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Nonlinear Approximation Method in Lagrangian Relaxation-Based Algorithms for Hydrothermal Scheduling

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***Abstract*–When the Lagrangian relaxation technique is used to solve hydrothermal scheduling problems, many subproblems have linear stage-wise cost functions. A well recognized difficulty is that the solutions to these subproblems may oscillate between maximum and minimum generations with slight changes of the multipliers. Furthermore, the subproblem solutions may become singular, i.e., they are un-determined when the linear coefficients become zero. This may result in large differences between subproblem solutions and the optimal primal schedule. In this paper, a nonlinear approximation method is presented which utilizes nonlinear functions, quadratic in this case, to approximate relevant linear cost functions. The analysis shows that the difficulty associated with solution oscillation is reduced, and singularity is avoided. Extensive testing based on Northeast Utilities data indicates that the method consistently generates better schedules than the standard Lagrangian relaxation method.**

Keywords: Lagrangian Relaxation, Power System Scheduling, Hydrothermal Coordination, Unit Commitment.

I. INTRODUCTION

Hydrothermal scheduling is one of the most important daily activities for a utility company. The goal is to minimize the total generation cost subject to system demand, reserve and individual unit constraints. The significant economic implications of scheduling have fostered much research on this difficult mixed-integer programming problem over the past decades. Among many methodologies developed, Lagrangian relaxation is one of the most successful optimization-based methods ([1], [2], [3]).

The objective of Lagrangian relaxation is to obtain a near optimal solution to the scheduling problem. By using Lagrange multipliers to relax the system-wide demand and reserve constraints, the problem is decomposed and converted into a two-level optimization problem. The low level consists of a number of subproblems, one for each generating unit, and the high level dual problem is to optimize the multipliers. The dual solution, however, is generally associated with an infeasible schedule (in the sense that the once relaxed system-wide constraints are not satisfied) even at the convergence of the high level problem. Some techniques, usually heuristics, are needed to modify the dual solution to obtain a good feasible schedule. In our previous work, the Lagrangian relaxation technique has been successfully applied to schedule thermal, hydro and pumped-storage units, and to make inter-utility purchase decisions ([4], [5], [6], [7]).

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Within this Lagrangian relaxation framework, some subproblems, such as hydro or purchase transaction subproblems ([5], [6], [7]), may have linear cost functions with coefficients determined by Lagrangian multipliers. Another example is a "schedulable contract" where power can be dispatched as a one-block thermal unit with linear cost. A well recognized difficulty for using the standard Lagrangian relaxation technique (SLR for short) is that the solutions of these subproblems may oscillate between the minimum and maximum generation or transaction levels with slight changes of the multipliers ([8], [9]). Furthermore, the solution may become singular when these coefficients become zero.

One approach to resolving this solution oscillation difficulty is to use the augmented Lagrangian decomposition and coordination technique ([7], [8], [9], [10]). In this method, quadratic penalties associated with system demand requirements are added to the Lagrangian. Since the Lagrangian has to be decomposed into individual subproblems, the quadratic penalties are approximated by a decomposable quadratic function with some variables replaced by the solution of the previous iteration. This procedure may create high order terms for some subproblems thereby complicating the corresponding subproblem algorithms. Furthermore, the convergence properties are related to penalty coefficients associated with system demand requirement, and it may be difficult to select the coefficients that would suit different types of subproblems at the same time.

In this paper, a nonlinear approximation (NA) method is developed. The key idea is to use a nonlinear function, quadratic in this case, to approximate a linear cost function within a subproblem. Instead of changing the entire Lagrangian as in the augmented Lagrangian decomposition and coordination technique, this method approximates the linear cost function of each subproblem "locally" as needed. This corresponds to solving an approximated primal problem where parts of the cost function and/or constraints are modified. Our analysis shows that the difficulties associated with solution oscillation and singularity have been overcome, and the method consistently generates better schedules than the standard Lagrangian relaxation method.

