Optimization-Based Scheduling of Hydrothermal Power Systems with Pumped-Storage Units

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Abstract. This paper presents an optimization-based method for scheduling hydrothermal systems based on the Lagrangian relaxation technique. After system-wide constraints are relaxed by Lagrange multipliers, the problem is converted into the scheduling of individual units. This paper concentrates on the solution methodology for pumped-storage units. A pumped-storage unit can be operated in generation, pumping or idle states. It can smooth peak loads and provide reserve, therefore plays an important role in reducing total generation costs. There are, however, many constraints limiting the operation of a pumped-storage unit, such as pond level dynamics and constraints, and discontinuous generation and pumping regions. Moreover, according to the current practice, the dynamic transitions among operating states (generation, pumping and idle) are not arbitrary. The most challenging issue in solving pumped-storage subproblems within the Lagrangian relaxation framework is the integrated consideration of these constraints.

The basic idea of our method is to relax the pond level dynamics and constraints by using another set of multipliers. The subproblem is then converted into the optimization of generation or pumping levels for each operating state at individual hours, and the optimization of operating states across hours. The optimal generation or pumping level for a particular operating state at each hour can be obtained by optimizing a single variable function without discretizing pond levels. Dynamic programming is then used to optimize operating states across hours with only a few number of states and transitions. A subgradient algorithm is used to update the pond level Lagrangian multipliers. This method provides an efficient way to solve a class of subproblems involving continuous dynamics and constraints, discontinuous operating regions, and discrete operating states. Testing results based on Northeast Utilities power system show that this algorithm is efficient, and near optimal solutions are obtained.

Keywords: Hydrothermal Power System Scheduling, Pumped-storage Unit Scheduling, Lagrangian Relaxation.

1. Introduction

A problem faced daily by a utility company in operating a hydrothermal power system is to determine the commitment and generation of all power resources to meet the system demand and reserve requirements on an hourly basis. The goal is to minimize the total generation cost. The economic consequence of operation scheduling is significant. Reducing the generation cost by 0.5% can result in savings of several million dollars per year for a large utility company ([4]). Consistently generating optimal schedules, however, has proved to be very difficult. The reason is that the problem belongs to the class of NP-hard (Non-Polynomial) combinatorial problems, i. e., the computational requirements increase exponentially with problem sizes, and is considered to be extremely difficult to solve for systems of practical sizes (e.g., 100 units).

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In this paper, an optimization-based method for obtaining near optimal schedules of hydrothermal systems is presented based on the Lagrangian relaxation technique ([4], [5], [10]). By using Lagrangian multiplier to relax the system-wide demand and reserve requirements, the problem is decomposed and converted into a two-level optimization problem. The low level consists of a number of subproblems, one for each unit, and the high level is to optimize Lagrange multipliers. The disadvantage of this method is that the dual solution is generally infeasible, i.e., the once relaxed system-wide constraints are generally not satisfied. Some techniques, usually heuristics, are needed to modify the dual solution to obtain a good feasible schedule. However, since the cost from the dual solution is a lower bound on the optimal cost, the quality of the feasible solution can be quantitatively evaluated.

In our previous work, the Lagrangian relaxation technique has been successfully used to schedule power systems with thermal and hydro units. The methods for solving hydro and thermal subproblems, the subgradient method for updating multipliers at the high level, and the heuristics for obtaining feasible solutions have been presented in [6], [7], [11]. This paper concentrates on the solution methodology for pumped-storage units.

Utilizing the economical energy generated by base-load units, a pumped-storage unit pumps water into its pond during the period of low demand and low cost, and discharges water for generation at hours of high demand and high cost. Pumping during the low demand period can also provide reserve so that no extra thermal units have to be committed just for providing reserve. A pumped-storage unit can therefore smooth peak loads, provide reserve, and play an important role in reducing total generation costs.

A pumped-storage unit has pond level dynamics and pond level limit constraints which couple the hourly generation or pumping decisions across the entire time horizon. It has minimum generation and minimum pumping requirements for efficient operation, and the unit can also stay idle, neither generating nor pumping. The unit therefore operates in discontinuous regions. A complicating problem is that the unit may generate or pump for less than an hour during a transient period between generation, pumping or idle. For modeling purpose, this can be conceived of as if the unit generated or pumped below minimum for the entire transition hour. According to the current practice, the transition dynamics is not arbitrary. The concept of "operating states" is therefore introduced to regulate the dynamic transitions among generation, pumping, idle, and "fringes" associated with less-than-minimum generation or pumping.

Integrated consideration of pond level dynamics and constraints, discrete operating regions and dynamic transitions among operating states is the most challenging issue in solving pumped-storage subproblems. Because of the discontinuous operating regions and dynamic transitions of operating states, it is difficult to formulate this subproblem as a constrained optimal control problem as in [10], where pond level constraints are handled by using the multiplier method. In view of the pond level dynamics and constraints, the subproblem would not be solved by using the merit order dispatch technique as in [1] either. Discretizing pond levels and solving the subproblem by using dynamic programming as in [3] may greatly increase computational requirements. Some of the pumped-storage issues mentioned above may have been encountered in solving thermal subproblems with fuel limits. However, they are generally simpler since there are no continuous-variable dynamics and constraints such as pond level dynamics and constraints. The techniques developed for solving fuel-limited thermal subproblems therefore may not be applicable to pumped-storage subproblems.

The basic idea of our method is to relax the pond level dynamics and constraints by using another set of multipliers. The subproblem is then converted into the optimization of generation or pumping levels for each operating state at individual hours, and the optimization of operating states across hours. The optimal generation or pumping level for a particular operating state at each hour can be obtained by optimizing a single variable function without discretizing pond levels. Dynamic programming is then used to optimize operating states across hours with only a few number of states and transitions. An intermediate level is created to update this set of pond level multipliers. This method eliminates the difficult trade-off between computational requirements and accuracy as needed by discretization approaches. It therefore provides an efficient solution to a class of subproblems involving continuous dynamics and constraints, discontinuous operating regions, and discrete operating states.

