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# Integrated Resource Scheduling and Bidding in the Deregulated Electric Power Market: New Challenges

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**Abstract.** Many challenging issues arise in the newly deregulated competitive electric power markets worldwide. Instead of centralized decision-making in a monopoly environment as in the past, many parties with different goals are now involved and competing in the market. The information available to a party may be limited, regulated, and received with time delay, and decisions made by one party may influence the decision space and well-being of others. These difficulties are compounded by the underlying uncertainties inherent in the system such as the demand for electricity, fuel prices, outages of generators and transmission lines, tactics by certain market participants, etc. Consequently the market is full of uncertainty and risk. Key questions to be addressed include how to predict load and market clearing prices, how to consider other parties' decisions in deciding one's own bids, and how to manage uncertainty and risk. Since finding an optimal solution to a traditional unit commitment problem is NP-hard even without considering multiple parties and uncertainties, it may be more practical to know which decision is good with confidence rather than looking for an optimal solution.

For an energy supplier, bidding decisions are coupled with generation resource scheduling or unit commitment since generator characteristics and how they will be used to meet the accepted bids in the future have to be considered before bids are submitted. For example, if starting up a thermal unit is expected, the associated startup cost should somehow be configured in the bid curves. The decisions, however, can be quite subtle since generally startup costs are not part of a bid. Bidding decisions should therefore be carefully made by considering the anticipated MCP, system demand, generation and startup costs, and competitor' decisions. What further complicates the issue is that some of the information is not available, or will be available but with significant delays.

In paper, two promising bidding strategies for power suppliers are discussed. The ordinal optimization method seeks "good enough" bids with high probability, and is an effective in handling market uncertainties with much reduced computational efforts. The basic idea of this method is to use a model to describe the influence of bidding strategies on the MCP. A nominal bid curve is obtained by solving optimal power generation for a given set of the MCPs within the Lagrangian relaxation framework. Then $N$ bids are generated by perturbing the nominal bid curve. The ordinal optimization method is applied to form a good enough bid set $S$, which contains some good bids with high probability, by performing rough evaluation. The best bid is then searched and selected over $S$ by solving generation scheduling or unit commitment problems within reasonable computational time. The game theoretic method aims at bidding and self-scheduling of a utility company in New England. The problem is investigated from the viewpoint of a particular utility bidder. The uncertainties caused by bids from other bidders and the ISO (Independent System Operator) bid selection process are explicitly considered. The problem is then solved within a reduced game theoretical framework, where the ISO has a closed-form solution for a given probabilistic description of the bids, and the utility's problem is solved by using Lagrangian relaxation. Although the two specific methods represent significant progress made thus far, the area is wide open for creative research to make the deregulated market a true success.

**Keywords:** power system scheduling, bidding, deregulated electric power market

## I. Introduction

The electric power industry is in revolution world-wide. This unprecedented restructuring of the industry started in South America and Europe, and is sweeping the United States (Dunn et al., 1995; Gross and Finlay, 1995; Halseth, 1997; Jacobs and Singh, 1997; Hao et al., 1998). The first state in the US with a deregulated power industry is California, with its new market operational in Spring 1998. The core of the deregulation includes structural and functional changes of key elements of the industry: generation, transmission, distribution, system operation, and power transactions. With the introduction of competition among power suppliers, the roles of these elements have been significantly changed, so are the rules governing their operations and interactions. The purposes of this paper are to raise several challenging issues in the deregulated power market, shed light on promising methods, and to present preliminary results. The discussion will be focused on energy generation bidding and scheduling.

In a traditional electric power system, a utility company is responsible for generating and delivering power to its industrial, commercial, and residential customers in its service area. It owns generation facilities and transmission and distribution networks, and obtains necessary information for the economical and reliable operation of its system. For instance, an important problem faced daily by a traditional utility company is to determine which and when generating units should be committed, and how they should be dispatched to meet the system-wide demand and reserve requirements. This *centralized resource scheduling problem* involves discrete states (e.g., on/off of units) and continuous variables (e.g., unit generation levels), with the objective to minimize the total generation cost. The economical impact of generation scheduling is significant. A one percent reduction of cost can result in more than ten million dollar savings per year for a large utility company. Various methods have been developed, and impressive results have been obtained (Rosenthal, 1981; Shaw and Bertsekas, 1985; Cohen and Sherkat, 1987; Ferreira et al., 1989; Guan et al., 1992; Renaud, 1993; Wang et al., 1995; Guan et al., 1997; Li et al., 1997). Under the new structure, resource scheduling is intertwined with bidding in the market, and power suppliers and system operators are facing a new spectrum of issues and challenges.

Many deregulated power markets are based on a pool-operation structure (Jacobs and Singh, 1997). For example, the California market contains a Power Exchange (PX) with "day-ahead" and "hour-ahead" energy markets, various energy and service suppliers, and a real-time market for energy balancing operated by the Independent System Operator (ISO). The PX and the ISO are independent and non-profit organizations with no commercial interest in the market. Electricity is primarily traded through bidding in the PX market. Independent from the PX, the ISO controls and operates the transmission grid, and facilitates transactions and transmission while avoiding influence on the generation schedules created by the PX. Electric power suppliers can sell energy to the PX, and ancillary services (including automatic generation control AGC, real-time load balance, spinning reserve, reactive power, and generating capacity required for grid congestion management) to the ISO. Energy is eventually distributed to end-customers through distribution networks belonging to "utility energy service companies," where ancillary services are used to support system operation. The energy markets are classified according to their time frames: day-ahead and