II. SOLUTION OSCILLATION AND SINGULARITY CAUSED BY LINEARITY: EXAMPLES

The oscillation and singularity caused by linear cost functions in some subproblems can be illustrated by the following simple example. Consider a system with only one thermal unit and one schedulable contract to be scheduled for hour one. The problem for this simplified system can be stated as:

$$\min\ C_t(p_t(1)) + C_c(p_c(1)), \tag{2.1}$$

subject to

$$p_t(1) + p_c(1) = P_d(1), \tag{2.2}$$

$$\underline{p}_t \le p_t(1) \le \overline{p}_t, \tag{2.3}$$

$$0 \le p_c(1) \le \overline{p}_c, \tag{2.4}$$

where $p_t(1)$ and $p_c(1)$ are the generation and contract levels of the thermal unit and the schedulable contract at hour one, respectively, $C_t(p_t(1))$ and $C_c(p_c(1))$ their associated costs, and $P_d(1)$ system demand. The thermal cost $C_t(p_t(1))$ is piecewise linear with four equal size segments, with q_{tl} as the rate of segment l. The schedulable contract cost $C_c(p_c(1))$ is linear with rate q_c. The two cost curves and associated parameters are given in Fig. 1. By performing economic dispatch with two possible commitments, it is easy to verify that for $P_d(1) = 160MW$, the optimal primal generation and contract levels are

$$\hat{p}_t(1) = 60MW,\ \hat{p}_c(1) = 100MW, \tag{2.5}$$

and the minimum total cost is \$5,228.

Oscillation and singularity will occur if this problem is solved by using the Lagrangian relaxation technique. Following the solution methodology of [4], [5], [6], the two subproblems are formulated as follows:

$$Min\, L_t\ \ with\, L_t = C_t(p_t(1)) - \lambda(1)p_t(1), \tag{2.6}$$

subject to

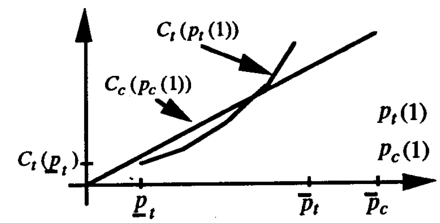
$$\underline{p}_t \le p_t(1) \le \overline{p}_t, \tag{2.7}$$

and

$$Min\, L_c\ \ with\, L_c = C_c(p_c(1)) - \lambda(1)p_c(1), \tag{2.8}$$

subject to

$$0 \le p_c(1) \le \overline{p}_c, \tag{2.9}$$



Thermal unit parameters:
$\underline{p}_t = 40$ MW, $q_{t1} = 32.0$ \$/MW, $q_{t3} = 38.4$ \$/MW
$\overline{p}_t = 120$ MW, $q_{t2} = 35.2$ \$/MW, $q_{t4} = 41.6$ \$/MW
Segment size = 20 MW, $C_t(\underline{p}_t) = \$1188$
Schedulable contract parameters:
$\overline{p}_c = 200$ MW, $q_c = 34$ \$/MW

Fig. 1 Cost curves of the two-unit problem

where $\lambda(1)$ is the Lagrangian multiplier associated with system demand at hour 1.

Given $\lambda(1)$, the optimal dual generation and contract levels, $p_t^*(1)$ and $p_c^*(1)$, respectively, can be obtained by solving corresponding subproblems. Since $C_c(p_c(1))$ is a linear function of $p_c(1)$, the cost function of the contract subproblem becomes

$$L_c = (q_c - \lambda(1))p_c(1), \tag{2.10}$$

where q_c is the contract rate. The optimal contract level is therefore

$$p_c^*(1) = \overline{p}_c \quad if\ \lambda(1) > q_c,$$
$$p_c^*(1) = 0 \quad if\ \lambda(1) < q_c. \tag{2.11}$$

It can be seen that p_c^* may oscillate between minimum and maximum contract levels when the multiplier is varying around the contract rate, and the subproblem is singular if the multiplier is exactly equal to the rate. In the latter case, p_c^* can take any value between the minimum and maximum contract levels, and the subproblem cost is zero.

Let $L_t^*(\lambda(1)) \equiv L_t(p_t^*(1))$ and $L_c^*(\lambda(1)) \equiv L_c(p_c^*(1))$, then the high level problem is to maximize the dual cost

$$\Phi(\lambda(1)) = L_t^*(\lambda(1)) + L_c^*(\lambda(1)) + \lambda(1)P_d. \tag{2.12}$$

By following a reasoning similar to the one presented in [11], $\Phi(\lambda(1))$ is piecewise linear, and its plot versus $\lambda(1)$, together with plots for the generation and contact levels, is shown in Fig. 2. It is observed that Φ is maximized at $\lambda(1) = q_c$, where the contract subproblem is singular. The maximal value of Φ is \$5,228, equal to the primal minimum cost. The duality gap, defined as the difference between the optimal primal cost and the optimal dual cost, is therefore zero. One can thus conclude that a good dual solution (even with zero duality gap) does not necessarily generate a good schedule since some of the subproblems may be singular.

