Optimization-Based Inter-Utility Power Purchases

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ABSTRACT

Power transactions are important activities among electric utility companies. Effective transaction decisions can result in significant savings since marginal generation costs of neighboring utilities could be quite different. Transactions, however, are coupled with the scheduling of units through system demand and reserve requirements, and are confined by many accustomed rules. How to make effective transaction decisions is therefore a very difficult problem. In this paper, an optimization-based method for the integrated consideration of power purchase transactions and the scheduling of thermal units is presented based on the augmented Lagrangian decomposition and coordination method. After the system-wide demand and reserve requirements are relaxed by using Lagrange multipliers and penalty coefficients, the overall problem is decomposed into purchase and thermal subproblems. For a purchase subproblem, the optimal purchase level for each purchase interval is first determined. The subproblem is then efficiently solved by using the dynamic programming approach without discretizing purchase levels. Thermal subproblems are solved by extending our previous method recently reported in the literature. The multipliers and penalty coefficients are then updated at the high level so that system demand and reserve requirements are gradually satisfied over iterations. The augmented Lagrangian decomposition and coordination method avoids the solution oscillation difficulties associated with linear cost functions of purchase subproblems and speeds up algorithm convergence. Numerical testing results based on modified Northeast Utility's data show that the algorithm is efficient, and significant savings can be obtained.

Key words: Inter-utility power transactions, Power purchases, Power system scheduling, Augmented Lagrangian relaxation.

1. Introduction

Inter-utility power transactions are common and important practices among electricity utility companies. Since utilities have various generators with different characteristics and must meet their time varying demand and reserve requirements, they usually have different marginal generation costs. It is often mutually beneficial for neighboring utilities to purchase or sell power if their marginal costs are substantially different. Proper transaction decisions can result in significant savings, and it is very important to make effective decisions to maximize the savings from power transactions.

A transaction can be triggered either by a sale utility or by a purchase utility. Usually the sale utility makes an offer including prices, available transaction periods and maximum purchase levels. The two utilities then negotiate and reach agreeable purchase intervals within available periods and the purchase levels restricted by the maximum. Decisions for both utilities should consider the savings obtained from the transaction and the scheduling of their own units, since the commitment and dispatch of units are directly affected by the transaction through system demand and reserve requirements. Decisions also have to follow accustomed rules based on standard load patterns and generator characteristics such as minimum up and down time. These rules result in time-varying constraints on power transaction intervals and levels. Since the scheduling problem alone is "NP hard," i.e., the computational requirements for obtaining an optimal solution grows exponentially with problem size, the resolution of this integrated power transaction and scheduling problem is very difficult. Furthermore, real-time response (e.g., within 15 minutes) is usually required for this kind of short-term transaction decisions.

A few results on inter-utility transactions have been reported in the literature. A limited power purchase problem is considered in [1], where the total amount of energy purchased within a period is given. The problem is to allocate this energy among the hours. In [2], energy and reserve transactions are considered for security reasons. Unfortunately, little work has been found to address the entirety of transaction problems for determining the most cost effective intervals, levels and prices while conforming with accustomed rules.

This paper concentrates on purchase transactions, and is based on transaction activities of Northeast Utilities Service Company (NU). Although a system usually consists of thermal, hydro and pumped-storage units, only thermal units and purchase transactions are considered at this stage of development. Furthermore, purchase is restricted to power only, and the purchase of reserve is not considered. Hydro and pumped-storage units as well as sale transactions will be incorporated in the future.

Since prices, available transaction periods and maximum purchase levels in a purchase transaction are usually set by the selling utility, the decision variables are restricted to be the intervals and levels of power transactions. The current practice imposes "minimum purchase time" constraint, i.e., a purchase should continue at least for a certain period of time, and the "purchase level" constraint, i.e., the purchase level should be the same within an on-peak load period or an off-peak load period. For different load periods, the minimum purchase times and purchase levels could be different. Equivalent to maximizing savings obtained from transactions, the objective is to minimize the total generation and purchase costs. This minimization is also subject to the system-wide requirements that the sum of generation and purchase should satisfy the overall system demand and reserve. The mathematical problem formulation is presented in Section 2.

