An Optimization-Based Algorithm for Scheduling Hydrothermal Power Systems with Cascaded Reservoirs and Discrete Hydro Constraints

Xiaohong Guan, Ernan Ni, Renhou Li
Institute of Systems Engineering
Xi'an Jiaotong University
Xi'an, Shaanxi, 710049, China

Peter B. Luh
Department of Electric & Systems Engineering
University of Connecticut
Storrs, CT, 06269-2157, USA

Abstract. **An optimization-based algorithm is presented for the short-term scheduling of hydrothermal power systems using the Lagrangian relaxation technique. This paper concentrates on the solution methodology for hydro subproblems with cascaded reservoirs and discrete hydro constraints. Continuous reservoir dynamics and constraints, discontinuous operating regions, discrete operating states, and hydraulic coupling of cascaded reservoirs are considered in an integrated fashion. The key idea is to substitute out the reservoir dynamics and to relax the reservoir level constraints by using another set of multipliers, making a hydro subproblem unit-wise and stage-wise decomposable. The optimal generation level for each operating state at each hour can be obtained simply by minimizing a single variable function. Dynamic programming is then applied to optimize the operating states across the planning horizon with a small number of well-structured transitions. A modified subgradient algorithm is used to update multipliers. After the dual problem converges, the feasible solution to the hydro power subsystem is obtained by using a network flow algorithm, with operating states obtained in the dual solutions, and possibly adjusted by heuristics. Numerical testing based on practical system data sets show that this method is efficient and effective for dealing with hydrothermal systems with cascaded reservoirs and discrete hydro constraints.**

***Key Words*:** Scheduling of hydrothermal power systems, Scheduling of cascaded reservoirs, Mixed-integer programming

I. INTRODUCTION

Hydrothermal scheduling is an important daily activity for utilities because of its significant economic impact. It aims at determining the commitment and generation of all schedulable power resources over a planning horizon to meet the system demands and reserve requirements. The goal is to minimize the total generation cost. To solve this NP-hard mixed integer programming problem, many algorithms have been developed. Lagrangian relaxation and its extension are among the most successful ([1-5]).

In Lagrangian relaxation, the problem is converted to a two-level optimization problem. The low level consists of a number of subproblems, one for each thermal unit or river catchment, and the high level is to optimize the multipliers. Generally hydro subproblems are more difficult to solve than thermal subproblems. A hydro unit has reservoir dynamics and constraints coupling the hourly generation across time. Operating in certain regions may not be permitted for security or efficiency reasons, resulting in discontinuous regions or even discrete generation levels. Furthermore, since the reservoirs in a river catchment are hydraulically coupled, the generation of an upstream unit affects the reservoir levels of the downstream units. Finally, to prevent wear-off caused by frequent starting up and shutting down, hydro units may also have minimum up/down time requirements resulting in discrete operating states, and start-up and shut-down costs as described in [17]. Integrated consideration of the above factors within the Lagrangian relaxation framework is a very challenging issue, and is the focus of this paper.

Many methods have been developed to solve hydro subproblems, including dynamic programming (DP), network flow, and standard mixed integer programming methods. DP is flexible and can handle the above mentioned constraints in a straightforward way ([6-7]). However, DP needs to discretize reservoir levels. For systems with cascaded reservoirs and discrete operating states, the state space expands exponentially with problem size, causing DP to suffer from the "curse of dimensionality" for practical applications. Network flow is the most widely used method for hydro power scheduling ([8-12]). Its major limitation, however, is its inability to deal with discontinuous operating regions and discrete operating states, although continuous non-network constraints can be approximated in a network flow formulation as in [8]. General linear and nonlinear programming methods encounter similar limitations as network flow ([13]). Heuristics are sometimes used to obtain hydro commitment, or to post-process solutions obtained by network flow. A combination of network flow, dynamic programming and heuristic method is reported in [17]. A genetic aided scheduling method is presented in [15], and a multi-pass dynamic programming method with special state approximation is developed in [16]. These two later methods do not consider discontinuous operating regions or discrete operating states. Recently, a commercial mixed-integer linear programming package is used to generate hydro schedules where integer variables are handled by partial enumeration such as branch-and-bound methods ([14]). When the problem is large and coordination with thermal units is needed, computational requirements may become too large for practical applications. In our previous work, efficient algorithms for hydro and pumped-storage subproblems were developed and embedded in the daily scheduling package of Northeast Utility Service Company (NU) ([20], [21]). However, the hydraulic coupling of cascaded reservoirs was not considered.