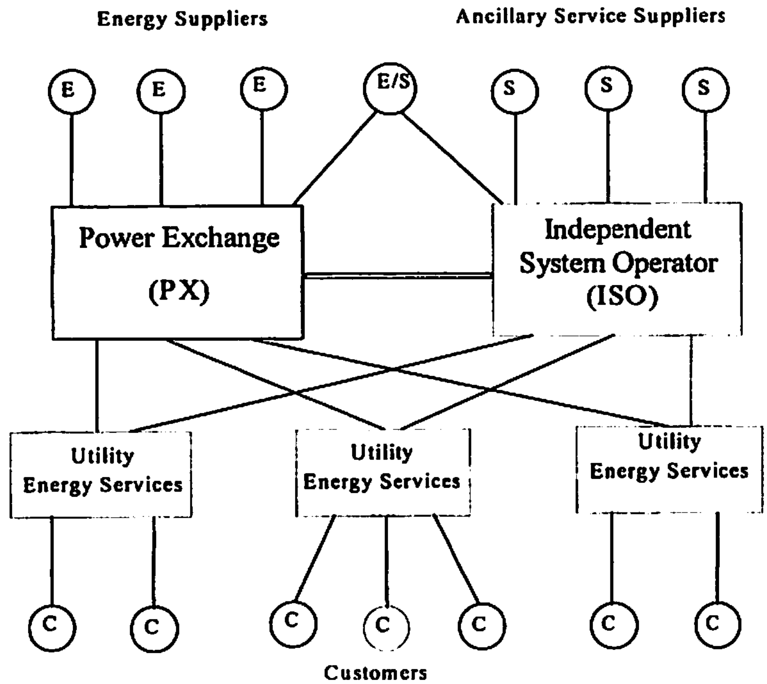


*Figure 1.* A deregulated market structure. E – Energy; S – Ancillary services; C – Customers.

hour-ahead in the PX, and real-time in the ISO to follow the load. The new market structure is illustrated in Figure 1.

In the day-ahead market, an energy supplier (either an independent power producer IPP or a generation company GENCO derived from or as a part of an electric utility company) submits to the PX a set of hourly power-price "supply bid curves" for each generator or for a portfolio of generating units, for the next day. These supply bid curves are aggregated by the PX to create a single "supply bid curve." On the other hand, an energy service company (either a utility energy service company UES or a non-utility energy service company ESCO) submits to the PX an hourly power-price "demand bid curve" reflecting its forecasted demand. These demand bid curves are also aggregated by the PX to create a single "demand bid curve." Based on the demand and supply bid curves, the PX determines a "Market Clearing Price" (MCP) for each hour as shown in Figure 2. The power to be awarded to each bidder is then determined, and all the power awards will be compensated at the MCP. After such an auction closes, each bidder can aggregate all its spower awards as its system demand, and performs a traditional unit commitment or hydrothermal scheduling to meet this obligations at the minimum cost over the bidding horizon. Although about 90% of the power supply is auctioned in the day-ahead market according to the 10-month operational experience of the California PX, a power supplier or demander can also bid in

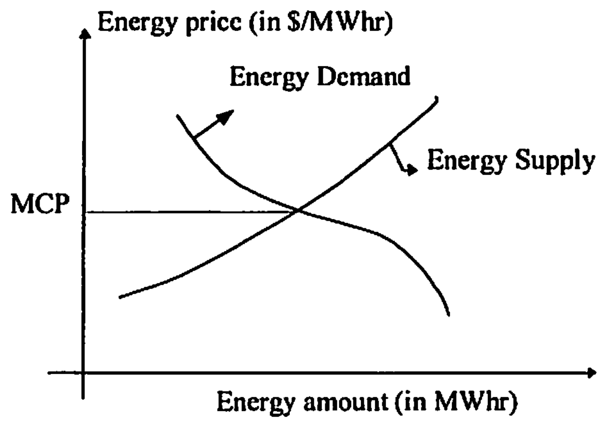


*Figure 2.* Market clearing price as the cross point of demand and supply bid curves.

the hour-ahead market and the real-time power market. This will add another dimension of difficulties for the bidders to trade-off bids submitted for different markets.

For an energy supplier, bidding decisions are coupled with resource scheduling since generators' characteristics and how they will be used to meet the accepted bids in the future have to be considered before bids are submitted. For example, if starting up a thermal unit is expected, the associated startup cost should somehow be configured in the bid curves. The decisions, however, can be quite subtle since generally startup costs are not part of a bid. Furthermore, it is clear that the higher the MCP, the higher the profit for accepted bids. A bidder might therefore want to raise bids to increase the profit. Doing this, however, is at the risk of losing the bids. To illustrate the idea, consider an aggregated staircase supply bid curve as shown in Figure 3, where Bid A determines the MCP and the adjacent bids belong to other bidders. Clearly if Bid A is changed to Bid B, the bidder's revenue will increase. However, the bid will be lost if it is changed to Bid C because of the competition from other bidders. Bidding decisions should therefore be carefully made by considering the anticipated MCP, system demand, generation and startup costs, and competitor's decisions. What further complicates the issue is that some of the information is not available, or will be available but with significant delays.

The ancillary services are sold at auctions in a different market managed by the ISO. Bidding decisions for these services, however, are often coupled with those for energy. For example, providing spinning reserve would means less energy generation for a thermal unit. Decisions have to be made regarding what percentage of a unit's capacity should bid for energy and what for reserve based on profitability and other considerations. These decisions involve uncertainties of both the energy and the ancillary service markets such as their market clearing prices.

The power market in New England of the US is based on the New England Power Pool, and is similar to that of California with the following differences. Organizational-wise, the functions of PX and ISO are combined under the auspices of a single ISO, the

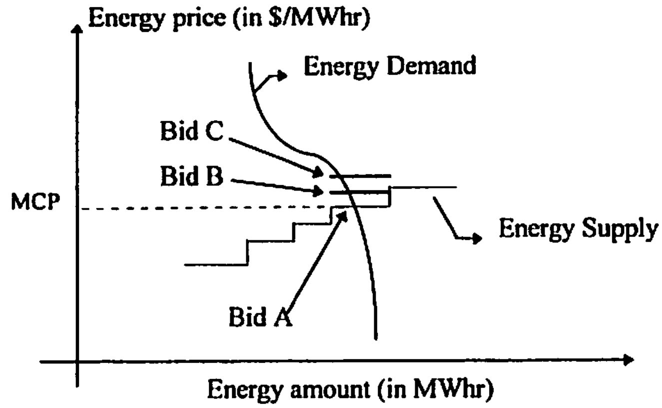


*Figure 3.* Bids determining the market clearance prices.