One of the potential approaches for solving the problem is Lagrangian relaxation. The basic idea of Lagrangian relaxation is to relax the system-wide demand and reserve requirements by using Lagrange multipliers. The "relaxed" problem is then decomposed and converted into a two-level optimization problem. This technique, however, may result in solution oscillation since the cost functions of purchase subproblems are linear. The purchase levels will oscillate between minimum and maximum values with a slight change of multipliers. The convergence of the overall algorithm is poor, and effective purchase decisions may not be obtainable.

In this paper, the augmented Lagrangian decomposition and coordination method of [3] and [4] is used to solve the integrated purchase and thermal scheduling problem. In this method, quadratic penalty terms associated with system demand requirements are added to the Lagrangian. The cost function of a purchase subproblem becomes quadratic, and the optimal purchase level for each purchase interval can be obtained by optimizing a single variable quadratic function. Dynamic programming with a few states and well structured transitions at each hour can then be used to solve the subproblem efficiently without discretizing purchase levels. The thermal subproblems are solved by extending our previous work ([5]). The multipliers and penalty coefficients are updated at the high level until a fixed number of iterations has been reached or until the

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convergence of the algorithm. The heuristics described in [5] are then used to modify the solution into a feasible one. The augmented Lagrangian decomposition and coordination method avoids the solution oscillation difficulties, speeds up algorithm convergence, while maintaining decomposibility of the augmented Lagrangian. The solution methodology is presented in Section 3.

The purpose of the research is to develop a power purchase decision support system for NU. The maximum time horizon is ten days (240 hours), with seven days as the normal planning cycle. The contributions of hydro and pumped-storage units are obtained by using a heuristic method, and then remain fixed in the algorithm. Testing results presented in Section 4 are based on NU data sets with transaction data modified to test out various conditions. The results show that the significant savings can be obtained, and the algorithm is efficient to support real-time transaction decisions.

2. Problem Formulation

The objective of the integrated purchase and scheduling problem is to minimize the total cost, i.e., thermal generation costs plus purchase costs, subject to system-wide demand and reserve requirements and individual unit and transaction constraints. According to the current practice, a day is divided into four load periods: one on-peak, one off-peak and two shoulder periods as depicted in Figure 1. For convenience of presentation, an off peak period of a day slightly runs into the previous day.

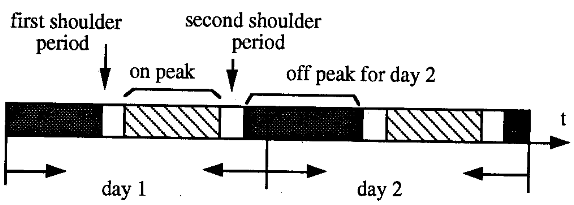


Figure 1. Load periods

Consider a system with M purchase transactions and I thermal units. It is required to determine the intervals and levels of power purchases, and the start-up, shut-down, and generation levels of thermal units over a specified planning horizon T. The time unit is one hour and the planning horizon may vary from one week to ten days. To formulate the problem mathematically, the following notation is first introduced:

$c_i(p_i^t(t))$: fuel cost of thermal unit i for generating power $p_i^t(t)$ at time t, a piece-wise linear function of $p_i^t(t)$, in dollars;

$c_m^b(t)$ price for purchase transaction m at time t, in dollars per MW;

D: number of days in the time horizon;

d(t): day index for hour t, d = [t/24] + 1, and d = 1,...,D;

I: number of thermal units;

i : index of thermal units, i = 1, ..., I;

M: number of purchase transactions;

m: index of power purchase transactions, m = 1, ..., M;

$p_d(t)$: system demand at time t, in MW ;

$p_i^t(t)$: generation of thermal unit i at time t, in MW;

$p_m^b(t)$: power level of transaction m at time t, in MW;

$p_{m-off}^b(d,j)$: power level of transaction m during the jth off-peak purchase interval of day d, in MW, to be explained in subsection 3.2.2;

$p_{m-on}^b(d,j)$: power level of transaction m during the jth on-peak purchase interval of day d, in MW;

$p_{m-max}^b(t)$: maximum power level offered by the selling utility in transaction m at time t, in MW;

$p_r(t)$: system reserve requirement at time t, in MW ;

$r_i^t(p_i^t(t))$: reserve contribution of thermal unit i at time t;

$s_i(t)$: start-up cost of thermal unit i, a linear function of time since last shut down, in dollars;

T : time horizon, in hours;

t : hour index, t = 1, ..., T;

$t_d(t)$: hour within day d corresponding to hour index t, t_d = t mod (24) with $1 \le t_d \le 24$;

$T_{off}(d,j)$: the jth off-peak purchase interval of day d;

$T_{on}(d,j)$: the jth on-peak purchase interval of day d;

$\tau_m^b(t_d)$: the minimum purchase time of purchase transaction m with τ_{off} for an off-peak period and τ_{on} for an on-peak period.

