A Fuzzy Optimization-Based Method for Integrated Power System Scheduling and Inter-Utility Power Transaction with Uncertainties

Houzhong Yan
Energy Marketing Division
Southern California Edison Company
Rosemead, CA 91770, USA

Peter B. Luh
Department of Electrical & Systems Engineering
University of Connecticut
Storrs, CT 06269-3157, USA

Abstract

Electric utilities face many uncertainties in their daily scheduling and inter-utility transaction operations. The effects of these uncertainties can propagate through the time horizon, significantly affecting the economics of schedules and transactions. With deregulation in the utility industry and increasing competition in the electricity market, these uncertainties should be properly managed. In this paper, system demand, reserve requirements and prices of future purchase transactions are considered as uncertain, and the integrated scheduling and transaction problem is formulated as a fuzzy mixed integer programming problem for a power system consisting of thermal units and purchase transactions. Based on the symmetric approach of fuzzy optimization and the Lagrangian relaxation technique, a fuzzy optimization-based algorithm is developed. Testing results using fuzzy simulation show that the method produces robust scheduling and transaction decisions to hedge against uncertainties.

1. Introduction

Power system scheduling and inter-utility transactions are important activities faced daily by electric utilities. These two activities form an integrated problem since they are coupled through system-wise constraints such as system demand, reserve requirements, and transmission line capacity constraints. There are many uncertainties in the integrated problem, including future transaction opportunities, system demand, fuel prices, unit availability, etc. A typical example is in the case of purchase transactions, where opportunities emerge randomly. When a utility is determining whether to take a particular transaction opportunity within the ten or fifteen minutes after an offer was received, it does not know whether a better offer would come soon after. Another example is the fluctuation of system demand. Scheduling and transaction decisions are generally determined based on predicted system demand, and if the demand changes, the decision made may no longer be economical. The effects of uncertainties can propagate through the time horizon, significantly affecting the economics of schedules and transactions. With increased emphasis on competition in the utility industry, these uncertainties can no longer be ignored, but should be properly managed.

96 SM 496-0 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the 1996 IEEE/PES Summer Meeting, July 28 - August 1, 1996, in Denver, Colorado. Manuscript submitted January 2, 1996; made available for printing June 27, 1996.

In the integrated problem, the total generation of units plus purchased amount minus sales should equal system demand. Since predicted demand is usually subject to 2% to 5% variation, it is better to consider possible ranges of demand instead of crisp numbers in the planning stage to have robust scheduling and transaction decisions. Based on the above thought, the total generation plus purchased amount minus sales is required to be "essentially" or "roughly" equal to the predicted demand. This requirement is therefore "fuzzy" in nature, and crisp treatment of it (requiring to be satisfied exactly or crisply all the time) may lead to uneconomic scheduling and transaction decisions. For example, to meet predicted demand at one hour crisply in the planning stage, a gas turbine with a limited capacity may be committed for a minimum total cost. However, as actual demand varies from the predicted one, the demand may not be met by the committed units and confirmed transactions. This might result in purchasing emergency power with very high prices, and the resulting cost would be worse than that of committing a steam unit with a larger capacity or purchasing power with a slightly higher price in the planning stage.

A similar situation can be found in power transactions. There is always a possibility of losing a better opportunity because of signing the current contract without considering the future transaction opportunities. Based on system operators' experience and the current market information, the prices of potential future transactions for the next few hours or the next few days can be subjectively estimated within possible ranges, e.g., between $15 and $18 per megawatt. This subjective estimation of prices is therefore "fuzzy" in nature, and crisp representation may result in uneconomical decisions.

Some of the above uncertainties, especially future system demand, might be handled by using frequency-based probabilities. A stochastic programming method was reported in [1] to analyze the effect of demand uncertainty on unit commitment risk, where a Gauss-Markov load model was used for system demand uncertainty. Theoretically, it might be more accurate to model future system demand by using probability distribution function, but the scheduling problem may become too complicated to resolve. On the other hand, some of the uncertainties, such as the prices of future purchases, without frequency information, might not be properly handled by frequency-based probabilities.

Fuzzy set theory provides a natural platform to model fuzzy relationships such as "essentially" or "roughly" as described above, and adds the dimension of fuzziness, vagueness, or uncertainty to the conventional set theory. A brief literature review of fuzzy optimization is provided in the next section.

As a step towards incorporating uncertainties in power system scheduling and transaction problems, this paper concentrates on the problem formulation and solution methodology for a power system consisting of thermal units and purchase transactions subject to system demand and reserve requirements. Future purchase transactions, predicted system demand and reserve requirements are considered as uncertain. By using fuzzy relations to model system demand and reserve requirements and fuzzy numbers to approximate the possible prices of future transaction opportunities, the integrated problem is formulated as a fuzzy mixed integer programming problem in Section 3.

Power system scheduling without considering uncertainties (a brief literature review is provided in the next section) is believed to be NP-hard, i.e., the computational requirements to obtain an optimal solution increase exponentially as the problem size increases. The problem with uncertainties becomes even more complicated. In order to solve the problem in a meaningful and practical way, the problem is first converted to a crisp optimization problem. To efficiently solve the resulting crisp problem, the Lagrangian relaxation technique is used to relax the complicating constraints and to decompose the problem into individual unit and transaction subproblems and a membership subproblem, which are easier to solve and have intuitive appeal as described in Section 4. To compare the performance of the fuzzy algorithm with a corresponding crisp algorithm, fuzzy simulation is developed as described in Section 5. Testing results indicate that the fuzzy algorithm performs better than the crisp one, and robust scheduling and transaction decisions are obtained to hedge against uncertainties as presented in Section 6.

2. Literature Review

A brief literature review of deterministic scheduling and transaction problems is presented in subsection 2.1, fuzzy optimization and its applications to power systems in subsection 2.2, and fuzzy simulation in subsection 2.3.