Independent System Operator which was split from the New England Power Pool. A utility company may have a generation branch and a transmission/distribution branch, though information exchange between the two branches is restricted. Operational-wise, the ISO estimates system demand and determines bids to be accepted, with demand bidding dropped altogether. In addition, an energy supplier is not required to submit all its generation capacities to the ISO. Rather, it can submit part of its generation to the ISO for the market, while "self scheduling" the remaining for a specified level of its own load or for bilateral transactions with other market participants.

It can be seen from the above that many challenging issues arise under the new competitive market structure. Instead of centralized decision-making in a monopoly environment as in the past, many parties with different goals are now involved and competing in the market. The information available to a party may be limited, regulated, and received with time delay, and decisions made by one party may influence the decision space and well-being of others. These difficulties are compounded by the underlying uncertainties inherent in the system such as the demand for electricity, fuel prices, outages of generators and transmission lines, tactics by certain market participants, etc. Consequently the market is full of uncertainty and risk. The recent experience learned from the California market has shown that MCPs are volatile and often out of bidders' expectation. Key questions to be addressed include how to predict load and market clearing prices, how to consider other parties' decisions in deciding one's own bids, and how to manage uncertainty and risk. Game theoretic framework may provide valuable insights, however, major progress must be made for the result to have practical impact. Simulation may also turn out to be valuable. Since finding an optimal solution to a traditional unit commitment problem is NP-hard even without considering multiple parties and uncertainties, it may be more practical to know which decision is good with confidence rather than looking for an optimal solution.

The remaining of the paper is organized as follows. The state of art and current research efforts will be summarized in the next Section. A simplified mathematical formulation will be developed to model the integrated bidding and scheduling process in Section III. Based on this formulation, an ordinal optimization method for bid selection will be presented in Section IV, and a game theoretic optimization method for integrated bidding and self-scheduling will be presented in Section V. The issue of generation scheduling and energy delivery capability will be briefly discussed in Section VI. Concluding remarks are then given in Section VII.

## II. Literature Review

### *II.1. Deregulated Power Markets*

Many approaches have been presented in the literature to address the deregulated power markets. The market structure model discussed the most is the "British Model," which is different from the "California Model" but the two share a similar framework (Gross and Finlay, 1995). The California Model, including specifications, bidding protocols, bid evaluation, and simulation results, was detailed in (London Economics Inc., 1997; PX Business Rules and Protocols Task Group, 1997; Wilson, 1997). The analysis and simulation were to test the rules and protocols for designing the market. A simplified bidding scheme for the California market was presented in Cazalet and Ellis (1996). Under this scheme, a participant in the PX bids a quantity of energy (generation or load) for each hour at the posted PX market price. The PX then adjusts the posted prices in response to supply surplus or un-met demand. In this way, the bid evaluation process is simple. Developing appropriate bidding strategies to maximize a bidder's profit, however, was not addressed in the above reports.

For each participant, bidding strategies ideally should be determined to maximize its profit. Game theory is a natural platform to model such an environment (Owen, 1995; Gross and Finlay, 1995; Krishna and Ramesh, 1997; Ferrero et al., 1997). Optimal bidding strategies to maximize a bidder's profit based on the pool model of England and Wales were presented in Gross and Finlay (1995) under the assumption that any particular bid has no effect on the MCP. For a market where a bid consists of start-up price, variable price, and generator capacity, it was demonstrated that profit will be maximized by bidding each generator at its physical cost curve and maximum capacity. This is done by showing that such a strategy is a "Nash equilibrium" for the market, i.e., there is no incentive for a bidder to unilaterally deviate from this strategy if everyone else bids this way. The conditions assumed in Gross and Finlay (1995), however, are not practical since a bid may affect MCP, and start-up prices are usually not included in a bid. A bidder, as mentioned earlier, may want to increase or decrease bidding prices from a generator's physical cost curve to maximize its profit.

Matrix games have been used in several papers, e.g., Krishna and Ramesh (1997) and Ferrero et al. (1997). Bidding strategies are discretized, such as "bidding high," "bidding low," or "bidding medium." With discrete bidding strategies, payoff matrices are constructed by enumerating all possible combinations of strategies, and an "equilibrium" of the "matrix bidding game" can be obtained. Because of inherent complexity, self-scheduling may not be easily incorporated.

Modeling and solving the bid selection problem of the PX have also been discussed. In Hao et al. (1998), bids are selected to minimize the total system cost, and the energy clearing price is determined as the highest accepted price for each hour. In Alvey et al. (1998), a bid clearing system in New Zealand is presented. Detailed models are used, including network constraints, reserve constraints, and ramp-rate constraints, and linear programming is used to solve the problem. Other approaches addressing various aspects of generation and ancillary service bidding can be found in Dekrajangpetch et al. (1998), Sheble (1998), and Singh and Papalexopoulos (1998), where Lagrangian relaxation, decision tree, and expert systems were used to analyze and support the bidding process. For example, a bidding strategy considering revenue adequacy was presented in Li et al. (1998) based on Lagrangian relaxation and an iterative bid adjustment process, which might be too complicated for the current PX market.

### *II.2. Stochastic Optimization*

One way to model the bidding process from an individual bidder's point of view is to model competitors' behaviors as uncertainties. Therefore the bidding problem can be converted to a stochastic optimization problem. One of the widely used approaches in stochastic optimization is *stochastic dynamic programming* (Contaxis, 1990; Li et al., 1990). The basic idea is to extend the backward dynamic programming procedure by having probabilistic input and probabilistic state transitions in place of deterministic input and transitions, and by using *expected* costs-to-go in place of deterministic costs-to-go. The direct consequence is the significant increase of the input space and the number of possible transitions. For example, when stochastic dynamic programming is used to solve a hydro scheduling problem with uncertain inflows, one more dimension is needed to consider probable inflows in addition to reservoir levels, significantly worsen the "curse of dimensions." Another approach is *scenario analysis* (Carpentier et al., 1996; Takriti et al., 1996). Each scenario (or a possible realization of random events) is associated with a weight representing the probability of its occurrence. The objective is to minimize the expected cost over all possible scenarios. Since the number of possible scenarios and consequently the computational requirements increase drastically as the number of uncertain factors and the number of possibilities per factor increase, this approach can only handle problems with a limited number of uncertainties. Recently, stochastic dynamic programming has been embedded within the Lagrangian relaxation framework for manufacturing scheduling problems, where stochastic dynamic programming is used to solve uncertain subproblems after system-wide coupling constraints are relaxed. Since dynamic programming for each subproblem can be effectively solved without encountering the curse of dimensionality, good schedules are obtained without a major increase in computational requirements (Luh et al., 1998).