As stated earlier, the objective of the integrated purchase and scheduling problem is to minimize the total cost, i.e., thermal generation cost plus purchase cost:

Objective function:

$$J=\sum_{t=1}^{T}\left\{\sum_{i=1}^{I}\left[c_i(p_i^t(t))+s_i(t)\right]+\sum_{m=1}^{M}c_m^b(t)p_m^b(t)\right\}. \qquad (2.1)$$

The decision variables are transaction intervals and levels ($p_m^b(t)$), and power generated by thermal units ($p_i^t(t)$). This minimization is subject to the following system-wide demand and reserve requirements:

System-wide constraints:

-- system demand:

$$\sum_{i=1}^{I}p_i^t(t)+\sum_{m=1}^{M}p_m^b(t)=p_d(t); \qquad (2.2)$$

-- system reserve:

$$\sum_{i=1}^{I}r_i^t(x_i(t),p_i^t(t)) \ge p_r(t). \qquad (2.3)$$

Individual thermal constraints include capacity, minimum up/down time and ramp rate, as detailed in [5]. Power purchase constraints are described below.

Power Purchase Constraints:

As mentioned earlier, the on-peak and off-peak load pattern of a day can be clearly recognized. In the current practice, transaction decisions for a load period are usually made based on the average marginal cost of the period. The prices and maximum purchase levels offered are constant over a load period. A minimum purchase time per load period is required since thermal units with "minimum up time" constraints are generally used to provide the power. The minimum purchase time may be different for off-peak and on-peak periods. Allowable purchase patterns include an off-peak period with one purchase level, or an on-peak period with one purchase level. Shoulder periods can be connected with either an on-peak period or an off-peak period or both, but they generally cannot stand alone. If power is purchased for consecutive on-peak and off-peak periods, power should also be purchased for the shoulder period in between, and no gap is allowed. Three allowable purchase patterns in a day are shown in Figure 2, and a purchase transaction for a day should comply with these patterns. A purchase transaction across two consecutive days should also comply with similar patterns.

Any interval in a load period complying with the above purchase patterns is called a "purchase interval." There could be many purchase intervals with different starting and ending hours associated with one load period. The jth off-peak (or on-peak) purchase intervals of day d is indexed by (d, j). Purchase levels within a purchase interval should be the same, as described by the following constraints:

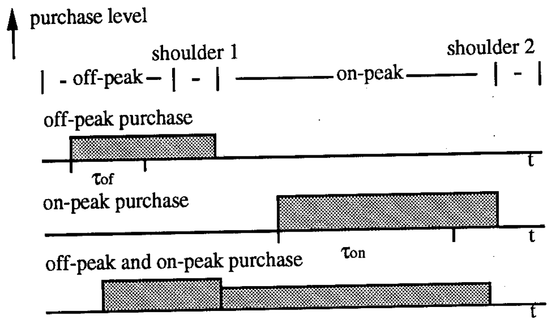


Figure 2. Allowable purchase patterns

if $p^b_m(t) > 0$, then

$$p^b_m(t) = p^b_{m-off}(d,j), \qquad t_d \in T_{off}(d,j); \tag{2.4}$$

$$p^b_m(t) = p^b_{m-on}(d,j), \qquad t_d \in T_{on}(d,j); \tag{2.5}$$

where $p^b_m(t)$ is the power level of transaction m at time t, $p^b_{m-off}(d,j)$ and $p^b_{m-on}(d,j)$ are, respectively, the power level of transaction m across the jth off-peak or on-peak purchase intervals of day d. The purchase level is also constrained by the maximum level offered by the selling utility as described below:

$$0 \le p^b_m(t) \le p^b_{m-max}(t) \tag{2.6}$$

3. Solution Methodology

3.1 The Augmented Lagrangian Relaxation Approach

From the problem formulation (2.1), (2.2) and (2.3), it can be seen that individual power transactions and generating units are independent, but together they must meet the system demand and reserve requirements. The cost function to be minimized is also additive with respect to individual transactions and units. The problem is therefore ideal for the Lagrangian relaxation approach that exploits the decomposability of the problem. In the standard Lagrangian approach, however, a decomposed purchase subproblem has a linear cost function with coefficients determined by multipliers. The optimal solution of a subproblem is therefore obtained only at boundary points: maximum or minimum purchase values (the minimun value is assumed to be zero). The solution may oscillate between these maximum and minimum values with slight changes of multipliers. The convergence of the overall algorithm is poor, and effective purchase decisions may not be obtainable. This difficulty can be overcome by appending quadratic penalty functions associated with system demand requirements to the Lagrangian, and this is known as the augmented Lagrangian approach. The quadratic penalty terms can also improve algorithm convergence ([6]).

To maintain the decomposibility of the augmented Lagrangian, the "auxiliary problem technique" is applied based on the augmented Lagrangian decomposition and coordination method of [3] and [4]. The non-separable quadratic penalty terms are replaced by their linear approximations around the solution obtained from the previous iteration, taking advantage of the iterative nature of the solution process. An auxiliary function which is strongly convex, differentiable and separable is also introduced to the cost function to improve the convergence property.

The augmented Lagrangian for our problem is given by:

$$\begin{aligned} L \equiv \sum_{t=1}^{T} \Bigg[& \sum_{i=1}^{I} \left[c_i(p^t_i(t)) + s_i(t)\right] + \sum_{m=1}^{M} c^b_m(t) p^b_m(t) \\ & + \lambda(t)\{p_d(t) - [\sum_{i=1}^{I} p^t_i(t) + \sum_{m=1}^{M} p^b_m(t)]\} \\ & + \mu(t)\{p_r(t) - \sum_{i=1}^{I} r^t_i(x_i(t), p^t_i(t))\} \\ & + \frac{c}{2}\{p_d(t) - [\sum_{i=1}^{I} p^t_i(t) + \sum_{m=1}^{M} p^b_m(t)]\}^2 \Bigg], \end{aligned} \tag{3.1}$$

where $\lambda(t)$ and $\mu(t)$ are multipliers associated with demand and reserve requirements at time t, respectively, and c is a positive penalty coefficient. The decomposition and coordination algorithm is formed by linearizing the last term of L and adding quadratic terms of decision variables as auxiliary functions. The objective function of the k+1th iteration is then given by:

$$\begin{aligned} \sum_{t=1}^{T} \Bigg[& \sum_{i=1}^{I} \left[c_i(p^t_i(t)) + s_i(t)\right] + \sum_{m=1}^{M} c^b_m(t) p^b_m(t) \\ & + \lambda(t)\{p_d(t) - [\sum_{i=1}^{I} p^t_i(t) + \sum_{m=1}^{M} p^b_m(t)]\} \\ & + \mu(t)\{p_r(t) - \sum_{i=1}^{I} r^t_i(x_i(t), p^t_i(t))\} \\ & + \frac{1}{2\varepsilon}[\sum_{i=1}^{I}[p^t_i(t) - \bar{p}^t_i(t)]^2 + \sum_{m=1}^{M}[p^b_m(t) - \bar{p}^b_m(t)]^2] \\ & + c[p_d(t) - \sum_{i=1}^{I} \bar{p}^t_i(t) - \sum_{m=1}^{M} \bar{p}^b_m(t)][p_d(t) - \sum_{i=1}^{I} p^t_i(t) - \sum_{m=1}^{M} p^b_m(t)] \Bigg], \end{aligned} \tag{3.2}$$

where $\bar{p}^t_i(t)$ and $\bar{p}^b_m(t)$ are the results obtained from the previous iteration, and ε is a positive number.

A two-level max-mini optimization problem can then be formed. At the low level, the subproblems consist of individual purchase and thermal subproblems with given multipliers and penalty coefficients:

Power Purchase Subproblems (P-m), m = 1, ..., M:

$$\begin{aligned} \min_{p^b_m(t)} L^b_m \quad \text{with } L^b_m \equiv \sum_{t=1}^{T} \{ & c^b_m(t)p^b_m(t) - \lambda(t)p^b_m(t) \\ & + \frac{1}{2\varepsilon}[p^b_m(t) - \bar{p}^b_m(t)]^2 - c[p_d(t) - \sum_{i=1}^{I} \bar{p}^t_i(t) - \sum_{m=1}^{M} \bar{p}^b_m(t)]p^b_m(t) \}, \end{aligned} \tag{3.3}$$

subject to individual power purchase constraints (2.4)-(2.10).