Another difficulty in solving the pumped-storage subproblem is associated with its piece-wise linear cost function with coefficients determined by the multipliers. The optimal generation or pumping may therefore change from one corner point to another with slight modification of multipliers. This solution oscillation may result in numerical instability. To overcome this difficulty, the piece-wise linear water-power conversion is approximated in the paper by a quadratic function. A lower bound to the original problem, however, can still be obtained by re-solving the pumped-storage subproblems using the original water-power conversion function.

2. Problem Formulation

Consider a hydrothermal power system with I thermal units, J hydro units and K pumped-storage units. It is required to determine the operating status and generation/pumping levels of all units over a specified time period T. The objective is to minimize the total generation cost subject to system demand and spinning reserve requirements, and other individual unit constraints. The time unit is one hour and the planning horizon may vary from one day to ten days.

Notations

To formulate the problem mathematically, the following notation is first introduced:

$C_i(p_{ti}(t))$: fuel cost of thermal unit i for generating power $p_{ti}(t)$ at time t, a piecewise linear function of $p_{ti}(t)$, in dollars;
I: number of thermal units;
J: number of hydro units;
K: number of pumped-storage units;
$P_d(t)$: system demand at time t, in MW;
$p_{hj}(t)$: power generated by hydro unit j at time t, in MW;
$p_{pk}(w_k(t))$: power generated (positive values) or power used for pumping (negative values) by pumped-storage unit k at time t, in MW;
$\bar{p}^g_{pk}(t)$: maximum generation level of pumped-storage unit k at time t, in MW;
$\underline{p}^g_{pk}(t)$: nominal minimum generation level of pumped-storage k;
$\bar{p}^p_{pk}(t)$: maximum pumping level of pumped-storage k;
$\underline{p}^p_{pk}(t)$: nominal minimum pumping level of pumped-storage k;
$P_r(t)$: system spinning reserve requirement at time t, in MW;
$p_{ti}(t)$: power generated by thermal unit i at time t, in MW;
$r_{hj}(p_{hj}(t))$: spinning reserve contribution of hydro unit j at time t, in MW;
$r_{pk}(p_{pk}(w_k(t)))$: spinning reserve contribution of pumped-storage unit k at time t, in MW;
$\bar{r}_{pk}$: maximum spinning reserve contribution of pumped-storage unit k, in MW;
$r_{ti}(p_{ti}(t))$: spinning reserve contribution of thermal unit i at time t, in MW;
$S_i(t)$: startup cost of thermal unit i, in dollars;
T: time horizon under consideration, in hours;
$v_k(t)$: pond level of pumped-storage unit k at time t, converted to MWhr;
V^o_k: initial pond level of pumped-storage unit k, converted to MWhr;
V^T_k: terminal pond level of pumped-storage unit k, converted to MWhr;
$\bar{V}_k$: maximum pond level of pumped-storage unit k, converted to MWhr;
$w_k(t)$: water discharged (positive values) or refilled (negative values) by pumped-storage unit k at time t, converted to MW;
ρ: efficiency of pumped-storage units.

Cost function and system-wide constraints

The problem is formulated as the following mixed integer programming problem:

(P):

$$\min_{\substack{p_{ti}(t)\\ p_{hj}(t)\\ p_{pk}(t)}} J, \quad with\ J \equiv \sum_{t=1}^{T}\sum_{i=1}^{I}[C_i(p_i(t)) + S_i(t)], \tag{2.1}$$

subject to
system-wide constraints including

-- system demand:

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{j=1}^{J} p_{hj}(t) + \sum_{k=1}^{K} p_{pk}(w_k(t)) = P_d(t); \tag{2.2}$$

-- spinning reserve:

$$\sum_{i=1}^{I} r_{ti}(p_{ti}(t)) + \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) + \sum_{k=1}^{K} r_{pk}(p_{pk}(w_k(t))) \geq P_r(t), \tag{2.3}$$

and individual constraints.

Thermal and hydro unit constraints

For a thermal unit, individual constraints may include:
-- capacity and minimum generation;
-- minimum up/down time;
-- ramp rate;
-- minimum generation for the first and last hour;
-- must-run or must-not-run.

For a hydro unit, individual constraints include:
-- capacity and minimum generation;
-- total weekly or daily hydro energy.

Detailed descriptions of individual constraints for thermal and hydro units can be found in [6], [7] and [11].

Pumped-storage unit constraints

The constraints for pumped-storage units can be classified into two categories: those associated with pond levels and those associated with generation or pumping levels. The first category includes pond level dynamics:

$$v_k(t+1) = v_k(t) - w_k(t), \tag{2.4}$$

and pond level limits and terminal pond level:

$$0 \leq v_k(t) \leq \bar{V}_k, \tag{2.5}$$

$$v_k(T) = V^T_k. \tag{2.6}$$

The initial pond level is given as

$$v_k(0) = V^o_k. \tag{2.7}$$

Following the practice of the billing rules of New England Power Pool (NEPOOL), the initial pond level is assumed to be full. The terminal pond level usually equals the initial pond level so that the pond is ready to be used for the next cycle.