Simulation is another widely used approach for stochastic optimization. Since such problems are generally associated with inherent computational difficulties especially when discrete variables are involved, it is more appropriate to ask which solution is better as opposed to looking for an optimal solution based on a limited number of simulation runs.

Recently, an intelligent computational method—Ordinal Optimization (OO) has been developed to solve complicated optimization problems possibly with uncertainties (Deng and Ho, 1997; Ho, 1994; Ho and Larson, 1995; Ho, 1997a, 1997b, 1997c; Lau and Ho, 1997). Ordinal optimization is based on the following two tenets: (1) It is much easier to determine "order" than "value." To determine whether A is larger or smaller than B is a simpler task than to determine the *value* of (A − B) especially when uncertainties exist. (2) In stead of asking the "best for sure," we seek the "good enough with high probability." This softening the goal of optimization should also make the problem easier.

Consider, for example, a search on a bidding strategy space $\Theta$. Suppose that the "good enough" subset, $G \subset \Theta$, is defined as the top 1% of the strategy space, and a selected subset, $S \subset \Theta$, is the space to be searched. The goal of OO is to construct a small search space $S$ while maintaining a high alignment probability that $|G \cap S| \neq 0$. If an efficient method can be developed to construct a small but "good" $S$ for detailed search, then major speedup can be achieved. An iterative OO method was presented in Deng and Ho (1997) to narrow the search to favor good subsets of the search space through limited sampling. The method has been applied to the famous unsolved Whitsenhausen problem in the optimal control of linear systems with quadratic objective functions and Gaussian noises (an LQG problem). A solution that is 50% better than the best known solution has been obtained. To apply this conceptual framework to integrated resource scheduling and bidding, major efforts are needed to build power market simulation models and to construct a small but good search space $S$.

It can be seen from above that tools to support the bidding process are far from satisfactory in view of the inherent complexity (multiple participants with their own objectives in a dynamic and uncertain environment) and the sizes of practical problems (tens or hundreds of generators with various constraints). High quality and computationally efficient approaches are critically needed to address the new challenges and to develop effective bidding and self-scheduling strategies.

## III. Problem Formulation

### *III.1. Individual Bidding Formulation*

To simplify the presentation but without loss of generality, assume that there are $I$ supply bidders, each could own a single generating unit or a portfolio of units. The objective of Bidder $i$ is to select its supply bid curves $\{B_{it}(\cdot)\}_{t=1}^{T}$ to maximize its profit over a time horizon $T$, i.e.,

$$\underset{\{B_{it}(\cdot)\}_{t=1}^{T}}{Max}\ J_i,\ \text{with}\ J_i = \sum_{t=1}^{T}\left[\sum_{m=1}^{M_i}\lambda(B_{lt}(\cdot)\mid l=1,2,\ldots,I)p_{im}^{a}(B_{lt}(\cdot)\mid l=1,2,\ldots,I)\right.$$
$$\left. -C_i(p_i^a(\cdot,t)) - S_i(p_i^a(\cdot,t))\right], \qquad (1)$$

where

$B_{it}(\cdot) =$ Price-generation supply bid curve of Bidder $i$ as shown in Figure 1 and Figure 3;

$C_i(p_i^a(\cdot)) =$ Generation cost of Bidder $i$ for delivering generation award $p_i^a(\cdot)$;

$I =$ Number of bidders;

$M_i =$ Number of generating units of Bidder $i$;

$p_{im}^a(\cdot) =$ Generation award of unit $m$ of Bidder $i$;

$p_i^a(\cdot) =$ Aggregated generation award of Bidder $i$, i.e., $p_i^a(\cdot, t) = \sum_{m=1}^{M_i} p_{im}^a(\cdot, t)$;

$S_i(p_i^a(\cdot)) =$ Costs associated with the up/down state transitions for delivering generation award $p_i^a(\cdot)$;

$T =$ Time horizon;

$\lambda(\cdot) =$ Market clearing price (MCP) determined by the aggregated supply bid curve and the aggregated demand bid curve.

According to the PX rule, if a bid is accepted with $p_i(t)$ as the amount of energy to be generated, Bidder $i$ will be compensated by the dollar amount $\lambda(t) \cdot p_i(t)$ no matter how the bid was originally submitted. Startup costs should be embedded in bid curves since there is no direct startup compensation. The above (1) is thus a functional optimization problem to determine the optimal supply bid curves $\{B_i(t)\}_{t=1}^{T}$ to maximize the profit subject to relevant operating constraints such as the minimum down/up time, ramp-rate constraints, etc. Note that MCPs are determined by the bids submitted by all the bidders, and when submitting the bids, a bidder does not know the bid curves submitted by others. There are thus two ways to look at the problem. The first is to treat the object function in (1) as inherently uncertain, and solve the problem by using stochastic optimization. The second is to explicitly consider other bidders within the problem formulation from a game theoretical point of view. Both approaches will be highlighted later.

### *III.2. PX Economic Dispatch Formulation*

Given supply bid curves submitted by energy suppliers and demand bid curves by UES or ESCO, the PX is to minimize the overall costs while satisfying the hourly demand. For simplicity, the aggregated demand bid curve for each hour is assumed to be represented by a single deterministic system demand $P_d(t)$. In this case, the PX's problem is a traditional *economic dispatch* problem since units are assumed to be committed and startup costs embedded in the supply bid curves. The problem is described by:

$$\min_{\{p_i(t)\}_{i=1}^{I}} C, \text{ with } C \equiv \sum_{i=1}^{I} \sum_{t=1}^{T} B_{it}(p_i(t)), \tag{2}$$

subject to

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_i(t) = P_d(t), \quad t = 1, 2, \ldots, T. \tag{3}$$

Since the objective function is separable in time, the dispatch can be performed for individual hours separately, and solved by using traditional nonlinear programming methods.