Thermal Subproblems (P-i), i = 1, ..., I:

$$\begin{aligned} \min_{p^t_i(t)} L^t_i \quad \text{with } L^t_i \equiv \sum_{t=1}^{I} \{ & [c_i(p^t_i(t)) + s_i(t)] \\ & - [\lambda(t)p^t_i(t) + \mu(t)r^t_i(x_i(t), p^t_i(t))] \}, \end{aligned} \tag{3.4}$$

$$+\frac{1}{2\varepsilon}[\,p_i^t(t) - \bar{p}_i^t(t)\,]^2 - c\,[\,p_d(t) - \sum_{i=1}^{I}\bar{p}_i^t(t) - \sum_{m=1}^{M}\bar{p}_m^b(t)\,]\,p_i^t(t)\,\},$$

subject to individual thermal unit constraints.

The resolution of the subproblems are presented in subsections 3.2 and 3.3. The multipliers and penalty coefficients are then updated at the high level, as described in subsection 3.4. The solutions obtained form this two-level framework, called dual solutions, are usually infeasible, i.e., the once relaxed system-wide constraints are not satisfied. A heuristic method is needed to modify the dual solutions to obtain feasible results, as briefly described in subsection 3.4.

3.2 Solving Purchasing Subproblems

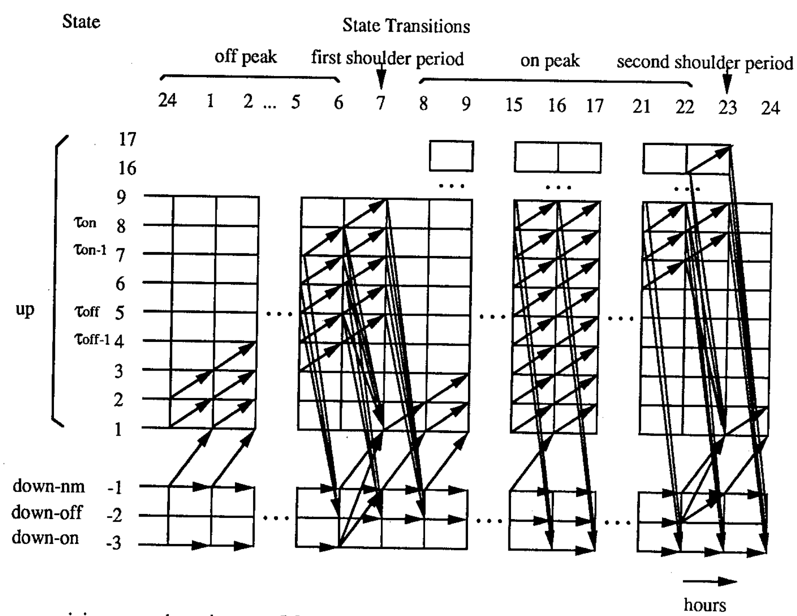
A purchase subproblem is to determine a transaction's optimal purchase intervals and levels with λ, c and ε given to minimize the cost of the transaction (eq.(3.3)), subject to the minimum purchase time and purchase level constraints. Although the cost is a quadratic function of $p_m^b(t)$ and is stage-wise additive, the purchase level at an individual hour cannot be obtained by simply minimizing the cost at that hour since the purchase level constraints (2.4) to (2.6) couple levels across hours. Discretizing levels and solving the subproblem by using dynamic programming may greatly increase the computation requirements.

The basic idea of our method is to find first the optimal purchase level for each purchase interval. Since the levels are the same across each purchase interval, this optimization can be efficiently done by minimizing a single variable quadratic cost function. A state transition diagram can be constructed where a state represents the number of hours that the purchase has been continued up to and include this hour. Each state, however, may be associated with several purchase intervals, and are therefore related to several purchase levels and costs. The optimal purchase intervals and levels for the entire planning horizon can nevertheless be obtained by using the dynamic programming approach. The states and state transition diagram are defined first.