Generation or pumping level constraints

The "normal" generation and pumping level constraints for unit k are given, respectively, by:

$$\underline{p}^g_k(t) \leq p_{pk}(w_k(t)) \leq \bar{p}^g_{pk}(t), \tag{2.8}$$

and

$$-\bar{p}^p_{pk}(t) \leq p_{pk}(w_k(t)) \leq -\underline{p}^p_k(t). \tag{2.9}$$

When the unit is idle, one has

$$p_{pk}(w_k(t)) = 0. \tag{2.10}$$

As mentioned in the Introduction, the unit may generate or pump below minimum at a transition hour between generation, pumping or idle. Such a below-minimum transition hour is called a "fringe hour," and satisfies

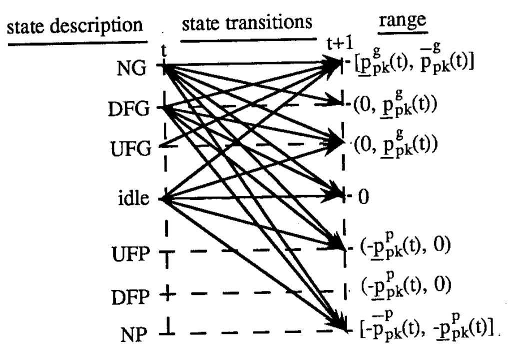
$$0 \le p_{pk}(w_k(t)) \le \underline{p}^g_{pk}(t), \tag{2.11}$$

and

$$-\underline{p}^p_{pk}(t) \le p_{pk}(w_k(t)) \le 0, \tag{2.12}$$

for fringe generation or fringe pumping, respectively. The unit, however, may go directly from normal generation to idle or normal pumping. Fringes are therefore not required transition operations, but are introduced to model those below-minimum transition hours.

To describe the dynamics involved, the concept of "operating states" is introduced to regulate the dynamic transitions among generation, pumping, idle, and fringes. The following operaing states are defined based on NEPOOL billing rules: generation between nominal minimum and maximum as "Normal Generation (NG);" generation between zero and nominal minimum as "Down Fringe Generation (DFG)" if the generation at the previous hour is normal, and "Up Fringe Generation (UFG)" if the generation at the next hour is normal. The reason to distinguish two fringe generation states is that the valid transitions of DFG are different from those of UFG. A particular hour, however, could belong to DFG and UFG at same time. "Normal Pumping (NP)," "Down Fringe Pumping (DFP)," and "Up Fringe Pumping (UFG)" are similarly defined. A state transition diagram specifying states, their range of generation or pumping and valid transitions is given in Fig. 1. It can be seen that there are only a few number of states and transitions and the transitions are well structured.



(The transitions from three pumping states are not depicted to prevent the figure from being too crowded. They are, however, symmetric with respect to those from three generation states.)

Fig. 1 State transition diagram

Water-Power Conversion and Spinning Reserve

The water volume in the pond is measured by the energy needed to pump that amount of water. If 1 MWhr is used to pump a certain amount of water into the pond, the amount of energy available for generation is ρ MWhr, where $\rho < 1$ is the efficiency coefficient, and is assumed to be a constant across generation or pond levels. The water power conversion of the unit is therefore piece-wise linear as shown in Fig. 2. According to the NEPOOL billing rules, the spinning reserve contribution of a pumped-storage unit is a piece-wise linear function of the power generated or the power used for pumping as shown in Fig. 3†.

†. There are physically four generating units associated with one reservoir in the NU power system. Based on the billing rule, they are modeled as a single unit with minimum generation of one physical unit and capacity of four. The reserve contribution of a physical unit in normal generation is the difference between its maximum generation and generation level. When it generates below its nominal minimum, the reserve contribution is proportional to its generation level as set by the rule. When generation level of the aggregated unit is between its nominal minimum and maximum, the physical units are dispatched in the way to give maximum reserve contribution. The reserve contribution in a pumping state is always equal to the pumping level.

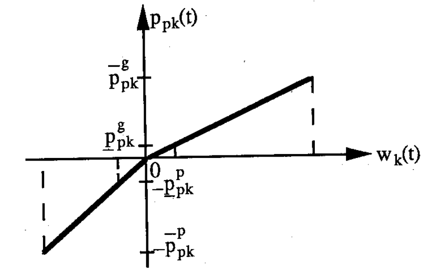


Fig. 2 Water-power conversion of a pumped-storage unit

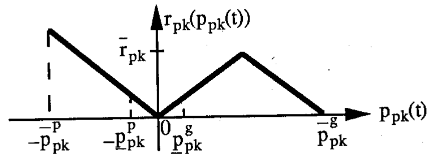


Fig. 3 Spinning reserve of a pumped-storage unit

3. Solution Methodology

3.1 The Lagrangian Relaxation Framework

The basic idea of the Lagrangian relaxation technique is to relax system-wide constraints on demand and spinning reserve (eqs. (2.2) and (2.3)) by using Lagrange multipliers and to formulate a two-level structure. According to the cost function (2.1) and constraints (2.2), (2.3), the Lagrangian is formulated as follows:

$$\begin{aligned} L = \sum_{t=1}^{T} \{ & \sum_{i=1}^{I} [C_i(p_i(t)) + S_i(t)] \\ & + \lambda(t)[P_d(t) - \sum_{i=1}^{I} p_{ti}(t) - \sum_{j=1}^{J} p_{hj}(t) - \sum_{k=1}^{K} p_{pk}(w_k(t))] \\ & + \mu(t)[P_r(t) - \sum_{i=1}^{I} r_{ti}(p_{ti}(t)) \\ & - \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) - \sum_{k=1}^{K} r_{pk}(p_{pk}(t))] \}, \end{aligned} \tag{3.1}$$

where $\lambda(t)$ and $\mu(t)$ are Lagrange multipliers associated with demand and spinning reserve requirements at time t, respectively. For notational convenience, define

$$\lambda \equiv [\lambda(1), \lambda(2), \ldots\ldots, \lambda(T)]^T, \tag{3.2}$$

$$\mu \equiv [\mu(1), \mu(2), \ldots\ldots, \mu(T)]^T. \tag{3.3}$$

By using the duality theorem ([2], [9]) and exploiting the decomposable structure of (3.1), a two-level max-mini optimization problem can be formed. Given multipliers λ and μ, the low level consists of individual thermal, hydro and pumped-storage subproblems:

Thermal subproblems (Pt-i), i=1,2,...,I:

$$\begin{aligned} \min_{p_{ti}(t)} L_{ti}, \ with\ L_{ti} \equiv \sum_{t=1}^{T} \{ & [C_i(p_{ti}(t)) + S_i(t)] \\ & - \lambda(t)p_{ti}(t) - \mu(t)\, r_{ti}(p_{ti}(t))\}, \end{aligned} \tag{3.4}$$

subject to individual thermal unit constraints.