Supplying reserve service is similar to supplying energy. Consequently, the bidding for *reserve services*—the capability to provide additional power within a specified time period, is not discussed for simplicity of presentation.

## IV. Ordinal Optimization Method for Bid Selection

As mentioned earlier, it is recognized that the pursuit of *optimal* bids and schedules under the new market structure is impractical because of problem complexity and the uncertainties involved. Instead, two near-optimal approaches are presented below. The first one treats MCP as uncertain, and seeks "good enough" bids and schedules with high probability based on ordinal optimization as presented next. The second addresses bidding and self-scheduling of a utility company in New England, where the ISO bid selection process and uncertainties about other bids are explicitly modeled. The problem is solved within a simplified game theoretical framework to be presented in Section V, where the ISO has closed form solution, and the utility's problem is solved by using Lagrangian relaxation.

### *IV.1. Ordinal Optimization Based Method for Power Generation Bidding*

For the rest of this section, the MCPs are treated as uncertain, and the bidding problem (1) for Bidder $i$ is considered as a stochastic functional optimization problem where a desired bid is a generation-price curve, or price as a function of generation. This kind of function optimization (as opposed to parameter optimization) is extremely difficult to handle, and a sensible approach is to solve a series of parameter optimization problems and then perform interpolation and extrapolation to generate a *nominal bid curve*. To do this, the bidding problem for bidder $i$ is first re-written as follows for a given series of estimated prices $\{\lambda_f(t)\}_{t=1}^{T}$:

$$\underset{\{p_{im}(t)\}}{Max}\ J_i, \text{ with } J_i = \sum_{t=1}^{T}\sum_{m=1}^{M_i}\left[\lambda_f(t)p_{im}(t) - C_{im}(p_{im}(t)) - S_{im}(z_{im}(t))\right], \tag{4}$$

where $\lambda_f(t)$ is the estimated MCP at time $t$, $p_{im}(t)$ the generation level of unit $m$ at time $t$, $C_{im}(\cdot)$ the generation cost of unit $m$, $S_{im}(z_{im}(t)$ the start-up cost of unit $m$, and $z_{im}(\cdot)$ the up/down state of unit $m$. The optimization is subject to relevant operating constraints. The actual MCP can be viewed as $\lambda_f(t)$ plus an error or a noise. For the given series of estimated MCPs $\{\lambda_f(t)\}_{t=1}^{T}$, (4) is a *parameter optimization problem* as opposed to a functional optimization problem. It is similar to a unit commitment subproblem within the hydrothermal scheduling context when solved by using the Lagrangian relaxation technique. The problem can thus be efficiently handled by dynamic programming as done in our previous work (Guan et al., 1992; Guan et al., 1997). The results are optimal generation levels $\{p_{im}(t)\}$ for all Bidder $i$'s units and for all the hours. From there, aggregated generation levels $\{p_i^*(t)\}_{t=1}^T$ can be calculated, and a series of generation-price pairs $\{p_i^*(t), \lambda_f(t)\}_{t=1}^T$ can be obtained. Nominal bid curves $\{\overline{b}_i(\overline{p}_i)\}_{t=1}^T$ can then be obtained by interpolation or extrapolation with multiple series of $\{p_i^*(t), \lambda_f(t)\}_{t=1}^T$ pairs. The bid curves for different hours obtained by solving (4) may be inter-temporally related. This is reasonable since there may be inter-temporal constraints such as minimum up/down times, ramp-rate constraints imposed on the system.

Based on the above nominal bid curves, $N$ sets of bid curves can be generated by perturbing the nominal bid curves as

$$\hat{b}_i^n(p_i(t)) = \overline{b}_i(\overline{p}_i) + \Delta b_i^n(p_i(t)), \quad n = 1, 2, \ldots, N, t = 1, 2, \ldots, T, \tag{5}$$

where $\Delta b_i^n(p_i(t))$ is a perturbation function. These $N$ sets of bid curves are evaluated and ranked by ordinal optimization. The estimated profit of each set of bid curves is calculated as

$$J_i^n = \sum_{t=1}^{T} \left[ \hat{\lambda}^n(b_i^n(t)) p_i^a(b_i^n(t)) - \sum_{m=1}^{M_i} (C_{im}(p_{im}^a(b_i^n(t)) - S_{im}(z_{im}(t))) \right], \tag{6}$$

where $\{\hat{\lambda}^n(b_i^n(t))\}$ is the MCPs associated with bid curves $\{b_i^n(\cdot)\}$.

To evaluate the profit $J_i^n$, a forecast model is needed to estimate the MCPs $\{\hat{\lambda}^n(b_i^n(t))\}$ based on available information such as bid curves $\{b_i^n(\cdot)\}$, weather forecast, available hydro energy, and possible strategies of competitors, etc. Since a bidder usually has a good record on its own historical bid curves and weather information, a regression or neural network-based forecasting model can be created. Game theoretic methods can also be developed to take into account market factors on top of the forecast model.

The major task in applying ordinal optimization is to construct the selected subset $S$ containing "good enough" bidding strategies with high probability, including the determination of its size $s$. There are two ways to pick $s$ sets of bidding strategies from a space of $N$ sets of perturbed bid curves: Blind Pick (BP) and Horse Race (HR). In Blink pick, $s$ sets of bidding strategies are randomly selected, whereas in Horse race $N$ sets of bid curves are preliminarily evaluated and the best $s$ sets of strategies are selected. For the BP method, the size $s$ can be determined in closed form (Ho, 1997a). The HR method, however, is often preferable since it can make use of results based on a simplified problem, and generally ends up with a smaller $s$ as compared to the BP method. To select $s$ good ones from the $N$ perturbed bidding strategies generated by (5), the profits defined by (6) are evaluated, and $s$ is determined by a regressed nonlinear equation to satisfy certain confidence requirement (Lau and Ho, 1997). Note that profit evaluation using (6) is a rough estimation since it is assumed that the generation award to a unit will be delivered by that unit. This is not necessary and may even be infeasible because individual operating constraints are not considered in the bidding process. A generation company can reallocate all its resource to meet its total generation award while satisfying individual constraints. Evaluating $N$ bidding strategies by using (6) is computationally efficient and the ordinal optimization method can guarantee that good enough strategies will be among the $s$ selected strategies.