To illustrate what is happening in the dual space as subproblem solutions oscillate, consider the above example scheduling over two hours. Assume that the demand at hour 1 and hour 2 are

$$P_d(1) = 160\,MW,$$

$$P_d(2) = 100\,MW,$$

respectively, and $\lambda(2)$ is the multiplier associated with the demand constraint at hour 2. The level curves of the piecewise linear dual cost function $\Phi(\lambda(1),\lambda(2))$ are plotted in Fig. 3, where "sharp edges" are observed as in [12]. If a subgradient method ([11], [13]) is used at the high level to update the Lagrangian multipliers as in most Lagrangian relaxation-based approaches, a "zig-zag" path along a sharp edge may be generated as shown in Fig. 4. One side of a sharp edge with a smaller $\lambda(1)$ corresponds to the minimal contract level, and the other side with a larger $\lambda(1)$ corresponds to the maximal contract level. While the multipliers zig-zag across a sharp edge, solution to the contract subproblem oscillates between minimal and maximal contract levels over the iterations. Singular solutions correspond to multipliers lying exactly on these edges. These sharp edges and the associated solution oscillation may result in slow convergence of the SLR algorithm ([12]). It will be shown in Section IV that the sharp edges can be "smoothed" by using the NA method.

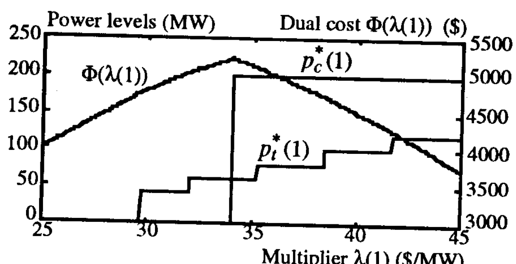


Fig. 2 Optimal power levels and dual cost versus multiplier

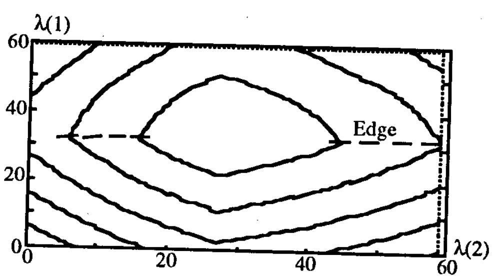


Fig. 3 Contours of dual cost function $\Phi(\lambda(1),\lambda(2))$

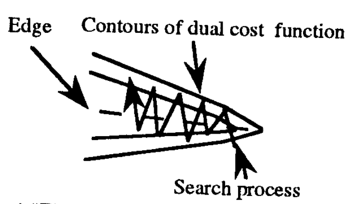


Fig. 4 "Zig-zag" search process in dual space

III. Problem Formulation

Consider a power system with I thermal units, J hydro units, K pumped-storage units and M schedulable contracts. The objective is to minimize the total generation and contract cost. This minimization is subject to system-wide demand and reserve requirements, and individual unit and contract constraints. The time unit is one hour and the planning horizon may vary from one day to ten days.

To formulate the problem mathematically, the following notation is introduced:

$C_{ti}(p_{ti}(t))$: cost of thermal unit i at hour t, in dollars;

$C_{cm}(p_{cm}(t))$: cost of schedulable contract m at hour t, in dollars;

E_{hj}: total available energy for hydro unit j (pond level and flow dynamics not considered following the rules of New England Power Pool), converted in MWhr;

I: number of thermal units;

J: number of hydro units;

K: number of pumped-storage units;

M: number of schedulable contracts;

$p_{cm}(t)$: contract power level of schedulable contract m at hour t, in MW;

$\bar{p}_{cm}(t)$: maximum contract level of schedulable contract m;

$P_d(t)$: system demand at hour t, in MW;

$p_{hj}(w_{hj}(t))$: power generated by hydro unit j at hour t, in MW;

$\bar{p}_{hj}(t)$: maximum generation level of hydro unit j;