Hydro subproblems (Ph-j), j=1,...,J:

$$\begin{aligned} \min_{p_{hj}(t)} L_{hj}, \ with\ L_{hj} \equiv \sum_{t=1}^{T} \{ & - \lambda(t)p_{hj}(t) \\ & - \mu(t)\, r_{hj}(p_{hj}(t))\}, \end{aligned} \tag{3.5}$$

subject to individual hydro unit constraints.

Pumped-storage unit subproblems (Pp-k), k=1,...,K:

$$\min_{w_k(t)} L_{pk}, \ \ with \ L_{pk} \equiv \sum_{t=1}^{T} \{ -\lambda(t) p_{pk}(w_k(t)) - \mu(t) \, r_{pk}(p_{pk}(w_k(t))) \}, \quad (3.6)$$

subject to constraints (2.4) - (2.12) and operating state dynamics.

Let $L^*_{ti}(\lambda, \mu)$, $L^*_{hj}(\lambda, \mu)$ and $L^*_{pk}(\lambda, \mu)$ denote, respectively, the optimal Lagrangian for (Pt-i), (Ph-j) and (Pp-k) for the given λ and μ. Then the high level dual problem is

(P-D):

$$\max_{\lambda, \mu} \Phi(\lambda, \mu), \ \ with \ \Phi(\lambda, \mu) \equiv \sum_{i=1}^{I} L^*_{ti}(\lambda, \mu) + \sum_{j=1}^{J} L^*_{hj}(\lambda, \mu) + \sum_{k=1}^{K} L^*_{pk}(\lambda, \mu) + \sum_{t=1}^{T} [\lambda(t) P_d(t) + \mu(t) P_r(t)], \quad (3.7)$$

subject to

$$\mu(t) \geq 0, \quad t = 1, 2, \ldots\ldots, T. \quad (3.8)$$

To obtain a near optimal solution, efficient algorithms are needed for solving three types of subproblems, solving the dual problem, and also for constructing a feasible solution.

3.2 Quadratic Approximation of the Water-Power Conversion Function

Since $p_{pk}(w_k(t))$ is piece-wise linear with respect to $w_k(t)$ as shown in Fig. 2 and $r_{pk}(p_{pk}(w_k(t)))$ is piece-wise linear with respect to $p_{pk}(w_k(t))$ as shown in Fig. 3, $L_{pk}(w_k(t))$ in (3.6) is a piece-wise linear function of $w_k(t)$ with coefficients determined by Lagrangian multipliers. When the pond level limit in (2.5) is not active, the optimal generation or pumping is generally obtained at one of the corner or boundary points: maximum generation/pumping, minimum generation/pumping, maximum reserve or idle (zero). The optimal generation or pumping therefore changes from one of these points to another as the multipliers change. This solution oscillation may lead to numerical instability. To overcome this difficulty, a quadratic function is used to approximate the water-power conversion as shown in Fig 4. The power $p_{pk}(w_k(t))$ will then be a quadratic function of $w_k(t)$, and this in turn leads to a piece-wise quadratic $L_{pk}(w_k(t))$. The optimal generation or pumping therefore no longer jumps from one corner point to another over the iterations, and the difficulties caused by solution oscillation can be avoided. To differentiate from the original water-power conversion, the new conversion is denoted as $\hat{p}_{pk}(w_k(t))$, new cost as $\hat{L}_{pk}$ and new subproblem as $(\hat{P}p\text{-}k)$.

3.3 Solving Pumped Storage Subproblems

Given λ and μ, the pumped-storage subproblem $(\hat{P}p\text{-}k)$ is to determine the generation or pumping at each hour so as to minimize the cost function (3.6) subject to individual constraints (2.4)-(2.12) and operating state dynamics. Although this cost function is stage-wise additive, the generation or pumping at hour t cannot be determined by simply minimizing the stage-wise cost function at that hour since pond level constraints (2.4)-(2.6) couple decisions across hours. Furthermore, operating regions are discontinuous, and dynamic changes of operating states across hours are not arbitrary. The basic idea of our algorithm is to relax the pond level dynamics and constraints by using another set of multipliers. The optimal generation or pumping for each operating state at a particular hour can be easily obtained by optimizing a single variable function. Dynamic programming can then be used to obtain the optimal generation or pumping levels for the entire time horizon without discretizing pond levels.

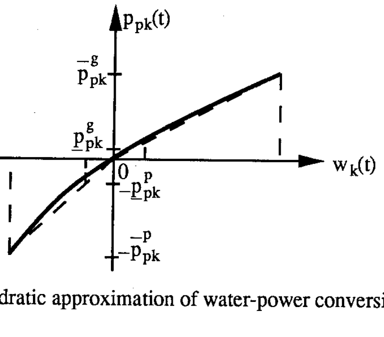


Fig. 4 Quadratic approximation of water-power conversion

Optimal generation or pumping level for each operating state

To obtain the optimal generation or pumping level associated with each state at each hour, pond level dynamics and constraints are first relaxed. From the pond level dynamics (2.4), the pond level can be determined based on the initial pond level, and water discharged or pumped as:

$$v_k(t) = V^o_k - \sum_{n=1}^{t} w_k(n). \quad (3.9)$$

By substituting the above equation into the pond level limits and terminal condition (2.5)-(2.6), the pond level dynamics and constraints can be rewritten as

$$V^o_k - \bar{V}_k \leq \sum_{n=1}^{t} w_k(n) \leq V^o_k, \quad t = 1, \ldots, T-1, \quad (3.10)$$

and

$$\sum_{n=1}^{T} w_k(n) = V^o_k - V^T_k. \quad (3.11)$$

By using additional sets of multipliers β_k, γ_k and ζ_k to relax (3.10) and (3.11), the cost function in (3.6) becomes:

$$\hat{L}'_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k) = \hat{L}_{pk}(\lambda, \mu) + \sum_{t=1}^{T-1} \{ \beta_k(t) [\sum_{n=1}^{t} w_k(n) - V^o_k] + \gamma_k(t) [V^o_k - \bar{V}_k - \sum_{n=1}^{t} w_k(n)] \} + \zeta_k [\sum_{n=1}^{T} w_k(n) - (V^o_k - V^T_k)]. \quad (3.12)$$