2.1 Crisp Power System Scheduling and Transactions

"Crisp" or deterministic power system scheduling has been an active research subject for more than two decades because of significant cost saving potential. In a crisp problem formulation, system demand, fuel prices of thermal units, and unit availability are assumed to be known. The approaches reported in the literature can be classified into five categories: partial enumeration (such as branch and bound), dynamic programming, Benders partitioning, Lagrangian relaxation, and heuristics ([2]). Lagrangian relaxation has been successfully used to obtain near optimal solutions ([2, 3, 4, 5, 6, 7, 8, 9, 10]). It is a mathematical technique for solving constrained optimization by exploiting the separable structure of the problem. The basic idea is to use Lagrange multipliers to relax system-wise demand and reserve requirements. The problem can then be decomposed into the scheduling of individual units, which is much easier to solve, and the multipliers are iteratively adjusted at the high level. Recently, scheduling and transactions in a crisp environment were considered as an integrated problem, and solved by using the Lagrangian relaxation technique in [11, 12].

2.2 Fuzzy Optimization and Its Applications in Power System

In fuzzy optimization, the objective may not be optimized exactly, and constraints can be satisfied to varying degrees. This is opposed to crisp optimization where an optimal solution is sought satisfying all the constraints crisply. Fuzzy optimization reported in the literature can roughly be classified into two categories. In the first category, problems have crisp coefficients in their objective functions and constraints, however, constraints can be satisfied to varying degrees. Most methods reported in the literature transform a fuzzy problem into a crisp one by using the symmetric approach of Bellman & Zadeh ([13, 14]). The basic idea is that the objective function should be essentially smaller than or equal to some "aspiration level," and this can be regarded as a constraint. Bellman & Zadeh treat this "objective constraint" and other constraints symmetrically, and define fuzzy optimization as maximizing the minimum degree of satisfaction among all the constraints -- a crisp optimization problem.

Problems in the second category have fuzzy coefficients in their objectives and/or constraints. The presence of fuzzy coefficients makes the optimization problems much more difficult. In the literature, these problems are generally transformed into problems of the first category, e.g., by converting into multiobjective optimization as in [15], or by using alpha cut as in [16]. The converted problems are then transformed into crisp optimization based on the symmetric approach.

Several applications of fuzzy optimization in power systems have been reported. A recursive "fuzzy dynamic programming" method has been developed to obtain the commitment and dispatch of thermal and hydro units ([17]). Fuzzy optimization methods for problems of the first category have been developed for optimal power flow with uncertain system loads in [18], for multi-area scheduling with fuzzy demand and tie capacity limits in [19], and for power system scheduling with fuzzy reserve requirements in [20]. Very few papers belonging to the second category have been found in the literature. The major difficulty is that after a problem is transformed twice, its original structure (e.g., separability) is likely to be lost, and efficient optimization might be difficult. This is especially critical for the problem considered here in view of the computation complexity even for the crisp version of the problem.

2.3 Simulation

To evaluate the performance of a fuzzy optimization method in an uncertain environment, simulation becomes mandatory. In the literature, fuzzy numbers are usually generated by extending random number generation mechanisms in [21, 22, 23]. In general, these approaches utilize the frequency-based interpretation of random numbers [24] to approximate the vagueness of fuzzy numbers.

3. Problem Formulation

To clearly present the problem formulation, the crisp version is first introduced in subsection 3.1, and followed by the fuzzy version in subsection 3.2.

3.1 Crisp Problem Formulation

A power system with I thermal units and M purchase transactions is considered for conciseness of presentation. For detailed formulation of a power system with thermal, hydro, and pumped-storage units, and purchase and sale transactions, please refer to our previous work [7, 8, 9, 11, 12]. The problem is to determine the start-up, shut-down, and generation levels of all thermal units, and durations and megawatt levels of purchase transactions over a specified time period T to minimize the total cost subject to system demand and reserve requirements and individual thermal unit and transaction constraints.

The objective function to be minimized is the total cost, i.e., the fuel and start-up costs of thermal units and purchase transaction costs:

$$\min \quad J \equiv \sum_{t=1}^{T} \{ \sum_{i=1}^{I} \{ C_{ti}(p_{ti}(t)) + S_i(t) \} + \sum_{m=1}^{M} C_{bm}(t) p_{bm}(t) \}, \quad (3.1)$$

where $C_{ti}(p_{ti}(t))$ is the fuel cost for generating power $p_{ti}(t)$ by thermal unit i at time t, $S_i(t)$ the start-up cost of thermal unit i, $C_{bm}(t)$ is the price of purchase transaction m for buying one megawatt of power at time t, and $p_{bm}(t)$ the power purchased by transaction m in MW at time t. The price of a purchase transaction may vary from one hour to the next, or may vary from a peak load period to an off-peak load period (remaining constant within a load period). The latter is the current practice among the utilities in New England and Western States, and is assumed in this paper.

The fuel cost of a thermal unit is usually modeled as a quadratic function or a piecewise linear function of the generation level, and the start-up cost as an exponential or linear function of time since last shut down. In this paper, the fuel cost $C_{ti}(p_{ti}(t))$ and start-up cost $S_i(t)$ are assumed to be piecewise linear and linear functions, respectively, following the rules of New England Power Pool.

System demand constraints require that the total generation of units plus purchased amount should equal system demand $P_d(t)$ at each time t, i.e.,

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{m=1}^{M} p_{bm}(t) = P_d(t). \tag{3.2}$$

Reserve requirements state that the total reserve contribution of all units should be greater than or equal to the required reserve (usually obtained as a percentage of system demand) at each time t, i.e.,

$$\sum_{i=1}^{I} R_i(p_{ti}(t)) \geq P_r(t). \tag{3.3}$$

In the above, $R_i(p_{ti}(t))$ is the reserve contribution of thermal unit i at generation level $p_{ti}(t)$, and $P_r(t)$ the required reserve at time t.

Individual thermal unit constraints include capacity constraints, minimum up/down time and ramp rate constraints. Purchase transaction constraints include minimum/maximum power to be purchased and allowable purchase patterns. The detailed mathematical descriptions of these constraints are presented in [7] and [11], respectively.