More accurate evaluation is applied to the s selected bidding strategies. For each bidding strategy with the associated forecasted MCPs, a traditional generation scheduling problem is solved to estimate Bidder $i$'s profit as follows (Guan et al., 1992; Guan et al., 1997):

$$\underset{\{p_{im}(t)\}_{m=1}^{M_i}}{Min} \; C_i, \text{ with } C_i \equiv \sum_{t=1}^{T} \sum_{m=1}^{M_i} [c_{im}(p_{im}(t)) + S_{im}(z_{im}(t), t)], \tag{7}$$

subject to system demand constraints

$$\sum_{m=1}^{M_i} p_{im}(t) = P_d^n(t), \quad t = 1, 2, \ldots, T, \tag{8}$$

and other individual unit constraints. In the above, $p_{im}(t)$ is generation level of unit $m$ of Bidder $i$ at time $t$, and $P_d^n(t) = p_i^a(\cdot, t)$ the aggregated energy awarded by the PX at hour $t$. The estimated profit is then given by

$$\hat{J}^n = \sum_{t=1}^{T} \lambda^n(t) P_d^n(t) - C_i^n. \tag{9}$$

The best strategy is then selected from by evaluating those strategies in the subset $S$ based on the estimates of the MCP. Consequently, much less computational efforts are required to search through $S$ as opposed to searching through $\Theta$ since the size of $S$ is much smaller than that of $\Theta$. Note that the strategies in $S$ are selected by estimating roughly profit of each generation unit in (6) rather than solving the scheduling problem (7,9), where generation resources of the entire company are utilized to maximize the total profit. The ordinal optimization, however, can guarantee that good strategies are in $S$ with a high probability. Numerical testing is being performed to demonstrate the effectiveness of this ordinal optimization approach.

### *IV.2. Energy-Reserve Trade-off Decision*

As mentioned earlier, the capacity of a generating unit can be used to provide energy or reserve. The profit for providing a certain amount of reserve can be estimated based on forecasted reserve prices from the ISO ancillary market. The profit for providing the same amount of energy can also be estimated based on the MCP of the PX energy market and the actual generation costs. Both calculations, however, involve significant forecast errors, and can only be performed for a limited number of simulation runs. Ordinal optimization can thus be applied to effectively compare the two options similar to what was presented in the previous subsection. In this comparison, we only need to determine the preference order of these two options as opposed to the value of the difference between the two profits. However, the decision space is still very large since the amount of reserve to bid has to be determined. Ordinal optimization method can help reduce the computational efforts as in the case of energy bidding.

## V. A Game Theoretical Approach for Bidding and Self-Scheduling

### *V.1. Problem Description*

This section highlights bidding and self-scheduling of a *utility company* in New England, where the relationship between the ISO and bidders has been described in Section 1. The problem will be investigated from the viewpoint of a particular utility bidder, say Bidder 1, where it bids part of the energy to the ISO, and self-schedules the rest. For simplicity of presentation, it is assumed that Bidder 1 has only $M_1$ thermal units. Hydro and pumped-storage units, however, can be easily incorporated. The uncertainties caused by bids from other bidders and the ISO bid selection process are explicitly considered. The problem is then solved within a reduced game theoretical framework, where the ISO has a closed-form solution for a given probabilistic description of the bids, and the utility's problem is solved by using Lagrangian relaxation (Zhang *et al.*, 1998b).

To manage complexity, it is assumed that Bidder $i$'s bid curve for hour $t$ is represented by a *quadratic function*:

$$B_i(p_i(t)) = a_i(t)p_i^2(t) + b_i(t)p_i(t), \quad t = 1, 2, \ldots, T, \tag{10}$$

where $a_i(t)$ and $b_i(t)$ are nonnegative coefficients, and $p_i(t)$ the aggregated generation level for the market satisfying

$$0 \leq p_i(t) \leq \overline{p}_i(t). \tag{11}$$

In the above, $\overline{p}_i(t)$ is the maximum bid level at hour $t$. Bidder $i$'s load can be supplied by either the market or through self-scheduling. The part to be supplied by the market is denoted as $p_{iM}(t)$, a decision variable to be optimized. A bid for hour $t$ is thus represented by $\{a_i(t), b_i(t), \overline{p}_i(t), p_{iM}(t)\}$.

Bidder 1 does not have exact information about the bids submitted by others, but does have their probabilistic descriptions. It is assumed that for hour $t$ the market has $J$ *scenarios*, each described by

$$\beta^j(t) = \left\{ \left\lfloor a_i^j(t), b_i^j(t), \overline{p}_i^j(t), p_{iM}^j(t) \right\rfloor, \quad i = 2, 3, \ldots, I \right\}, \quad j = 1, \ldots, J. \tag{12}$$

The probability of event $\beta^j(t)$ is $\rho^j(t)$, satisfying $\sum_{j=1}^{J} \rho^j(t) = 1$. For simplicity, the scenarios for different hours are assumed to be independent.

The New England ISO model is similar to the California PX model as described by (2) and (3), i.e., to minimize the total cost subject to system demand constraints. In terms of the current notation, it is described by

$$\underset{\{p_i(t)\}}{Min}\ C, \text{ with } C = \sum_{i=1}^{I} \sum_{t=1}^{T} C_i(p_i(t)), \tag{13}$$

subject to (11) and

$$\sum_{i=1}^{I} p_i(t) = \sum_{i=1}^{I} p_{iM}(t), \quad t = 1, 2, \ldots, T. \tag{14}$$

Since Bidder 1 bids part of the energy to the ISO and self-schedules the rest, its model is a variation of (1):

$$\min_{\{a_1(t),b_1(t),\bar{p}_1(t),p_{1M}(t),\{p_{1m}(t)\}\}} J_1, \text{ with}$$

$$J_1 \equiv E\left[\sum_{t=1}^{T}\sum_{m=1}^{M_1}\{C_{1m}(p_{1m}(t)) + \lambda_M^*(t)\,(p_{1M}(t) - p_1(t))\}\right]. \tag{15}$$

In the above, $p_{1m}(t)$ is the generation level of Bidder 1's unit $m$ at hour $t$, $C_{1m}(\cdot)$ the cost function of the unit, and $\lambda_M^*(t)$ the MCP at hour $t$. The expectation is taken with respect to uncertain bidding parameters reflected through $\lambda_M^*(t)$ and $p_{1M}(t)$. The above minimization is subject to individual unit constraints [see, e.g., Guan et al. (1992)] and the following load balance constraint:

$$\sum_{m=1}^{M_1} p_{1m}(t) + E\,(p_{1M}(t) - p_1(t)) = P_{1d}(t). \tag{16}$$

In the above, $p_{1d}(t)$ is the demand for Bidder 1 at time $t$, and is assumed to be known for simplicity.