$\underline{p}_{hj}(t)$: minimum generation level of hydro unit j;

$p_{pk}(t)$: power generated (positive) or used for pumping (negative) by pumped-storage unit k at hour t, in MW;

$P_r(t)$: system reserve at hour t, in MW;

$p_{ti}(t)$: power generated by thermal unit i at hour t, in MW;

$q_{cm}(t)$: rate or price of schedulable contract m at hour t, in \$/MW;

$r_{hj}(p_{hj}(w_{hj}(t)))$: reserve contribution of hydro unit j at hour t, in MW;

$r_{pk}(p_{pk}(t))$: reserve contribution of pumped-storage unit k at hour t, in MW;

$r_{ti}(p_{ti}(t))$: reserve contribution of thermal unit i at hour t, in MW;

$S_i(t)$: start up cost of thermal unit i at hour t, in dollars;

T: scheduling horizon, in hours;

$w_{hj}(t)$: water discharged for generation by hydro unit j at hour t, converted to MW;

The scheduling problem considered is formulated as the following mixed-integer programming problem:

Object Function -- total generation and contract cost

$$\min_{\substack{p_{ti}(t)\\p_{hj}(t)\\p_{pk}(t)\\p_{cm}(t)}} L,\ with\ L=\sum_{t=1}^{T}[\sum_{i=1}^{I}[C_{ti}(p_{ti}(t))+S_i(t)]+\sum_{m=1}^{M}C_{cm}(p_{cm}(t))] \tag{3.1}$$

System-wide Constraints

-- system demand

$$\sum_{i=1}^{I}p_{ti}(t)+\sum_{j=1}^{J}p_{hj}(w_{hj}(t))+\sum_{k=1}^{K}p_{pk}(t)+\sum_{m=1}^{M}p_{cm}(t)=P_d(t) \tag{3.2}$$

-- reserve requirement

$$\sum_{i=1}^{I}r_{ti}(p_{ti}(t))+\sum_{j=1}^{J}r_{hj}(p_{hj}(w_{hj}(t)))+\sum_{k=1}^{K}r_{pk}(p_{pk}(t))\ge P_r(t) \tag{3.3}$$

Individual Schedulable Contract Constraint

$$0\le p_{cm}(t)\le \overline{p}_{cm}(t) \tag{3.4}$$

Individual Hydro Constraints

-- Capacity and minimum generation

$$\underline{p}_{hj}(t)\le p_{hj}(w_{hj}(t))\le \overline{p}_{hj}(t) \tag{3.5}$$

-- Available hydro energy

$$\sum_{t=1}^{T}w_{hj}(t)\bullet 1\ hr=E_{hj} \tag{3.6}$$

According to the billing rules of New England Power Pool (NEPOOL), water discharged for generation is converted into MW, and the water-power conversion is ideal, i.e.,

$$p_{hj}(w_{hj}(t))=w_{hj}(t), \tag{3.7}$$

and the reserve contribution of hydro unit is

$$r_{hj}(p_{hj}(w_{hj}(t)))=\overline{p}_{hj}(t)-p_{hj}(w_{hj}(t)). \tag{3.8}$$

Individual Thermal Unit Constraints

-- Capacity and minimum generation
-- Minimum up/down time
-- ramp rate
-- minimum generation for the first & last hour generation
-- must-run and must-not-run

Individual Pumped-Storage Unit Constraints

-- Pond level constraints
-- Generation and pumping level constraints

The detailed descriptions of individual thermal and pumped-storage constraints are presented in [4] and [6], respectively.

IV. Solution Methodology

IV.1 The Lagrangian Relaxation Approach

The basic idea of the Lagrangian relaxation technique is to relax system-wide demand and reserve constraints in (3.2) and (3.3) by using Lagrange multipliers. Following the framework of [4], [5], and [6], a two-level structure is formed. Given the multipliers, the low level consists of individual thermal, hydro, pumped-storage and schedulable contract subproblems. The high level dual problem is to update the multipliers so as to maximize the dual function.

IV.2 Solving Schedulable Contract Subproblems

By arranging the terms in the Lagrangian according to individual units, the subproblem for schedulable contract m is written as follows:

$$Min\ L_{cm},\ with\ L_{cm}=\sum_{t=1}^{T}[C_{cm}(p_{cm}(t))-\lambda(t)p_{cm}(t)] = [q_{cm}(t)-\lambda(t)]p_{cm}(t), \tag{4.1}$$

subject to (3.4).