It should be mentioned that the objective function (3.1) is additive, and individual units and transactions are only coupled through the system demand and reserve requirements. This is an ideal separable structure for the Lagrangian relaxation technique as reported in [2, 3, 4, 5, 6, 7, 8, 9, 10].

3.2 Fuzzy Problem Formulation

As mentioned before, fuzzy set theory is a natural platform to model fuzzy or imprecise objects and/or constraints. Given a collection of objects Y, a fuzzy set $\tilde{A}$ is defined as

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) | y \in Y\}$$

$$\text{and } 0 \leq \mu_{\tilde{A}}(y) \leq 1, \tag{3.4}$$

where $\mu_{\tilde{A}}(y)$ is the membership function of y, representing the degree that y belongs to $\tilde{A}$ (ranging from zero to one for a normalized fuzzy set). If $\mu_{\tilde{A}}(y)$ could only be 0 or 1, the fuzzy set $\tilde{A}$ degenerates to a crisp set. A fuzzy or inexact relation, such as "essentially equal to" or "roughly less than or equal to," is also associated with a membership function representing the degree of certainty of that relation.

For a purchase opportunity that is yet to come, the prices can only be subjectively estimated based on the current market information and system operators' experience. Its unit prices are fuzzy in nature as discussed in the previous section, and the price at time t is approximated by a "fuzzy number" $\tilde{C}_{bm}(t)$. This fuzzy number is a fuzzy set describing the possible range of the price, e.g., in-between \$15 and \$18 per megawatt. The membership function, indicating the grade of the price in the set, is assumed to be piecewise linear and has a triangular shape (therefore called a "triangular fuzzy number") as depicted in Figure 1 and mathematically described by equation (3.5).

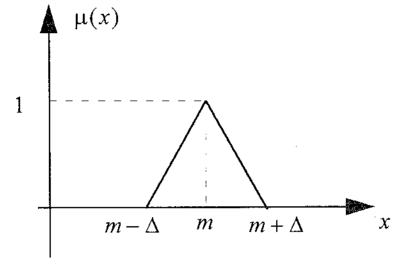

Figure 1. The membership function of a triangular fuzzy number

$$\mu(x) = \begin{cases} 1, & x = m, \\ 1-(m-x)/\Delta, & m-\Delta < x < m, \\ 1-(x-m)/\Delta, & m < x < m+\Delta, \\ 0, & elsewhere. \end{cases} \tag{3.5}$$

In the Figure 1, the parameters m is the nominal price having the maximum grade of membership, and $m+\Delta$ and $m-\Delta$ are, respectively, the maximum and minimum possible price of the transaction. It can be interpreted that the price becomes less possible as it increases above or decreases below m as indicated by reduced membership function.

As the prices of future purchases are approximated by fuzzy numbers, the objective in the fuzzy formulation is no longer crisp but is given by

$$\min \quad \tilde{J} \equiv \sum_{t=1}^{T} \{ \sum_{i=1}^{I} \{C_{ti}(p_{ti}(t)) + S_i(t)\} + \sum_{m=1}^{M} \tilde{C}_{bm}(t) p_{bm}(t)\}. \tag{3.6}$$

Since predicted system demand is not precise and usually contains 2% to 5% variation, system demand constraints are described as fuzzy equality relations, i.e., the total generation of units plus purchased amount should essentially equal system demand at each hour ([15]):

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{m=1}^{M} p_{bm}(t) \cong P_d(t). \tag{3.7}$$

The membership function of the above fuzzy equality relation "≅" is described by

$$\mu_{P_d(t)}(x) = \begin{cases} 1, & x = P_d(t), \\ 1-(P_d(t)-x)/\Delta_d(t), & P_d(t)-\Delta_d(t) < x < P_d(t), \\ 1-(x-P_d(t))/\Delta_d(t), & P_d(t) < x < P_d(t)+\Delta_d(t), \\ 0, & elsewhere, \end{cases} \tag{3.8}$$

with the same triangular shape as shown in Figure 1, where $P_d(t)$ is the "nominal" demand having the maximum grade of membership function (i.e., mean value of the predicted demand), and $\Delta_d(t)$ denotes the maximum range of variation of the predicted demand. This membership function $\mu_{P_d(t)}(x)$ indicates that it is less acceptable as the total generation of units plus purchased amount increases above or decreases below $P_d(t)$ as indicated by reduced membership in (3.8). Equation (3.7) can also be interpreted as that a solution should satisfy the system demand as much as possible -- not fall short or go over $P_d(t)$ "too much."

Reserve requirements can be also described as fuzzy inequality relations, i.e., the total reserve contribution at time t should be essentially greater than or equal to the required reserve at each hour:

$$\sum_{i=1}^{I} R_i(p_{ti}(t)) \gtrsim P_r(t), \tag{3.9}$$

where "$\gtrsim$" is the fuzzy inequality relation "essentially greater than or equal to." The membership function of this relation is assumed to be

$$\mu_{P_r(t)}(x) = \begin{cases} 1, & x \geq P_r(t), \\ 1-(P_r(t)-x)/\Delta_r(t), & P_r(t)-\Delta_r(t) < x < P_r(t), \\ 0, & elsewhere, \end{cases} \tag{3.10}$$

where $P_r(t)$ is the "nominal" reserve requirement, and $P_r(t)-\Delta_r(t)$ the minimum acceptable reserve. It can be interpreted that it is less acceptable as the total reserve contribution decreases below $P_r(t)$ as indicated by reduced membership in (3.10).

Individual thermal unit and purchase transaction constraints are the same as those for the crisp formulation. In fact, some of those constraints are crisp whereas others may not be crisp, but for simplicity of illustration, all these constraints are assumed to be crisp in this paper.

It can be seen from the above formulation that fuzzy coefficients (fuzzy prices of future purchase transactions) appear only in the objective function (3.6). It should also be mentioned that the piecewise linearity of membership functions ((3.5), (3.8) and (3.10)) plays a crucial role in retaining the separable structure of the integrated problem as will be seen in the next section. Finally, if all the spreads are zero, the problem degenerates to the crisp formulation presented in section 3.1.