3.2.1 State Transition Diagram

Similar to the concept of "up state" and "down state" of a thermal unit with minimum up/down time constraint as described in [5], the "up state" of a purchase transaction at a particular hour is defined as the number of hours that the purchase has been continued up to and include this hour. The purchase should be continued if the minimum purchase time within the load period has not been met, and it can be continued or discontinued after the minimum purchase time has been met. The down state, denoting that no power is purchased at this hour, is more complicated and includes three cases. If a purchase is discontinued during an off-peak period, the down state should be maintained until the next off-peak period comes according to the purchase pattern of Figure 2. This down state is defined as "down-off state." The "down-on state" is similarly defined for an on-peak period. Beyond these two down states, a "down-zero state" is defined as the case where no power has yet been purchased within this load period, but power can be purchased at next hour as long as there is enough time to satisfy the minimum purchase time constraint within the same off-peak or on-peak period.

The state transition diagram can then be constructed as in Figure 3 by combining the above analysis. In the figure, each node represents a state and each arrow indicates a possible state transition. When the purchase is continued, the number of hours being "up" is accumulated. The only exception is at the beginning of a shoulder period, where the number of hours being up should be reset to one since a minimum purchase time has to be satisfied for the next on-peak or off-peak period. A down-off state should be maintained except at the beginning of the following second shoulder period as explained above, and a down-on state should be maintained except at the beginning of the following first shoulder period. A down-zero state usually can go to an up state, but this down-zero state should be maintained if there is not enough time to satisfy the minimum purchase time within the on-peak or off-peak period (for example, from hour 16 to hour 21 in Figure 3). It can be seen that there may be several purchase intervals associated with an up state since the purchase can be continued or discontinued after the minimum purchase time requirement has been met. Since the optimal purchase levels for different purchase intervals may be different, the corresponding costs are therefore different. Purchase levels and costs for all the down state nodes are zero.



minimum purchase time τ_{off}=5 for off-peak minimum purchase time τ_{on}=8 for on-peak

Figure 3. State transition diagram

3.2.2 Optimal Purchase Level for a Purchase Interval

As mentioned earlier, the levels during an off-peak purchase interval should be kept the same. The decision variable $p_m^b(t)$ for hour t belonging to this interval is therefore a constant. By using $p_{m-off}^b(d,j)$ to denote the purchase level of the jth purchase interval of day d for transaction m, its optimal value can be obtained by minimizing a quadratic cost function.

Let $L_m'^b$ denote the cost of the jth purchase interval, and t_b and t_e denote the beginning hour and the ending hour of this interval, respectively. The cost function for this interval can be rewritten as:

$$L_m'^b(\lambda,\mu,c,p_{m-off}^b(d,j)) = \sum_{t_b}^{t_e}\{\ f(t)p_{m-off}^b(d,j) + \frac{1}{2\varepsilon}[\ p_{m-off}^b(d,j) - \bar{p}_m^b(t)\ ]^2\ \}, \quad (3.5)$$

where

$$f(t) = c_m^b(t) - \lambda(t) - c\left[p_d(t) - \sum_{i=1}^{I}\bar{p}_i^t(t) - \sum_{m=1}^{M}\bar{p}_m^b(t)\right]. \quad (3.6)$$

The optimal level $p_{m-off}^b(d,j)$ of purchase interval j of day d can then be obtained by minimizing the quadratic function (3.5). It is clear that this level is not necessarily zero or at the maximum purchase level as in the case with linear cost function. The optimal level for an on-peak purchase interval can be obtained similarly.

3.2.3 Dynamic Programming

After obtaining all purchase levels associated with an up state at hour t, the corresponding costs for this up state can be calculated based on these levels. The optimal intervals and levels for transaction m across the entire planning horizon can then be determined by using the dynamic programming approach.

3.3 Solving Thermal Subproblems

A thermal subproblem is to determine the generations of a thermal unit with λ, c and ε given so as to minimize the cost function (3.4). The problem is solved by following the method of [5] with the new cost function, where the old cost function is the first four terms of (3.4). It can be seen that (3.4) is piece-wise quadratic with respect to generation levels. The optimal generation for an up state at each hour can be obtained by minimizing a piece-wise quadratic cost function. The optimal commitment and generation of the thermal unit across the entire planning horizon can then be obtained by using dynamic programming without discretizing generation levels. For details please see [5].

Comparing with the standard Lagrangian relaxation approach, the generation obtained by using augmented Lagrangian is not necessarily at corner or boundary points as in [5].