Define the stage-wise function $h_k(w_k(t))$ as

$$h_k(w_k(t)) \equiv -\lambda(t) \hat{p}_{pk}(w_k(t)) - \mu(t) r_{pk}(\hat{p}_{pk}(w_k(t))) + [\sum_{n=t}^{T-1} (\beta_k(n) - \gamma_k(n)) + \zeta_k] w_k(t), \quad t = 1, \ldots, T-1, \quad (3.13)$$

and

$$h_k(w_k(T)) \equiv -\lambda(T) \hat{p}_{pk}(w_k(T)) - \mu(T) r_{pk}(\hat{p}_{pk}(w_k(T))) + \zeta_k w_k(T), \quad (3.14)$$

where β_k and γ_k are stack vectors like λ and μ. Regrouping terms in (3.12) according to hours and using the above defined stage-wise function $h_k(w_k(t))$, one can rewrite subproblem $(\hat{P}p\text{-}k)$ as:

$$\min_{w_k(t)} \hat{L}'_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k), \ \ with \ \hat{L}'_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k) \equiv \sum_{t=1}^{T} h_k(w_k(t)) + \sum_{t=1}^{T-1} \{ \gamma_k(t) [V^o_k - \bar{V}_k] - \beta_k(t) V^o_k \} - \zeta_k [V^o_k - V^T_k], \quad (3.15)$$

subject to (2.8)-(2.12) and operating state dynamics. The optimal water discharged or pumped for a particular state can then be obtained by

$$w^*_k(t) = arg \min_{w_k(t)} h_k(w_k(t)), \quad (3.16)$$

subject to the range of this operating state. This optimization process is shown in Fig. 5, where $h_k(w_k(t))$ and the ranges of all operating states are depicted. It is can seen that the optimal generation or pumping for each operating state can be obtained either at the minimum point of a quadratic function, a corner point or at the boundary of its range.

Optimize generation or pumping across hours

After the optimal generation or pumping level for each operating state is obtained, the stage-wise cost $h_k(w_k(t))$ for that state can be calculated. Based on the well structured state transition diagram of Fig. 1,

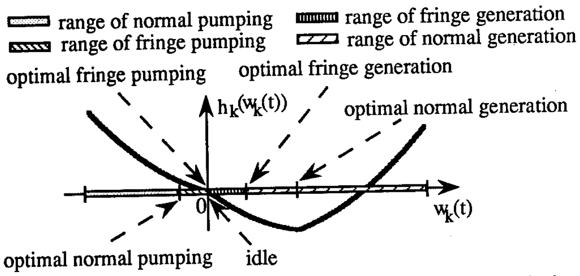


Fig. 5 Stage-wise cost function and optimal generation or pumping

dynamic programming can be used to optimize states across hours. The generation or pumping level over the entire time horizon can thus be efficiently obtained without discretizing pond levels.

Optimize the multiplier for pond levels

An intermediate level is created to update the multipliers associated with pond level dynamics and constraints. Let $\hat{L}'^{*}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k)$ denote the optimal Lagrangian for (3.15). The multipliers $\beta_k(t)$, $\gamma_k(t)$ and ζ_k are updated at the intermediate level by a subgradient algorithm to maximize the Lagrangian, i.e.,

$$\max_{\beta_k(t), \gamma_k(t), \zeta_k} \hat{L}'^{*}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k) \quad (3.17)$$

The subgradient algorithm to update λ, μ, β_k, γ_k, ζ_k is presented in subsection 3.5.

3.4 Solving Thermal and Hydro Subproblems

With λ and μ given, the optimal generation level of thermal subproblem (Pt-i) at hour t for an up state is obtained by minimizing a single variable stage-wise cost function. Based on this, a state transition diagram for (Pt-i) can be constructed with a small number of states and well structured state transitions. Dynamic programming can then be applied to obtain the optimal commitment and generation without discretizing generation levels. Detailed method is given in [7].

With λ and μ given , the optimal generation level for hydro subproblem (Ph-j) can be obtained by using a merit order allocation method. The total available hydro energy is allocated to individual hours at the maximum generation level $\bar{p}_{hj}(t)$ according to the descending order of $(\lambda(t) - \mu(t))$ until all the energy is allocated. For details please see [11].

3.5 Solving the Dual Problem

The high level dual problem is to update the multipliers λ and μ associated with demand and reserve requirements so as to maximize the dual function (3.7). Since discrete decision variables are involved at the low level, the objective function $\Phi(\lambda, \mu)$ in (3.7) may not be differentiable at certain points. Therefore, a subgradient algorithm ([7], [8]) is used to update λ and μ as follows:

$$\lambda^{l+1}(t) = \max\,[0, \lambda^l(t) + \alpha^l g_\lambda(t)], \quad (3.18)$$

$$\mu^{l+1}(t) = \max\,[0, \mu^l(t) + \alpha^l g_\mu(t)], \quad (3.19)$$

where l is the high level iteration index, α is the step size,

$$g_\lambda(t) \equiv P_d(t) - \sum_{i=1}^{I} p_{ti}(t) - \sum_{j=1}^{J} p_{hj}(t) - \sum_{k=1}^{K} p_{pk}(w_k(t)) \quad (3.20)$$

is the subgradient of $\Phi(\lambda, \mu)$ with respect to $\lambda(t)$, and

$$g_\mu(t) \equiv P_d(t) - \sum_{i=1}^{I} r_{ti}(p_{ti}(t)) - \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) - \sum_{k=1}^{K} r_{pk}(p_{pk}(w_k(t))), \quad (3.21)$$

the subgradient of $\Phi(\lambda, \mu)$ with respect to $\mu(t)$. As is explained in [7], the multiplier $\lambda(t)$ is positive since it is the marginal cost for demand at hour t.