4. Solution Methodologies

In the above, the integrated problem is formulated as a fuzzy mixed integer programming problem having a nonlinear objective function with fuzzy coefficients, fuzzy equality/inequality constraints, and other crisp individual constraints. For a method to be useful, the solution should recommend system operators what to do precisely, implying that the solution should be crisp. To develop an efficient methodology, the following three steps will be presented in the sequel: convert the problem to a fuzzy optimization problem with crisp coefficients in subsection 4.1, convert the resulting problem into a crisp optimization problem in subsection 4.2, and solve the resulting crisp problem by using the Lagrangian relaxation approach in subsection 4.3.

4.1 Convert to A Fuzzy Optimization Problem with Crisp Coefficients

From equation (3.6), it can be seen that only addition and scalar multiplication operations of fuzzy numbers are involved in the objective function. Based on fuzzy arithmetic [13, pp. 65], the fuzzy objective function $\tilde{J}$ is a triangular fuzzy number, with its mean and spread determined by the means and spreads of fuzzy coefficients and decision variables as given by

$$m_{\tilde{J}} = \sum_{t=1}^{T} \{ \sum_{i=1}^{I} \{ C_{ti}(p_{ti}(t)) + S_i(t) \} + \sum_{m=1}^{M} m_{\tilde{C}_{bm}} p_{bm}(t) \}, \tag{4.1}$$

and

$$\Delta_{\tilde{J}} = \sum_{t=1}^{T} \{ \sum_{m=1}^{M} \Delta_{\tilde{C}_{bm}} p_{bm}(t) \}. \tag{4.2}$$

It is conceptually difficult to minimize an objective which is a fuzzy set. One meaningful and practical way is to utilize the idea that a good solution should have a cost with a small mean and a small spread. The problem is thus transformed to the minimization of the mean plus a weighted spread of the fuzzy objective subject to the same set of constraints. With a weight w, the new objective function is given by

$$\min \quad J \equiv m_{\tilde{J}} + w\Delta_{\tilde{J}}. \tag{4.3}$$

This transformation is consistent with the stochastic optimal control concept, and is similar to the method presented in [16, pp. 203], where a fuzzy objective was converted into crisp but multiple objectives. After this transformation, the resulting problem is a fuzzy optimization problem with crisp coefficients and belongs to the first category as presented in Section 2.2. Separability is preserved since the new objective is a linear combination of mean and spreads, and is still additive in terms of individual units and transactions. In the next subsection, the transformed problem will be solved by using the symmetric approach of Bellman and Zadeh ([14]).

4.2 Second Transformation to Crisp and Separable Optimization

Based on the symmetric approach, the objective function in equation (4.3) should be essentially smaller than or equal to some "aspiration level" J_o:

$$J \equiv m_{\tilde{J}} + W\Delta_{\tilde{J}} \lesssim J_o, \tag{4.4}$$

where "$\lesssim$" denotes a fuzzy "essentially smaller than or equal to" relationship. The membership function of the fuzzy inequality relation is depicted in Figure 2 and given by equation (4.5).

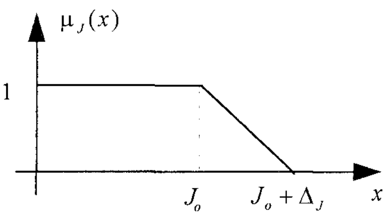

Figure 2. Membership function of the fuzzy inequality relation

$$\mu_J(x) = \begin{cases} 1, & x \leq J_o, \\ (J_o+\Delta_J-x)/\Delta_J, & J_o < x \leq J_o+\Delta_J, \\ 0, & elsewhere. \end{cases} \tag{4.5}$$

The aspiration level J_o represents the ideal cost for the power system. The schedule and transaction decisions become less acceptable as the total cost J increases above the ideal value as indicated by the reduced membership in Figure 2. The highest acceptable cost is $J_o+\Delta_J$. Selecting the aspiration level may be subjective and dependent on specific practice. One good candidate for the ideal cost J_o is the cost of the crisp problem with nominal system demand and reserve requirements. The highest acceptable cost $J_o+\Delta_J$ can be determined by choosing Δ_J as a certain percentage of J_o based on experience.

The problem is then to maximize the degree to which all the constraints (including the objective function constraint (4.4)) are satisfied. This is the "symmetric" approach that treats the objective and actual constraints symmetrically. Mathematically,

$$z^* = \max \mu_D = \max \min(\mu_J, \mu_{P_d(1)}, \ldots, \mu_{P_d(T)}, \mu_{P_r(1)}, \ldots, \mu_{P_r(T)}), \tag{4.6}$$

where μ_J is the membership function of the objective, and $\mu_{P_d}(t)$ and $\mu_{P_r}(t)$ are the membership functions associated with the system demand and reserve requirements for t=*1,...,T*, respectively. This problem can also be rewritten as

$$\max_{0\le z\le 1, p_{ti}(t), p_{bm}(t)} z \tag{4.7}$$

subject to

$$z \le \mu_J, \tag{4.8}$$

$$z \le \mu_{P_d(t)}, \qquad t=1,...,T. \tag{4.9}$$

$$z \le \mu_{P_r(t)}, \qquad t=1,...,T. \tag{4.10}$$

Substituting membership functions into the above equations (4.8, 4.9 and 4.10) yields the following crisp optimization problem:

$$\min_{0\le z\le 1, p_{ti}(t), p_{bm}(t)} -z \tag{4.11}$$

subject to

$$z\Delta_J - J_o - \Delta_J + J \le 0, \tag{4.12}$$

$$(z-1)\Delta_d(t) - g(t) + P_d(t) \le 0, \qquad t=1,...,T, \tag{4.13}$$

$$(z-1)\Delta_d(t) + g(t) - P_d(t) \le 0, \qquad t=1,...,T, \tag{4.14}$$

$$(z-1)\Delta_r(t) - r(t) + P_r(t) \le 0, \qquad t=1,...,T, \tag{4.15}$$

where $g(t) = \sum_{i=1}^{I} p_{ti}(t) + \sum_{m=1}^{M} p_{bm}(t)$ and $r(t) = \sum_{i=1}^{I} R_i(p_{ti}(t))$.