The solution to this subproblem is similar to (2.11), and the issues of oscillation and singularity exist. The basic idea of NA is to approximate the linear cost function in (4.1) by a strictly convex function, e.g., a quadratic function

$$q_{cm}(t)p_{cm}(t)\approx a_{cm}(t)p_{cm}^2(t)+b_{cm}(t)p_{cm}(t), \tag{4.2}$$

as shown in Fig. 5, where $a_{cm}(t)$ and $b_{cm}(t)$ are corresponding coefficients. It is easy to see that the optimal contract level,

$$p_{cm}^*(t)=\arg\min_{p_{cm}(t)}\{a_{cm}(t)p_{cm}^2(t)+[b_{cm}(t)-\lambda(t)]p_{cm}(t)\}, \tag{4.3}$$

subject to (3.4), will no longer oscillate between 0 and $\overline{p}_{cm}(t)$ as $\{\lambda(t)\}$ are updated over iterations since $p_{cm}^*(t)$ is a linear function of $\lambda(t)$. If the NA method is used to solve the schedulable contract subproblem of the example presented in Section 2, the dual cost $\Phi(\lambda(1))$ will be piecewise quadratic (with $p_c^*(1)$ substituting $p_c(1)$ in (2.8)) as shown in Fig. 6. The discontinuous step-up of $p_c^*(1)$ in Fig. 2 no longer exists, and the solution oscillation and singularity difficulties have been avoided. Correspondingly, the "sharp edges" in the level curves of the dual function have been "smoothed" as shown in Fig. 7, and the zig-zagging behavior of the multipliers in Fig. 4 is improved.

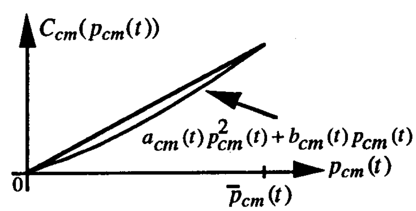


Fig. 5 Nonlinear approximation to contract cost function

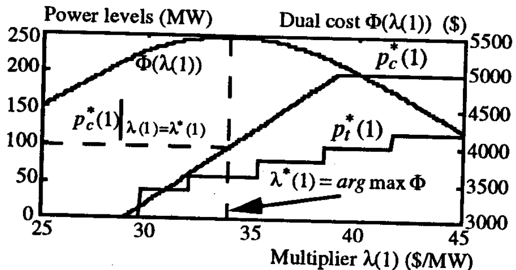


Fig. 6 Optimal power level and dual cost in NA method

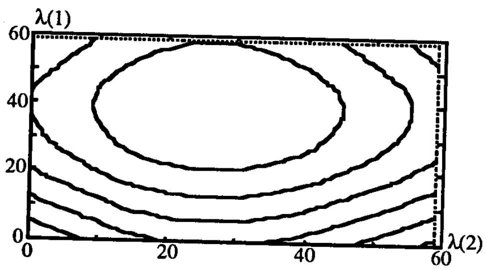


Fig. 7 Contours of dual cost function in NA method

IV.3 Solving Hydro Subproblems

A hydro subproblem is to determine the generations of a hydro unit so as to minimize the cost function

$$Min\, L_{hj},\ with\, L_{hj} = \sum_{t=1}^{T}[-\lambda(t)p_{hj}(w_{hj}(t)) -\mu(t)r_{hj}(p_{hj}(w_{hj}(t)))], \quad (4.4)$$

subject to (3.5) and (3.6). Substituting the reserve calculation (3.8) into (4.4), the cost function of hydro subproblem j becomes

$$L_{hj} = \sum_{t=1}^{T}[\mu(t)-\lambda(t)]p_{hj}(w_{hj}(t)) - \sum_{t=1}^{T}\mu(t)\overline{p}_{hj}(t). \quad (4.5)$$

Since the water power conversion in (3.7) is linear, the merit order allocation method of [5] can be used. This method allocates water to hours at the maximum capacity in the ascending ordering of $\mu(t)$-$\lambda(t)$, until all the hydro energy has been allocated. Solution oscillation and singularity difficulties similar to schedulable contracts may occur if the values of $(\mu(t)$-$\lambda(t))$ are close at several border hours.