The adaptive step sizing method of [8] is modified to obtain the step size α at iteration l. The high level iterations terminate when the dual cost L cannot be improved, or a preset number of high level iterations has been reached. The same subgradient algorithm is also used to update β_k, γ_k and ζ_k for pumped-storage subproblems. The subgradients for these three sets of multipliers are

$$g_{\beta_k(t)} \equiv V_k^o - \bar{V}_k - \sum_{n=1}^{t} w_k(n), \quad (3.22)$$

$$g_{\gamma_k(t)} \equiv \sum_{n=1}^{t} w_k(n) - V_k^o, \text{ and} \quad (3.23)$$

$$g_{\zeta_k} \equiv \sum_{n=1}^{T} w_k(n) - (V_k^o - V_k^T), \quad (3.24)$$

respectively. It should be emphasized that these multipliers are needed for pumped-storage subproblems only, and are updated at the intermediate level.

3.6 Obtaining Feasible Solutions

The dual solution obtained from Lagrangian relaxation is generally associated with an infeasible schedule, i.e., the once relaxed demand and reserve constraints, and the pond level dynamics and constraints are generally not satisfied ([5]). A heuristic method is developed to obtain a good feasible solution based on the dual results. The generation levels of pumped-storage, hydro, and thermal units with ramp rate are difficult to adjust in the heuristics since these units have constraints coupling their generation across hours such as pond level, total energy and ramp rate. The schedules of these units are therefore first modified to satisfy their individual constraints and then remain fixed. The generation levels of the thermal units without ramp rate constraint are adjusted to meet the system demand and reserve requirements according to the method presented in [7]. The method for obtaining a feasible solution for a pumped-storage unit is presented next.

The pond level of a pumped-storage unit is checked starting from hour 1 and working forward in time. If the pond level is less than zero at hour t_1, the total amount of the "over-used" water is divided into a number of small quanta. Pumping is increased and/or generation is decreased by a quantum at selected hours with small marginal costs $\lambda(t) - \mu(t)$ before t_1. However, if decreasing generation or increasing pumping at an hour would cause the pond level upper bound $\bar{V}_k$ to be violated before t_1, this hour will not be selected. In this way, curing "over-generation" will not result in "over-pumping." Similar procedure is performed if the pond level is greater than $\bar{V}_k$. The above procedure then repeats starting from $t_1 + 1$ until the pond levels for all hours satisfy the pond level constraints. Note that the original piece-wise linear water-power conversion of Fig. 2 is used to calculate generation or pumping level $p_{pk}(t)$ from water volume $w_k(t)$ in the heuristic.

3.7 Constructing a Lower Bound

Since the water-power conversion has been approximated by a quadratic function in the optimization process, the original problem has been changed. The dual cost, therefore, is not a lower bound to the optimal cost. To obtain a lower bound, the pumped-storage subproblems are re-solved based on the original piecewise linear water-power conversion function. The resulting duality gap, defined as the relative difference between the cost of the feasible schedule and the lower bound thus obtained, can be used to evaluate the quality of the feasible schedule.

4. Implementation and Testing Results

The algorithm is implemented in FORTRAN on a Sun Sparc Station 2. All the billing rules of NEPOOL are complied and many practical considerations are included. There are about 70 thermal units and dispatchable contracts, 7 hydro units and 1 large pumped-storage unit. This pumped-storage unit can generate about 20% of the peak load, and provide 100% of reserve requirements.

Numerical testing was performed using six NU billing data sets in 1991: February Week 2, February Week 3, March Week 3, April Week 1, April Week 2 and October Week 2. They are randomly selected from NU billing data files by NU engineers without any modification. Cogenerators and non-dispatchable contracts are deducted from system demand and reserve requirements. The scheduling horizon is either 7 days (168 hours) or 8 days (192 hours), with hour 1 being 8 AM, Monday morning. The system characteristics and parameters associated with the data sets tested are summarized in Table 1.

Table 1. Summary of NU system characteristics

System characteristics	Number of units	Total capacity or requirement (MW)
Steam units	14 - 22	1230 - 1565
Turbine units	24 - 34	274 - 473
Nuclear units	8	1062 - 2024
Hydro units	7	247 - 254
Pumped-storage unit (generation/pumping)	1	565 / 544 - 640 / 616
Dispachable contracts	0 - 2	0 - 300
All units	56 - 74	3534 - 4723
Peak load		3376 - 3934
Minimum load		1357 - 1887
Maximum reserve		152 - 189

Numerical results are summarized in Table 2. The number of high level iterations represents how many times high level multipliers λ and μ are updated for convergence. It can be seen that the number of high level iterations, though varying across data sets, is not sensitive to variations in system size and time horizon. Based on our testing experience, the computational time increases about linearly as the number of units or scheduling horizon increases. Note that the CPU times for all data sets are only a few minutes on a Sun Sparc Station 2, the algorithm is therefore efficient to be used on a daily basis.

Table 2. Numerical testing results based on NU data

	Feb. W2, 1991	Feb. W3, 1991
Time Horizon (hour)	168	192
Number of high level iterations	50	44
CPU (sec)	408	449
Best feasible cost ($)	4,617,464	7,883,044
Max . dual cost ($)	4,596,055	7,831,425
Duality gap (%)	0.47	0.66

	Mar. W3, 1991	Apr. W1, 1991
Time Horizon (hour)	168	168
Number of high level iterations	43	30
CPU (sec)	394	302
Best feasible cost ($)	6,067,742	4,970,567
Max . dual cost ($)	6,027,903	4,938,250
Duality gap (%)	0.66	0.65

	Apr W2, 1991	Oct. W2, 1991
Time Horizon (hour)	168	192
Number of high level iterations	29	56
CPU (sec)	298	435
Best feasible cost ($)	5,479,120	5,240,164
Max . dual cost ($)	5,472,981	5,225,502
Duality gap (%)	0.11	0.28

The duality gaps of all data sets (and others tested but not reported here) are below 0.7%. The results thus demonstrate consistent convergence of the algorithm, and support our claim that near optimal solutions are obtained. These gaps are slightly higher than those reported in [7] and [11], as might be caused by the additional relaxation of pond level constraints.