The problem thus becomes maximizing a scalar value z such that the membership values of all constraints should be greater than or equal to this z. This results in a crisp optimization problem, and the set of new constraints ((4.12) through (4.15)) will be referred to as the "membership constraints." It should be noted that the piecewise linearity of membership functions plays a crucial role in this step to retain a separable structure of the problem as mentioned before.

4.3 A Lagrangian Relaxation Approach

To exploit the decomposable structure of the above crisp problem, the "objective constraint" (4.12), the system demand constraints (4.13, 4.14), and reserve requirement constraints (4.15) are relaxed by using Lagrange multipliers λ_J, $\lambda_l(t)$, $\lambda_r(t)$ and $\gamma(t)$, respectively. A two level max-min optimization problem can then be formed. The low level consists of the membership subproblem, thermal and purchase subproblems, and the high level problem updates Lagrange multipliers as explained below. Before going into the details, the objective function $-z$ in (4.11) is replaced by $b(z-2)^2 + J$ where b is a large positive number such that $b>>J$ following [20]. This is to make the problem approximately equivalent to the crisp case when all constraints are satisfied (i.e., when $z=1$. In this case, the original objective J will be minimized.). Also, if $z=1$ cannot be achieved, the term $b(z-2)^2$ will dominate the objective function. This term is equivalent to $-z$ of (4.11) but can reduce the bang-bang nature of the membership subproblem solutions as will be explained below.

With the new objective, the relaxed problem R is

$$R: \quad \min_{0\le z\le 1, p_{ti}(t), p_{bm}(t)} b(z-2)^2 + J + \lambda_J(z\Delta_J - J_o - \Delta_J + J)$$
$$+\sum_{t=1}^{T}\{\lambda_l(t)[(z-1)\Delta_d(t) - g(t) + P_d(t)]$$
$$+\lambda_r(t)[(z-1)\Delta_d(t) + g(t) - P_d(t)]$$
$$+\gamma(t)[(z-1)\Delta_r(t) - r(t) + P_r(t)]\}, \tag{4.16}$$

subject to the individual thermal unit and purchase transaction constraints. With λ_J, $\lambda_l(t)$, $\lambda_r(t)$ and $\gamma(t)$ given and the objective J of (4.3) plugged in (4.16), three kinds of subproblems are obtained after re-grouping relevant terms:

Membership subproblem:

$$L_z(\lambda_J,\lambda_l,\lambda_r,\gamma) \equiv \min_{0\le z\le 1} b(z-2)^2 + \lambda_J\Delta_J z$$
$$+\sum_{t=1}^{T}(z-1)\{(\lambda_l(t)+\lambda_r(t))\Delta_d(t) + \gamma(t)\Delta_r(t)\}. \tag{4.17}$$

Thermal subproblems:

$$L_i(\lambda_J,\lambda_l,\lambda_r,\gamma) \equiv \min_{p_{ti}(t)} \sum_{t=1}^{T}\{(1+\lambda_J)(C_{ti}(p_{ti}(t)) + S_i(t))$$
$$+(\lambda_r(t)-\lambda_l(t))p_{ti}(t) - \gamma(t)R_i(p_{ti}(t))\}. \tag{4.18}$$

Purchase subproblems:

$$L_m(\lambda_J,\lambda_l,\lambda_r,\gamma) \equiv \min_{p_{bm}(t)} \sum_{t=1}^{T}\{(1+\lambda_J)[m_{\tilde{C}_{bm}}(p_{bm}(t))$$
$$+w\Delta_{\tilde{C}_{bm}} p_{bm}(t)] + (\lambda_r(t)-\lambda_l(t))p_{bm}(t)\}. \tag{4.19}$$

Compared with the subproblems obtained from the crisp version in [11], only one additional subproblem needs to be solved for the fuzzy optimization problem here. The membership subproblem is to maximize the minimum degree of satisfaction among all the fuzzy constraints. Since the objective function of equation (4.17) is quadratic, this subproblem can be solved without difficulties.

A thermal subproblem is to determine the generations of a thermal unit so as to minimize the cost function (4.18) subject to its individual constraints. The problems are solved by following the method presented in [7] with the slightly modified cost function (generation and start-up costs are multiplied by $1+\lambda_J$). When the cost constraint (4.12) is not satisfied, the multiplier λ_J becomes positive. The thermal units would be more expensive, therefore tend to generate less to reduce the cost.

A purchase subproblem is to determine the megawatt levels and durations for a purchase transaction with the given multipliers so as to minimize the cost function (4.19). The problem can be solved by following the method of [11] with the modified cost function. It can be seen that if the spread $\Delta_{\tilde{C}_{bm}}$ is zero, the subproblem degenerates to the crisp subproblem presented in [11]. The weight w in (4.19) can be interpreted as a risk factor to reduce the range of cost uncertainties.

The high level dual problem is to update the multipliers λ_J, $\lambda_l(t)$, $\lambda_r(t)$ and $\gamma(t)$ so as to maximize the dual function as follows:

$$(D) \quad \max_{\lambda_J,\lambda_l,\lambda_r,\gamma\ge 0} L(\lambda_J,\lambda_l,\lambda_r,\gamma) \equiv \sum_{i=1}^{I} L_i(\lambda_J,\lambda_l,\lambda_r,\gamma)$$
$$+\sum_{m=1}^{M} L_m(\lambda_J,\lambda_l,\lambda_r,\gamma) + L_z(\lambda_J,\lambda_l,\lambda_r,\gamma)$$
$$-\lambda_J(J_o+\Delta_J) + \sum_{t=1}^{T}\{(\lambda_l(t)-\lambda_r(t))P_d(t) + \gamma(t)P_r(t)\}. \tag{4.20}$$

Since discrete decision variables are involved in low level subproblems, the objective function $L(\lambda_J,\lambda_l,\lambda_r,\gamma)$ of (4.20) may not be differentiable at certain points. These multipliers are thus updated by using the subgradient method with adaptive step sizing as presented in [7]. Comparing with our previous work [7], the number of multipliers is increased as indicated by equation (4.16) since more constraints are relaxed by Lagrange multipliers.