### V.2. *The ISO Solution*

From the ISO's viewpoint, its problem is deterministic since the ISO solves the problem after all the bids have been submitted. From the game theoretical framework, however, Bidder 1 needs to "solve" the ISO problem in the absence of complete information. One way to overcome this lack of information is to solve the ISO problem for each possible scenario and then aggregates the results. Solution under a particular scenario $\{\beta^j(t)\}_{t=1}^T$ is derived first, where the index $j$ is omitted when appropriate.

For simplicity of presentation, bounds on the maximum bid levels (11) are assumed to be inactive and therefore ignored in the following derivation [See Zhang et al. (1998b) for more general derivation]. The ISO bid selection process can then be solved by using Lagrangian relaxation. Using multipliers $\{\lambda_M(t)\}$ to relax (14), the ISO Lagrangian is formed as

$$L_{ISO} = \sum_{t=1}^{T}\left\{\sum_{i=1}^{I} C_i(p_i(t)) + \lambda_M(t)\left(\sum_{i=1}^{I} p_{iM}(t) - p_i(t)\right)\right\}. \tag{17}$$

With $\{\lambda_M(t)\}$ given, (17) can be decomposed into individual subproblems, one for each bidder. The Bidder $i$'s subproblem is

$$L_i = \min_{\{p_i(t)\}} \sum_{t=1}^{T}\left\{a_i(t)p_i^2(t) + b_i(t)p_i(t) - \lambda_M(t)p_i(t)\right\}. \tag{18}$$

The solution for (18) is

$$p_i^*(t) = \frac{\lambda_M(t) - b_i(t)}{2a_i(t)}. \tag{19}$$

With the above closed form solution for each subproblem, it is not necessary to iteratively update the multipliers $\{\lambda_M(t)\}$. Rather, closed form solution for $\{\lambda^*_M(t)\}$ can be obtained by substituting (19) into (14). After several steps of derivation, one obtains

$$\lambda^*_M(t) = \frac{(c_0(t) + 2p_{1M}(t))a_1(t) + b_1(t)}{c_1(t)a_1(t) + 1}, \quad \text{and} \tag{20}$$

$$p^*_1(t) = \frac{c_0(t)/2 + p_{1M}(t) - c_1(t)b_1(t)/2}{c_1(t)a_1 + 1}, \tag{21}$$

where

$$c_0(t) \equiv \sum_{i=2}^{I}\left[2p_{iM}(t) + \sum_{i=2}^{I}\frac{b_i(t)}{a_i(t)}\right], \quad \text{and} \tag{22}$$

$$c_1(t) \equiv \sum_{i=2}^{I}\frac{1}{a_i(t)}. \tag{23}$$

Bidder 1 may be a buyer or a seller depending on the sign of $(p_{1M}(t) - p^*_1(t))$ (positive for buying).

### V.3. *The Bidding and Self-Scheduling Strategy*

Given the above results on $\{\lambda^*_M(t)\}$ and $\{p^*_1(t)\}$ for a particular scenario as functions of all the bids submitted, Bidder 1's problem (15) is similar to a traditional unit commitment problem. It can be solved by introducing another set of multipliers $\{\lambda_1(t)\}$ to relax Bidder 1's its system demand constraints (16). Bidder 1's Lagrangian can be written as

$$\begin{aligned} L_1 = {} & E\left\{\sum_{t=1}^{T}\sum_{m=1}^{M_1}\left[C_{1m}(p_{1m}(t)) + \lambda^*_M(t)\left(p_{1M}(t) - p^*_1(t)\right)\right]\right\} \\ & + \sum_{t=1}^{T}\lambda_1(t)\left[P_{1d}(t) - \sum_{m=1}^{M_1}p_{1m}(t) - E\left(p_{1M}(t) - p^*_1(t)\right)\right]. \end{aligned} \tag{24}$$

In the above, $\{\lambda^*_M(t)\}$ and $\{p^*_1(t)\}$ are from (20) and (21), respectively, for a particular scenario, and the expectation is taken across all possible scenarios. Since the RHS of (24) is additive for a given set of multipliers $\{\lambda_1(t)\}$, a two-level algorithm can be developed. At the low level, individual thermal subproblems are formed, one for each unit. These thermal subproblems are similar to those in traditional hydro-thermal scheduling, and can be solved by using dynamic programming as presented in Guan et al. (1992). On additional subproblem, the "bidding subproblem," is obtained as

$$\min_{a_1(t),b_1(t),p_{1M}(t)} L_{1B}, \quad \text{with } L_{1B} \equiv E\left[\sum_{t=1}^{T}\left[\lambda^*_M(t) - \lambda_1(t)\right]\cdot\left[p_{1M}(t) - p_1(t)\right]\right]. \tag{25}$$

At the high level, the multipliers $\{\lambda_1(t)\}$ are iteratively updated based on the degrees of demand constraint violation.

*Table 1.* Bidding parameters of Bidder 2.

| | $b_2(t)$(%) | | | $p_{M2}(t)$(%) | | |
|---|---|---|---|---|---|---|
| Case | L | M | H | L | M | H |
| 1 | 80 | 100 | 120 | 20 | 30 | 40 |
| 2 | 80 | 100 | 120 | 10 | 30 | 50 |
| 3 | 60 | 100 | 140 | 20 | 30 | 40 |
| 4 | 100 | 120 | 140 | 20 | 30 | 40 |
| 5 | 20 | 40 | 60 | 20 | 30 | 40 |

$b_2(t)$(%): $b_2(t)$ as a percentage of Bidder 1's self-scheduling marginal cost without market. $p_{2M}(t)$(%): $p_{2M}(t)$ as a percentage of Bidder 1's load.