The pumped-storage unit usually generates during day time with high demand and pumps at nights with low demand so as to smooth the peaks and fill the valleys. The original demand, the demand with pumped-storage unit generation deducted and pumping added, and pond level for the February Week 2 data set are shown in Fig. 6. The role of the pumped-storage unit in smoothing peaks and filling valleys is obvious. It is also observed that the pond is gradually depleted during the weekdays and filled up on the weekend since demand and average marginal cost in the weekdays are generally higher than those on the weekend. Another benefit of pumping at night is to provide reserve so that no extra thermal units have to be committed just for providing reserve. The reserve requirement and contribution of the pumped-storage unit for the February Week 2 data set are plotted in Fig. 7. It can be seen that the reserve requirement is almost covered by the pumped-storage unit alone for entire time horizon.

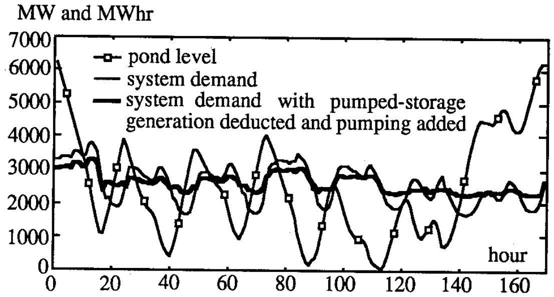


Fig. 6 Pond level and system demand for Feburary Week 2, 1991

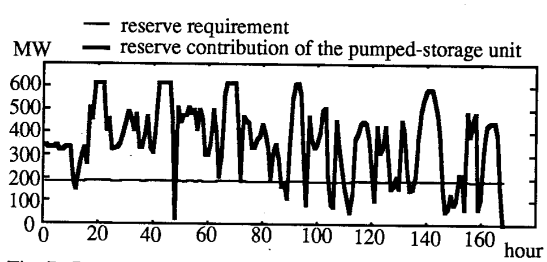


Fig. 7 Reserve requirement and pumped-storage contribution for Feburary Week 2, 1991

Our near optimal schedule (S_{opt}) for February Week 3 data set is compared with the NU heuristic schedule (S_{heu}) obtained by NU engineers with the aid of a priority-list dispatch package. The cost of S_{opt} is 26,607 dollars lower than that of S_{heu}. The hourly marginal costs and pond levels for two schedules are depicted in Fig. 8. In this week, a nuclear unit is scheduled out on the weekend so that the average marginal cost on the weekend is not lower than that in the weekdays as can be seen in Fig. 8. Therefore, a good pumped-storage schedule may not conform with the usual pattern of depleting pond in weekdays and filling it up on the weekend. It can be seen from Fig. 8 that during the weekdays, much less hydro energy in the pond is used for generation in S_{opt} than in S_{heu}. As a result, the near optimal schedule is overall more economic.

5. Conclusions

An optimization-based method for scheduling hydrothermal power systems with pumped storage units has been presented. Relaxing pond level dynamics and constraints and using dynamic programming without discretizing pond levels have proved to be an efficient and effective

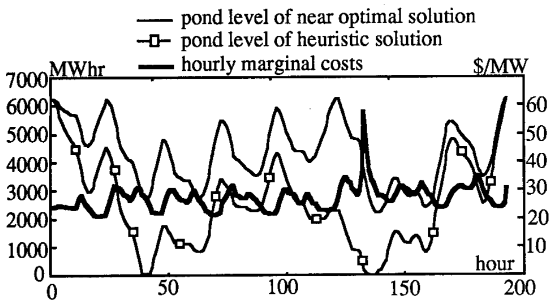


Fig. 8 Pond level comparison of two solutions

method to solve the pumped-storage subproblems involving continuous dynamics and constraints, discontinuous operating regions, and discrete operating states. Testing results based on Northeast Utilities power system show that this algorithm is efficient, and near optimal schedules are obtained. The results also indicate the importance of the pumped-storage unit in smoothing peaks and filling valleys of system demand and providing reserve. This algorithm has been embedded in the scheduling package of NU, and used by NU engineers on a daily basis.

Acknowledgement

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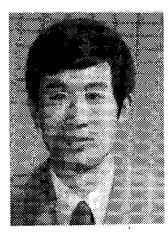

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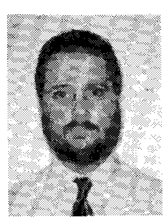

Peter M. Rogan was born in New Britain, CT on April 22, 1961. He graduated Magna Cum Laude from the Loomis-Chaffee School in 1979 and received his A.S. degree in Electrical Engineering from Hartford State Technical College in 1982.

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Discussion

L.A.F.M. Ferreira, Instituto Superior Técnico, 1096 Lisbon, Portugal: I congratulate the authors for an interesting paper on the scheduling of pumped storage units. In this discussion, I would like to offer the following comments:

A. On the relationship of this paper with a closely-related, non-referenced paper [D1]

The problem of short-term scheduling of a pumped storage plant in which both reservoir constraints and plant dynamics (including "discontinuous operating regions and discrete operating states") are considered has been addressed in detail in [D1]. The method proposed in [D1] is basically the same as the one proposed here by the authors: dynamic programming (based on a state transition diagram for the plant) and relaxation of the reservoir constraints by using Lagrange multipliers.