The method presented above should produce robust solutions to hedge against uncertain system demand and future transaction opportunities. For example, suppose the system operator is evaluating a purchase offer with a unit price of $20 per megawatt. For the crisp method of [11], the decision is made based on the comparison of the multipliers and this unit price. If the average of multipliers in the time duration is slight higher than the price, the purchase transaction will be accepted and a contract will be signed. If a transaction opportunity of $16 per megawatt emerges soon after, the previous decision may become uneconomical. In the fuzzy algorithm, the decision of a purchase transaction is made not only based on the comparison of the multipliers and the price, but also affected by future transaction opportunities and possible ranges of system demand. In fact, the combined multipliers in (4.19) may not reach the point to accept this transaction. This is why the fuzzy method may make a conservative decision on the purchase which is on the cost margin.

4.4 Constructing A Feasible Solution

At the stop of iterations, subproblem solutions generally do not construct a feasible solution, i.e., constraints (4.12) to (4.15) generally are not satisfied. Compared with the crisp method of [11], one more constraints (the objective constraint (4.12)) has to be satisfied. The approach taken here is to first fix the membership variable to the value z^* obtained from the membership subproblem. The heuristics developed in [7, 11] are used to construct a solution satisfying (4.13), (4.14) and (4.15). The total cost J is then calculated. If (4.12) is satisfied, this solution is feasible, otherwise, membership variable z^* is adjusted as

$$z^* = (J_o - J + \Delta_J) / \Delta_J \qquad (4.21)$$

to satisfy the objective constraint.

5. Fuzzy Simulation

5.1 Realization of Fuzzy Numbers

To evaluate the performance of the fuzzy method in an uncertain environment, a fuzzy simulation shell has been developed. In the simulation, fuzzy numbers are approximately realized based on the frequency interpretation using two random numbers following the method of [21]. To realize a fuzzy number, a random number u_1 uniformly distributed in-between 0 and 1 is first generated, and a "level set" is formed consisting of all values whose memberships are greater than or equal to u_1. A second random number u_2 uniformly distributed within the level set is then generated, and u_2 is regarded as the realization of the fuzzy number. The realization of a triangular fuzzy number is depicted in Figure 3.

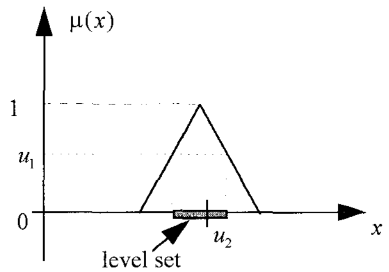

Figure 3. The realization of a triangular fuzzy number

5.2 Simulation Procedure

The simulation procedure consists of the following steps:

Step 1: Execute fuzzy algorithm with the fuzzy system demand, reserve requirements, and prices of future purchases. One set of scheduling and purchase decisions and a z^* are obtained. The solution is feasible with respect to the constraints relaxed (i.e., (4.12) to (4.15)).

Step 2: Realize fuzzy system demand and prices for future purchases by using the method presented above. Reserve requirements are obtained as 7% of system demand.

Step 3: Generate the level sets of fuzzy prices corresponding to the value of z^* obtained. If the realized price of a purchase in a load period fall in the associated level set, the purchase of the load period is accepted per the results obtained in Step 1. Otherwise, the opportunity for that period will be rejected.

Step 4: To meet the demand and reserve requirements realized in Step 2, the scheduling decisions are modified following the heuristics reported in [7,11]. The resulting decisions are feasible with respect to the realized fuzzy demand and reserve requirements in a crisp sense, and the total cost J of (3.1) is calculated.

To evaluate the performance of the fuzzy algorithm, the crisp algorithm of [11] is also simulated. The simulation procedure is similar to what described above, with the following minor modifications. In Step 1, the crisp algorithm is run with all fuzzy numbers replaced by their mean values. The decision for accepting/rejecting a transaction in Step 3 is based on system marginal costs and the realized prices, as described by equation (3.3) of [11].

6. Testing Results

The algorithm was implemented in FORTRAN on an IBM RISC6000 workstation. Testing results presented here are based on three Northeast Utilities (NU) data sets: week 1 in May 1993, week 2 in August 1993, and week 1 in September 1993. These data sets are selected from NU billing data files with prices and periods of purchase opportunities modified to test various scenarios. The contributions of hydro and pumped-storage units, together with power provided by co-generators and non-dispatchable contracts, are deducted from system demand. The mean values of system demand are obtained from the data sets, with maximum variation 4% of the demand on either side. The nominal required reserve is calculated as 7% of the demand at each hour, with maximum variation 4% of the nominal value. The system characteristics associated with these data sets are summarized in Table 1. The parameters for the testing are summarized in Table 2, with time horizon equal to one week (i.e., 168 hours).

Table 1. Summary of System Characteristics

System characteristics	Number of units or transactions	Total capacity or requirement (MW)
Steam units	22-27	2160-2447
Turbine units	19-23	453-498
Nuclear units	7-8	1974-2156
All units	48-58	4587-5118
Purchases	5-6	600-800
Peak load		3943-4635
Minimum load		1728-1831

Table 2. Parameters of Fuzzy Algorithm

Data sets	J_o	$J_o + \Delta_J$	w
Mayw193	5,528,270	6,081,097	0.5
Augw293	7,095,578	7,805,136	0.5
Sepw193	6,296,282	6,925,910	0.5

Testing results are obtained by running the simulation 100 times for both fuzzy and crisp algorithms, and are summarized in Table 3. The number of high level iterations and computational times in Table 3 are obtained after running the algorithms once as stated in Step 1. Based on the results, simulations are conducted 100 times according to Steps 2 to 4 to obtain the average costs of Table 3. It can been seen that the average total costs of the fuzzy method are consistently lower than the ones obtained by the crisp method. However, the number of high level iterations and computational times of the fuzzy method are larger than those of the crisp one since more Lagrange multipliers are needed to be updated.