3.4 Solving the Dual Problem

The high level dual problem is to update the multipliers λ and μ associated with demand and reserve requirements, penalty coefficient c so as to maximize the dual function as follows:

$$\max_{\lambda,\mu,c} \Phi(\lambda,\mu,c), \text{ with } \Phi(\lambda,\mu,c) \equiv \sum_{i=1}^{I} L_i^{t*}(\lambda,\mu,c) \quad (3.7)$$

$$+ \sum_{m=1}^{M} L_m^{b*}(\lambda,\mu,c) + \sum_{t=1}^{T}\ \{\ [\lambda(t)p_d(t) + \mu(t)p_r(t)] + c\ [\ p_d(t) - \sum_{i=1}^{I}\bar{p}_i^t(t) - \sum_{m=1}^{M}\bar{p}_m^b(t)\ ]p_d\ \},$$

where $L_i^{t*}(\lambda,\mu,c)$ and $L_m^{b*}(\lambda,\mu,c)$ denote, respectively, the optimal costs of subproblem (P-m) and (P-i) for the given multipliers and penalty coefficients. A subgradient method is adopted for updating λ and μ by the following formula ([7]):

$$\lambda^{k+1}(t) = \max[0, \lambda^k(t) + \rho^k g_\lambda(t)], \quad (3.8)$$

$$\mu^{k+1}(t) = \max[0, \mu^k(t) + \rho^k g_\mu(t)], \quad (3.9)$$

where k is the high level iteration index, and ρ^k is step size at the kth iteration. The subgradients of $\Phi(\lambda,\mu)$ with respect to $\lambda(t)$ and $\mu(t)$ are given by:

$$g_\lambda(t) = P_d(t) - \sum_{i=1}^{I} p_i^t(t) - \sum_{m=1}^{M} p_m^b(t), \quad (3.10)$$

$$g_\mu(t) = P_r(t) - \sum_{i=1}^{I} r_i^t(x_i(t), p_i^t(t)). \quad (3.11)$$

Penalty coefficients c and ε are selected and then remain fixed over iterations based on testing experience.

The solution to the dual problem is generally associated with an infeasible schedule, i.e., the demand and reserve constraints are generally not satisfied because of the discrete decision variables involved. A heuristic method is developed to generate a good feasible solution based on dual results. The power levels in purchase transactions are difficult to adjust in heuristics since the level constraints couple the levels across hours. The solutions of purchase subproblems therefore remain fixed in the heuristic. Thermal generations are adjusted by using the method presented in [5].

4. Implementation and Testing Results

The algorithm was implemented in FORTRAN on a SUN Sparc Station 2. All the billing rules of New England Power Pool are satisfied, and many practical considerations are included. To avoid proprietary issues, numerical results presented here are based on four after-the-fact billing data sets: week 3 in February 1991; week 4 in February 1991; week 3 in March 1991; and week 3 in May 1991. They are randomly selected from NU data files with purchase prices and periods modified to test various conditions. The contributions of hydro units, pumped storage units, together with power provided by co-generators and non-dispatchable contracts (including sale transactions), are removed from system demand and reserve requirements. The system characteristics and parameters associated with the data sets are summarized in Table 1.

Testing results for the four data sets are summarized in Table 2. For each data set, the total generation and purchase costs with and without purchase transactions are listed. It can be seen that the costs with purchase transactions are consistently lower than those without purchase transactions. The CPU times for the four data sets (and for others data sets tested but not reported here) are several minutes on a SUN Sparc Station 2 and about 1 minute on an IBM-3090 mainframe computer. According to the current NU practice, the response time required for a real-time purchase decision is about 15 minutes. The algorithm is therefore efficient to support real-time purchase decisions.

The available purchase periods and maximum purchase levels offered by a selling utility and the purchase intervals and levels obtained from our algorithm for February week 3, 1991 are depicted in Figure 4. It can be seen that purchase levels are not necessarily zero or at the maximum purchase levels, and purchase intervals may

Table 1. Summary of NU system characteristics

System characteristics	Number of units	Total capacity or requirement (MW)
Purchases	1-2	300 - 650
Steam units	14 - 22	1230 - 1565
Turbine units	24 - 34	274 - 473
Nuclear units	8	1062 - 2024
Hydro units	7	247 - 254
Pumped-storage unit (generation/pumping)	1	565 / 544 - 640 / 616
All units	56 - 74	3534 - 4723
Peak load		3376 - 3934
Minimum load		1357 - 1887
Maximum reserve		152 - 189