The NA method can be applied to solve hydro subproblems. Since $\mu(t)$-$\lambda(t)$ is generally negative, i.e., the marginal cost of demand is larger than that of reserve, a concave quadratic function is used to approximate the water power conversion as shown in Fig. 8, so that L_{hj} in (4.5) is strictly convex. The energy constraint (3.6) is then relaxed by using another set of multipliers $\{\rho_{hj}\}$, and the cost function (4.5) can be re-written as:

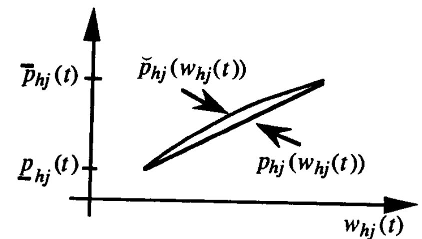


Fig. 8 Nonlinear approximation of water power conversion

$$\breve{L}_{hj} = \sum_{t=1}^{T}[\mu(t)-\lambda(t)]\breve{p}_{hj}(w_{hj}(t)) - \sum_{t=1}^{T}\mu(t)\overline{p}_{hj}(t) +\rho_{hj}[\sum_{t=1}^{T}w_{hj}(t)-E_{hj}], \quad (4.6)$$

where $\breve{p}_{hj}(w_{hj}(t))$ is the newly introduced nonlinear water power conversion function. Define the stage-wise function as

$$h_{hj}(w_{hj}(t)) \equiv [\mu(t)-\lambda(t)]\breve{p}_{hj}(w_{hj}(t))+\rho_{hj}w_{hj}(t). \quad (4.7)$$

The optimal generation at each hour can then be obtained as

$$w^{*}_{hj}(t) = arg \min_{w_{hj}(t)} h_{hj}(w_{hj}(t)), \quad (4.8)$$

subject to (3.5).

Let $\breve{L}^{*}_{hj}(\rho_{hj})$ denote the optimal Lagrangian for (4.6). Since $\breve{L}^{*}_{hj}(\rho_{hj})$ is piecewise quadratic, a quadratic fitting line search method ([14], pp. 206) is used to update the multiplier ρ_{hj} associated with the hydro energy constraint (3.6) at an intermediate level to maximize $\breve{L}^{*}_{hj}(\rho_{hj})$.

IV.4 Solving Thermal and Pumped-storage Subproblems

Since a thermal unit generally has several blocks, the solution oscillation difficulties are less severe as compared to schedulable contracts or hydro units. The method presented in [4] is therefore sufficient without the need for nonlinear approximation. Pumped-storage subproblems can be efficiently solved by relaxing pond level constraints with additional sets of multipliers and applying dynamic programming technique to optimize generation or pumping across hours ([6], [15]). The methodology in [6] is used to solve the pumped-storage subproblems, where some of the nonlinear approximation ideas were already incorporated.

IV.5 Updating Multipliers at High Level and Constructing Feasible Solutions

The high level dual problem is to update the multipliers associated with system demand and reserve. A subgradient method with adaptive step sizing is used to update $\lambda(t)$ and $\mu(t)$ ([1], [2], [4], [11], [13]). Since the dual solution is generally associated with an infeasible schedule, heuristic methods have been developed to obtain a good feasible schedule as presented in [4] and [6]. In the current case, the original linear schedulable contract cost functions and linear water-power conversion are used for obtaining feasible schedules and for calculating feasible costs.

V. Numerical Testing Results

The algorithm was implemented in FORTRAN on a SUN Sparc Station 2 and an IBM 3090 mainframe for scheduling the power system of Northeast Utilities Service Company (NU). All the billing rules of New England Power Pool are satisfied, and many practical considerations are included. There are about 65 thermal units, 7 hydro units, 1 large pumped-storage unit, and 2-10 schedulable contracts. There are two types of hydro units in the NU system: weekly hydro units and daily hydro units. Although the energy constraint in (3.6) is only for weekly hydro units, daily hydro units are modeled and solved by the same method except that the available hydro energies are specified for each day.

Extensive testing has been performed using NU billing data to compare the NA method with the standard Lagrangian relaxation (SLR) method. Eleven data sets in 1991 are randomly selected from NU billing data files without any modification. The scheduling horizon is either 7 days (168 hours) or 8 days (192 hours). The system characteristics and parameters for these data sets are summarized in Table 1.