To solve the new bidding subproblem (25), we note that $\lambda_M^*(t)$ and $p_1^*(t)$ are functions of all the bids submitted (including the ones submitted by Bidder 1) for each possible scenario under consideration. Bidder 1's expected cost across all scenarios $L_{1B}$ is therefore a function of Bidder 1's parameters $\{a_1(t), b_1(t), \overline{p}_1(t), p_{1M}(t)\}$. The optimal set of bidding parameters can thus be obtained by numerical optimization using, for example, a gradient method. The multipliers $\{\lambda_1(t)\}$ are then iteratively updated at the high level based on subproblem solutions by using, for example, the Bundle Trust Region Method (Schramm and Zowe, 1992; Zhang et al., 1998a). It can be shown that the bidding subproblem has inherent degeneracy with an infinite number of equivalent solutions. For details, please see Zhang et al. (1998b).

### V.4. *Highlights of Numerical Results*

The method presented above has been implemented in C++ based on our original hydrothermal scheduling code presented in Guan et al. (1992) and Zhang et al. (1998a). A data set provided by Northeast Utilities (NU) is used to demonstrate the capabilities of the method in handling various market situations. To simplify the testing, all other market bidders are aggregated as Bidder 2 with three possible bidding strategies, bidding low (L), bidding medium (M), and bidding high (H) with equal probability 1/3.

The value of Bidder 2's parameters $a_2(t)$ is 0.09 for the High strategy, 0.05 the Medium strategy, and 0.01 the Low strategy. With Case 1 as the base case, four additional cases are created by varying Bidder 2's parameters $b_2(t)$ and $p_{2M}(t)$ to test various market situations. The value of $b_2(t)$ as a percentage of Bidder 1's self-scheduling marginal cost (self-scheduling all its units without participating in the market) and the value of $p_{2M}(t)$ as a percentage of Bidder 1's load are provided in Table 1. The method is compared with the "mean method" that considers Bidder 2's bidding model as deterministic with each parameter set to its mean value. Comparison of the results based on 100 simulation runs is presented in Table 2.

Case 2 represents a volatile market with large variances on $p_{2M}(t)$, and the saving of the stochastic method over the mean method is increased as compared to the base case.

*Table 2.* Cost comparison of the mean method and the stochastic method.

| Case | Mean Method ($) | Stochastic Method ($) | Savings (%) |
|---|---|---|---|
| 1 | 106,169 | 105,759 | 0.39% |
| 2 | 100,930 | 100,365 | 0.56% |
| 3 | 102,420 | 101,431 | 0.97% |
| 4 | 103,076 | 102,682 | 0.38% |
| 5 | 105,334 | 104,800 | 0.51% |

Case 3 also represents a volatile market with large variances on $b_2(t)$, and the saving is also increased as compared to the base case. Cases 2 and 3 thus illustrate that the method works better than the mean method in volatile markets. Case 4 represents a high-cost market with the mean value of $b_2(t)$ increased 20% above the base case, and Case 5 represents a low-cost market with the mean value of $b_2(t)$ decreased 40% below the base case. The savings over the mean method for these two cases are also significant, illustrating that the method works well for both high-cost and low-cost market situations.

The average CPU time for the mean method is 70 seconds, and for the stochastic method 95 seconds. The CPU time requirements for the two methods are therefore close since the only stochastic subproblem is the bidding subproblem which is solved by using a gradient method. Thermal subproblems as well as high level multiplier updating are the same for both methods. Numerical testing therefore shows that the method is computationally efficient, and good bidding and self-scheduling results for practical problems are obtained.

The above results are presented within a "reduced" game theoretical framework since although the uncertainties of other bidders and the ISO's bid selection process are explicitly considered, the exact "gaming" phenomenon is not captured. How to effectively model the gaming situations, what is the appropriate equilibrium concept under the mixture of day-ahead and hour-ahead markets, and how to develop computationally efficient algorithms to obtain good strategies to maximize the profit while reducing risk are challenging issues. A different approach is to circumvent the gaming phenomenon by developing an intelligent MCP forecasting model, and using the forecasted MCPs to solve a bidder's problem (15). This MCP prediction model, however, will be much more complicated than a traditional load forecasting model in view of the complexity and volatility of the power markets.

## VI. Generation Scheduling and Energy Delivery Capability

As mentioned earlier, a generation company can treat its aggregated energy awards as demand and perform traditional generation scheduling to obtain *hourly generation levels* for its units. Two difficulties may occur in view that large steam or nuclear units generally have ramp rate constraints limiting the rate of change of generation levels. First, the generation scheduling problem may not admit a feasible solution since unit ramp rates are mostly ignored by the PX. Second and a more subtle issue is that even if the problem admits a solution, the energy delivery obligation as awarded by the PX may not be fulfillable. This is because traditional generation scheduling obtains *hourly* generation levels which are assumed to be constant for each hour. In the PX market, however, buy and sell are processed in terms of time-varying *energy* to meet the constantly changing system demand. A schedule satisfying the hourly ramp rate of a traditional scheduling problem may not be able to meet individual units' limits on actual energy delivery. Energy delivery capability or the realizability of a generation schedule and its relation to traditional unit ramp rate constraints have been investigated in Gaun et al. (1999). Based on the "Maximum Principle" of optimal control theory, necessary and sufficient conditions have been established to check if an energy delivery schedule can be realized.

## VII. Conclusions

The deregulation and reconstruction of electric power industry world-wide have raised many challenging issues for the economic and reliable operation of electric power systems. Traditional unit commitment or hydrothermal scheduling problems are integrated with resource bidding, and the development of optimization-based bidding strategies is at a very preliminary stage. Ordinal optimization seeks "good enough" bidding strategies with high probabilities, and will turn out to be effective in handling market uncertainties with much reduced computational efforts. Game theoretic framework combining with advanced optimization techniques shall allow us to directly model competition in the deregulated market, and provide much needed insights for the synthesis of effective bidding and self-scheduling strategies. Although the two specific methods presented in the paper represent significant progress made thus far, the area is wide open for creative research to make the deregulated market a true success.

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