Table 3. Comparison Results of Testing Cases

	Number of high level iterations		Computational time (sec.)		Total cost (K$)		z^*
Data sets	Fuzzy	Crisp	Fuzzy	Crisp	Fuzzy	Crisp	Fuzzy
Mayw193	51	29	197	133	5,640	5,689	0.595
Augw293	48	33	205	122	7,232	7,307	0.659
Sepw193	56	34	228	145	6,411	6,474	0.625

To analyze the reasons why the fuzzy method produces more economic solutions, two scenarios of the August week 2 data set have been examined. In the first scenario, there is no future purchase opportunities, and the goal is to examine the effects of fuzzy system demand and reserve requirements. Testing results by running the simulation 100 times are summarized in Table 4. The feasibility and infeasibility in the table refer to the satisfaction of realized demand and reserve requirements by using the solutions obtained in Step 1. It can be seen that the fuzzy algorithm commits less turbine units than what the crisp one does. The solution of the fuzzy method also contains less infeasible hours and lower infeasible MW levels, and needs less changes to be feasible when system demand varies. The reason is that the crisp algorithm commits units to meet fixed system demand and reserve requirements crisply, but once system demand varies as time evolves, the original economic solution may become uneconomic. In fact, the solution of the crisp method contains more isolated infeasible hours, therefore requires more changes and perhaps more turbine units at higher costs.

Table 4. Result Summary for the First Scenario

	Fuzzy	Crisp	Diff.
Total Number of Committed Turbine Units	1	8	7
Infeasible Hours	24	39	15
Infeasible MW Level	74.2	92.8	18.6
Number of Changes for Feasible Schedules	16	27	11
Total Cost (K$)	7,422	7,445	23

In the second scenario, the system demand and reserve requirements are assumed given after they are first realized, and the goal is to examine the effects of fuzzy prices of purchase opportunities. For the August week 2 data set, two purchase transactions are considered: one is a crisp purchase with $25.5/MWh fixed price and 300MW maximum level for peak hours (16 hours per day and five weekdays for the week) considered in Step 1; and the other is a future purchase opportunity covering the same time period with fuzzy prices in-between $14/MWh to $18/MWh and 200MW maximum level considered in Steps 2 to 4. The results are summarize in Figure 4 with costs provided in Table 5. It can be seen from Figure 4 that MW levels of the fuzzy algorithm are less than those of the crisp algorithm, and time durations to purchase in the fuzzy algorithm are also shorter. The reason is that the price of the crisp purchase is very close to the system marginal costs and cost saving is not significant, therefore the fuzzy algorithm decides not to purchase much. From Table 5, it can also be seen that the average total cost of the fuzzy method is less than the one obtained by using the crisp method, since the cheaper purchase opportunity emerged later, and affects the economics of the previous decisions. Therefore, the fuzzy method hedges against uncertainties in deciding the first crisp offer, and this results in flexibility when opportunities arise in the future. Because hydro and pumped-storage units have not been incorporated into the algorithm and the transactions tested were not based on actual opportunities emerged, it is hard to compare the results with actual purchase decisions.

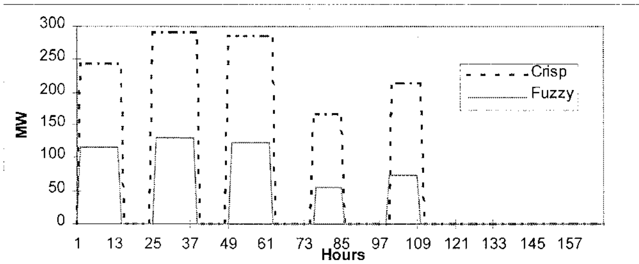

Figure 4. Decisions for the crisp purchase by the two methods

Table 5. Cost Results for the Second Scenario

	Fuzzy	Crisp
Average total costs (K$)	7,199	7,230

7. Conclusion

By using fuzzy relations to model uncertain system demand and reserve requirements and fuzzy numbers to approximate uncertain prices of future purchase opportunities, the integrated scheduling and transaction problem has been formulated as a fuzzy mixed integer programming problem. A fuzzy optimization algorithm has been developed, and simulation results show that the average total cost are reduced, and robust schedules and transaction decisions are obtained. It should be mentioned that both problem formulation and solution methodology can be extended to problems considering other selected uncertainties, e.g., uncertain fuel prices of thermal units. As a step towards the production use of the algorithm, hydro and pumped-storage units will be incorporated into the system.

Comparing with stochastic optimization, the fuzzy optimization-based method presented in this paper is computationally more tractable, while the probability distribution for future system demand might be more explicit. In order to properly model the uncertainties and further improve the results, the parameters of the fuzzy numbers, which are used to model uncertainties in the paper, need to be adjusted based on the historical data or human experience.

Acknowledgment

This work was supported in part by the National Science Foundation under Grant ECS-9420972, and by a grant from Northeast Utilities Service Company. The authors would like to thank Dr. Robert Tomastik of United Technology Research Center and anonymous reviewers for their valuable comments and suggestions.