For illustration purposes, Fig 1B is included in this discussion. Notice that the problem is somewhat more complex as there are three units in the pumped storage plant (as opposed to one-unit plant in the present paper), there are requirements on minimum up-time and state transition costs.

In this paper the authors propose a subgradient approach to update the Lagrange multipliers corresponding to the reservoir constraints. However, we have found that approach time-consuming and sometimes unreliable. Hence, upon discussing the advantages and disadvantages of that approach, the following was proposed: extract the value of the multipliers from the dual solution of a network flow program and then proceed to the adjustment of the water values to meet the reservoir constraints. Adjustment techniques and numerical results are presented in [D1].

B. On the test results presented

B1. The authors present results corresponding to a real system. However, I find the results surprising: the pumped storage unit is always committed. It is committed as a generator in the peak hours, and as a pump in the other hours.

To see this consider for example Fig. 6. Notice that the curve corresponding to "system demand" and the curve corresponding to "system demand with pumped-storage generation deducted and pumping added" intersect each other but there are no time intervals for which they are coincident. The decommitted state (off) would correspond to a time interval with coincident curves.

Our experience is that the commitment decision is a difficult decision to make and that pumped storage units are committed only a few hours a day (and in some days they are off all day long). A reason for this *dead band* is well known: only about 70% of the energy used in the pump mode can be recovered in the generation mode.

B2. In the problem formulation, the authors introduce the ideas of "fringe pumping", "fringe generation", and "fringe hours". However, none of these ideas are mentioned in the test results. An illustration would be welcome.

I will appreciate the authors' response to the foregoing comments.

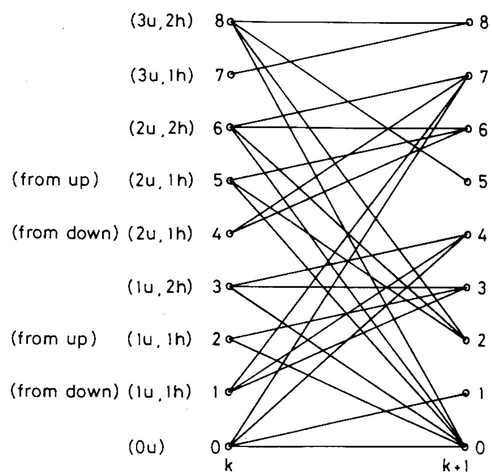

Fig. 1B *State transition function for a 3-unit pumped storage plant considering minimum up time and state transition costs*

*i*u, there are *i* units committed
1h, unit has been in this state for first hour
2h, unit has been in this state for two or more hours
'from down', at least a unit startup is involved
'from up', at least a unit shutdown is involved
State 0, no units committed
State 1 corresponds to one ('1u') unit committed, for the first hour ('1h') and whose previous state was 0 ('from down')
State 2 corresponds to one unit committed, for the first hour and whose previous state was 5, 6, or 8 ('from up')
State 3 corresponds to one unit committed, for the second hour (2h)
Similar interpretation applies to the remaining states

[D1] L.A.F.M. Ferreira, "Short-term scheduling of a pumped storage plant", *IEE Proceedings, Part C*, Vol. 139, No. 6, pp. 521-528, Nov. 1992.

Manuscript received August 19, 1993.

X. Guan, P.B. Luh, H. Yan (Department of Electrical and Systems Engineering, University of Connecticut, Storrs, CT 06269-3157, USA), and P. Rogan (Northeast Utilities Service Company, Berlin, CT 06037-1616, USA): The authors appreciate discusser's valuable comments. The response to these comments is as follows.

1 Response to comment A

The authors congratulate the discusser on the valuable work reported in D1 dated November 1992. The discusser's paper D1 should have been cited if it were published and available before the current paper was submitted and revised. It should be mentioned that solving pumped-storage subproblems by relaxing pond level constraints has been presented by the authors in R1 dated June 1992, although the discrete operating states and "fringe hours" were not considered at that time.

The authors agree that the coupling of cost functions among units considered in D1 is a complicated issue. For the problem considered in this paper, there is no coupling of cost functions among units, since the operating costs of pumped-storage units are negligible comparing with the generation costs of thermal units. Only shadow cost terms appear in the cost function of a decomposed pumped-storage subproblem. After the pond level constraints are relaxed at the intermediate level, the optimal generation or pumping level for each state at each hour can be obtained by optimizing a single variable function, as reported in the paper. The optimal operating states across hours are then efficiently obtained by using dynamic programming. Based on our testing experience, the subgradient method is efficient at the intermediate level. It usually takes 5 to 8 iterations to converge, the computational time for updating pond level multipliers is trivial, and the overall performance of the algorithm has been very good. Therefore, there is no need to employ network flow algorithms which may still generate infeasible schedules. It should be noted that although there is only one pumped- storage unit in the NU power system, the algorithm can be used for systems with multiple pumped-storage units.

2 Response to comment B1

The authors appreciate the discusser's comment on our numerical results. From our experience, the optimal or near optimal schedule of pumped-storage units depends heavily on the differences between on-peak and off-peak loads, system characteristics and pumped storage efficiency (74% for NU's pumped-storage unit). The pumped-storage unit in NU's system can contribute 100% of reserve requirements when the unit either generates or pumps. Since it is very expensive for thermal units to provide the reserve in NU's system, the pumped-storage unit may therefore generate or pump to provide the reserve between on-peak and off-peak periods. For the February Week 2 data set shown in Fig. 6, there are usually one or two idle hours (uncommitted hours) between on-peak and off-peak periods during weekday, and a few more idle hours during the weekend.

3 Response to comment B2

The authors appreciate and agree with the comment that fringe generation and pumping should be mentioned in numerical testing. The fringe generation and pumping provide an opportunity to reduce total generation cost since the minimum generation and pumping constraints are not required at fringe hours. Based on our testing experience, the unit is often scheduled for fringe generation or pumping. Cost savings are indeed observed in comparison with the case where no fringe hours are allowed.

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Manuscript received October 8, 1993.