOL) is the centralized scheduling and dispatching authority in New England, and Northeast Utilities Service Company (NU) has relinquished real-time scheduling and dispatching of all its generators to this authority. NEPOOL and member utility companies then determine how much individual companies should pay to meet their own loads based on NEPOOL billing rules. The scheduling problem considered in this paper has been designed for this billing purpose, and our method provides a valid solution for NU satisfying all the billing rules. If, however, NEPOOL desires to use this method for on-line scheduling, precise river models should be incorporated.

3. According to NEPOOL billing rules, the time varying start-up cost is a linear function of time since last shut down. This start-up cost function is different from the exponential start-up cost function used by some utilities. The effect of this difference is believed to be small. The algorithm, however, is not limited by the specific form of the start-up cost function. For more details please see [1].

4. The method for incorporating pumped-storage units within the overall scheduling framework has been finished, and our paper "Optimization-Based Scheduling of Hydrothermal Power Systems with Pumped-Storage Units" has been submitted to the IEEE PES Winter Meeting of 1993.

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