References

1. Zhai, D., A. M. Breipohl, F. N. Lee, and R. Adapa, "The Effect of Load Uncertainty on Unit Commitment Risk," *IEEE Trans. on Power Syst.*, Vol. 9, No. 1, February 1994, pp. 510-516.
2. Cohen, A., "Optimization-Based Methods for Operations Scheduling," *Proc. of IEEE,* Vol. 75, No. 12, 1987, pp. 1574-1591.
3. Bertsekas, D. P., G. S. Lauer, N. R. Sandell, and T. A. Posbergh, "Optimal Short-Term Scheduling of Large Scale Power Systems," *IEEE Trans. on Automatic Control*, Vol. AC-28, No. 1, 1983, pp. 1-11.
4. Bard, J. F., "Short-Term Scheduling of Thermal Electric Generators Using Lagrangian Relaxation," *Operations Research*, Vol. 36, No. 5, 1988, pp. 756-766.
5. Ferreira, L. A. F. M., T. Andersson, C. F. Imparato, T. E. Miller, C. K. Pang, A. Svoboda, and A. F. Vojdani, "Short-term Resource Scheduling in Multi-Area Hydrothermal Power Systems," *Electric Power & Energy Syst.*, Vol. 11, No. 3, July 1989, pp. 200-212.
6. Ruzic, S and N. Rajakovic, "A New Approach for solving Extended Unit Commitment Problem," *IEEE Trans. on Power Syst.*, Vol. 6, No. 2, February 1991, pp. 269-277.
7. Guan, X., P. B. Luh, H. Yan and J. A. Amalfi, "An Optimization Based Method for Unit Commitment," *International Journal of Electrical Power & Energy Syst.*, Vol. 14, No. 6, Feb. 1992, pp. 9-17.
8. Yan, H., P. B. Luh, X. Guan and P. Rogan, "Scheduling of Hydrothermal Power Systems," *IEEE Trans. on Power Syst.*, Vol. 8, No. 3, August 1993, pp. 1358-1365.
9. Guan, X., P. B. Luh, H. Yan and P. Rogan, "Optimization-Based Scheduling of Hydrothermal Power Systems with Pumped-Storage Units," *IEEE Transaction on Power Systems,* Vol. 9, No. 2, May 1994, pp. 1023-1031.
10. Wang, C., S. M. Shahidehpour, "Power Generation Scheduling for Multi-Area Hydro-thermal Systems with Tie Line Constraints, Cascade Reservoirs and Uncertain Data", *IEEE Trans. on Power Syst.*, Vol. 8, No. 3, August 1993, pp. 1333-1340.
11. Zhang, L., P. B. Luh, X. Guan and G. Merchel, "Optimization-Based Inter-Utility Power Purchases," *IEEE Trans. on Power Syst.*, Vol. 9, No. 2, May 1994, pp. 891-897.
12. Prasannan, B., P.B. Luh, H. Yan, J.A. Palmberg and L. Zhang, "Optimization-based Sale Transactions and Hydrothermal Scheduling," *Proceedings of the 1995 IEEE PICA Conference*, Salt Lake City, May 1995, pp. 137-142, also to appear in *IEEE Transactions on Power Systems*.
13. Zimmermann, H. J., *Fuzzy Set Theory and Its Applications*, second edition, Kluwer Academic Publishers, Boston, 1991.
14. Bellman, R., and L. A. Zadeh, "Decision-making in a Fuzzy Environment," *Management Science*, Vol. 17, 1970, pp. 141-164.5.
15. Lai, Y. J., and C. L. Hwang, "Fuzzy Mathematical Programming, Methods and Applications," *Lecture Notes in Econ. and Math. Syst.*, No. 394, Springer Verlag, 1992.
16. Sakawa, M., *Fuzzy Sets and Interactive Multiobjective Optimization*, Plenum Publishers, 1993.
17. Su, C. C., and Y. Y. Hsu, "Fuzzy Dynamic Programming: An Application to Unit Commitment," *IEEE Trans. on Power Syst.*, Vol. 6, No. 3, Aug. 1991, pp. 1231-1237.
18. Miranda, V. and J. T. Saraiva, "Fuzzy Modeling of Power System Optimal Load Flow," *IEEE Trans. on Power Syst.*, Vol. 7, No. 2, May 1992, pp. 843-849.
19. Wang, C., S. M. Shahidehpour and R. Adapa, "A fuzzy Set Approach to Tie Flow Computation in Optimal Power Scheduling," *Proc. of the 54th Annual Meeting of the American Power Conf.*, Chicago, Illinois, April 1992, pp. 1443-1450.
20. Guan, X, P. Luh and B. Prasannn, "Power System Scheduling with Fuzzy Reserve Requirements," *IEEE/PES* 1995 Summer Meeting, Portland, Oregon, July 1995; also to appear in *IEEE Transactions on Power Systems*.
21. Chanas, S., and M. Nowakowski, "Single Value Simulation of Fuzzy Variables," *Fuzzy Sets & Syst.*, Vol. 25, 1988, pp. 43-57.
22. Chanas, S., and S. Heilpern, "Single Value Simulation of Fuzzy Variables - Some Further Results," *Fuzzy Sets & Syst.*, Vol. 33, No. 1, Oct. 1989, pp. 29-36.
23. Negi, D. S. and E. S. Lee, "Analysis and Simulation of Fuzzy Queues," *Fuzzy Sets and Syst.*, Vol. 46, 1992, pp. 321-330.
24. Hansson, A., "A Stochastic Interpretation of Membership Functions," *Automatica*, Vol. 30, No. 3, March 1994, pp. 551-553.

Houzhong Yan received his B.S. degree in Mathematics and M.S. degree in Computer Sciences from East China Institute of Technology, Nanjing, P.R. China in 1982 and 1986, respectively. From 1986 to 1989, he was a lecturer at the Dept. of Computer Sciences, Southeastern University, Nanjing, P.R. China. Currently he is a Ph.D. candidate at the University of Connecticut, and senior engineer with Engery Marketing Division of Southern California Edison Company, California.

Peter B. Luh (S'77-M'80-SM'91-F'95) received his B.S. degree in Electrical Engineering from National Taiwan University, Taipei, Taiwan, Republic of China, in 1973, the M.S. degree in Aeronautics and Astronautics from M.I.T., Cambridge, Massachusetts, in 1977, and the Ph.D. degree in Applied Mathematics from Harvard University, Cambridge, Massachusetts, in 1980. Since 1980, he has been with the University of Connecticut, and currently is a Professor in the Department of Electrical and Systems Engineering. His major research interests include schedule generation and reconfiguration for manufacturing systems and power systems. Dr. Luh is an Editor of the *IEEE Transaction on Robotics and Automation*, was an Associate Editor of the *IEEE Transactions on Automatic Control*, and has served on program Committees and Operating Committees of many major national, international, and inter-society conferences. He is listed in *Who's Who in Engineering*, *Who's Who in the East*, and *Who's Who in American Education*.