Table2. Summary of Testing Results

Data sets	Number of high level iterations	CPU time (sec)	Operating cost without purchase ($)	Operating cost with purchase ($)	Savings ($)
Feb w3,91*	25	442.89	12,515,802	12,386,310	129,491
Feb w4,91*	35	471.70	7,831,191	7,810,129	21,062
Mar w3,91*	27	406.82	10,657,665	10,344,447	313,217
May w3,91*	31	466.02	9,446,602	9,404,352	42,249

*: transaction data are modified

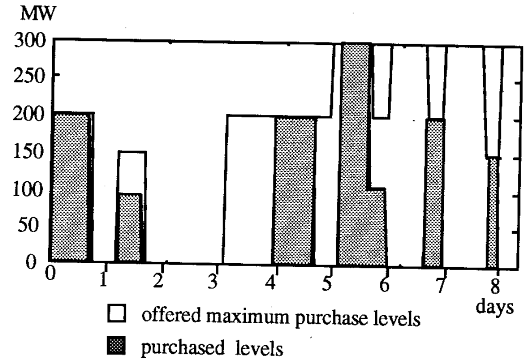


Figure 4. Purchase Intervals and Levels

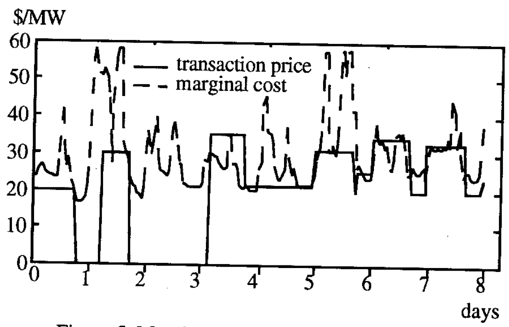


Figure 5. Marginal costs and transaction prices

not coincide with available purchase periods. The allowable purchase patterns in Figure 2 are also complied. The multipliers associated with system demand, interpreted as system marginal costs, and offered purchase prices are shown in Figure 5. Because hydro and pumped-storage units have not yet been incorporated into the algorithm and actual transactions were not based on the billing data, it is hard to compare the results with actual purchase decisions.

5. Conclusions

An effective algorithm based on the augmented Lagrangian decomposition and coordination technique has been developed to solve the integrated power purchase and thermal scheduling problem. After obtaining optimal levels for all purchase intervals, the optimal solutions for purchase subproblems are obtained by using the dynamic programming approach. The algorithm overcomes the solution oscillation problem caused by linear cost functions when the standard Lagrangian approach is used. Testing results show that this algorithm is efficient, and significant savings are obtained from purchase transactions. The algorithm will assist NU purchase decisions after hydro and pumped-storage units are incorporated.

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References

[1] R. N. Wu, T. H. Lee and E. F. Hill, "Effect of Interchange on Short-Term Hydro-Thermal Scheduling," *IEEE Transactions on Power Systems*, Vol. 6, No. 3, Aug. 1991, pp. 1217-1223.

[2] N. Chowdhury and R. Billinton, "Expert/Import of Spinning Reserve in Interconnected Generation Systems," *IEEE Transactions on Power Systems*, Vol. 6, No. 1, Feb. 1991, pp. 43-50.

[3] G. Cohen, D. L. Zhu, "Decomposition Coordination Methods in Large Scale Optimization problems: The non-differentiable case and the use of Augmented Lagrangian," *Advances in large scale systems, Theory and Application, Vol.* 1, 1988, pp. 203-266.

[4] J. Batut, R. P. Sandrin, "New Software for the Generation Rescheduling in the Future EDF National Control Center," *Proceedings of the 10th Power Systems Computation Conference*, Graz, Austria, 1990.

[5] X. Guan, P. B. Luh, H. Yan and J. A. Amalfi, "A Optimization-Based Method for Unit Commitment," *International Journal of Electrical Power & Energy Systems*, Vol. 14, No.1, 1992, pp. 9-17.

[6] D. P. Bertsekas, *Constrained Optimization and Lagrangian Multiplier Methods*, Academic Press, New York, NY, 1982.

[7] P. B. Luh, D. J. Hoitomt, E. Max, K. R. Pattipati, "Schedule Generation and Reconfiguration for Parallel Machines," *IEEE Transaction on Robotics and Automation*, Vol. 6, No. 6, Dec. 1990, pp. 687-696.

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