The selection of quadratic approximation parameters, such as a_{cm} and b_{cm} in (4.2), are based on experience. Usually b_{cm} is chosen as 0.85-0.95 of q_{cm}, slope of the linear function. The quadratic coefficient a_{cm} is then calculated to generate the same cost at $\overline{p}_{cm}(t)$ as in Fig. 5. In this case, the magnitude of the quadratic coefficient would be small, implying that the new cost would be close to the original linear cost. Similar method can be used for the water-power conversion as shown in Fig. 8.

Testing results for the 11 data sets are summarized in Table 2. For each data set, the feasible costs of the NA method and the SLR method are listed. It can be seen that the costs of the NA method are consistently and significantly lower than those of the SLR method. The generation levels of a daily hydro unit for January week 3, 1991 are depicted in Figure 9. For better viewing, only the first day and last day are shown. It can be seen that generation levels obtained by the NA method are not necessarily at the minimum or maximum, in contrast to the SLR method.

The CPU times of the NA method are similar to those of the SLR method, i.e., several minutes on a SUN Sparc Station 2 and about 1 minutes on an IBM 3090. The algorithm is therefore efficient for daily scheduling.

VI. Conclusions

An effective nonlinear approximation method is developed to resolve the solution oscillation and singularity difficulties of Lagrangian relaxation-based scheduling algorithms. The linear cost functions of schedulable contract and hydro subproblems are locally approximated by nonlinear functions so that solutions to subproblems and the high level problem can all be improved. This method provides a simple and effective way to resolve inherent difficulties of Lagrangian relaxation-based algorithms. Extensive testing shows that the method consistently and significantly generates better schedules than the SLR method. This method has been incorporated in the optimization-based scheduling package of NU, and is in production use on a daily basis.

TABLE 1
SUMMARY OF NU POWER SYSTEM

System characteristics	Number of Units	Total capacity or requirement (MW)
Steam units	14 - 22	1230 - 1565
Turbine units	24 - 34	274 - 473
Nuclear units	8	1062 - 2024
Hydro units	7	247 - 254
Pumped-storage unit (generation/pumping)	1	565 / 544 - 640 / 616
Dispachable contracts	2 - 10	0 - 800
All units	56 - 82	3534 - 5223
Peak load		3376 - 4434
Minimum load		1357 - 1887
Maximum reserve		152 - 189

TABLE 2
COST COMPARISON OF NA METHOD WITH SLR METHOD

Data set	SLR			NA			Savings ($)
	IT	Cost ($)	CPU (s)	IT	Cost ($)	CPU (s)	SLR-NA
Jan. W3	47	5,233,144	464	47	5,204,026	459	29,118
Feb. W2	24	4,601,450	279	45	4,588,461	444	12,989
Feb. W3	41	7,864,169	496	38	7,855,841	476	8,328
Feb. W4	35	4,422,011	323	24	4,410,706	246	11,305
Mar. W3	38	6,031,247	407	55	6,026,418	517	4,829
Mar. W4	18	4,837,316	239	15	4,829,335	227	7,981
Apr. W2	51	4,955,321	487	47	4,952,279	463	3,042
Apr. W3	20	5,462,575	264	33	5,459,533	357	3,042
Apr. W4	31	4,715,904	345	53	4,709,849	502	6,055
May W3	28	5,158,179	316	39	5,146,778	401	11,401
Oct. W2	44	5,194,753	402	62	5,174,970	532	19,783

IT: Number of high level iterations
CPU: CPU time in second on SUN Sparc 2

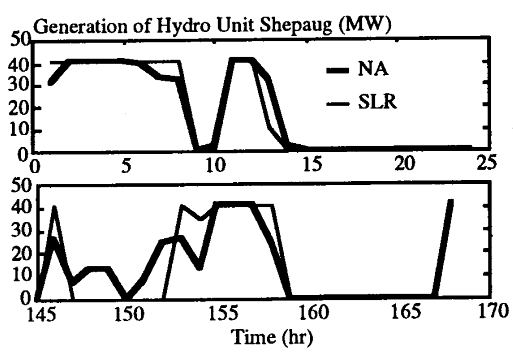


Fig. 9 Generation of a daily hydro unit

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