Relaxing reservoir dynamics and hydraulic coupling by using additional sets of Lagrangian multipliers is an efficient and systematic way to handle discontinuous operating regions, discrete operating states, and hydraulic coupling of

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reservoirs ([18-21]). By extending this idea, a new algorithm is presented in this paper for solving hydro subproblems with cascaded reservoirs within the Lagrangian relaxation framework of hyrdothermal scheduling. The basic idea is to substitute out the reservoir dynamics and relax reservoir level limits by using another set of Lagrangian multipliers. This relaxation is computationally efficient and numerically stable as compared to relaxing reservoir dynamics ([18]), since reservoir level limits are inequality constraints and many of them may not be active. The sub-Lagrangian associated with a river catchment then becomes unit-wise and stage-wise decomposable, and the cost function of an individual unit involves only the multipliers associated with its own and its direct downstream unit. For each discrete operating state of a unit at an hour, the optimal generation can be obtained by optimizing a single variable function. DP is then used to optimize the discrete operating states of the unit across the scheduling horizon with a very small number of well-structured transitions. The multipliers are updated at an intermediate level. A nonlinear network flow algorithm is then used to generate a feasible schedule at the convergence of the dual problem, with operating states obtained in the dual solution possibly adjusted by heuristics. Numerical testing based on practical size data sets shows that this method is computationally efficient to handle hydro subproblems with cascaded reservoirs and discrete hydro constraints.

II. Problem Formulation

Consider a hydrothermal power system with I thermal units, J hydro reservoirs and K pumped-storage units. Without loss of generality, assume that there is only one river catchment since with given Lagrange multipliers, hydro subproblems associated with different river catchments are independent. The hydro units in a plant are aggregated as one unit. It is required to determine the operating states and generation/pumping levels of all units over a specified period T. The goal is to minimize the total generation cost subject to system demand and reserve requirements, and individual unit constraints. The time unit is one hour and the planning horizon may vary from one day to a week.

To formulate the problem mathematically, the notation to be used is first introduced:

I: number of thermal units;
J: number of hydro units;
K: number of pumped-storage units;
$C_{ti}(p_{ti}(t))$: fuel cost of thermal unit i for generating power $p_{ti}(t)$ at time t, in dollars;
$P_d(t)$: system demand at time t, in MW;
$P_r(t)$: system spinning reserve requirement at time t, in MW;
$p_{hj}(w_j(t))$: power generated by hydro unit j at time t, in MW;
$p_{pk}(t)$: power generated or used for pumping by pumped-storage unit k at time t, in MW;
$p_{ti}(t)$: power generated by thermal unit i at time t, in MW;
$r_{hj}(p_{hj}(w_j(t)))$: spinning reserve contribution of hydro unit j at time t, in MW;
$r_{pk}(p_{pk}(t))$: spinning reserve contribution of pumped storage unit k, at time t, in MW;
$r_{ti}(p_{ti}(t))$: spinning reserve contribution of thermal unit i at time t, in MW;
$S_{ti}(t)$: start-up cost of thermal unit i at time t, in dollars;
$S_{hj}(x_{hj}(t))$: start-up cost of hydro unit j at time t, in dollars;
t: time index, $t = 1,2,\ldots T$.
T: time horizon under consideration, in hours;
$v_j(t)$: reservoir level of hydro reservoir j at time t;
$\bar{v}_j$: maximum reservoir level of hydro unit j;
$\underline{v}_j$: minimum reservoir level of hydro unit j;
v_j^0: initial reservoir level of hydro unit j;
v_j^T: terminal reservoir level of hydro unit j;
$w_j(t)$: water discharge of hydro unit j at time t;
$\bar{w}_j(t)$: maximum water discharge for hydro unit j at time t;
$\underline{w}_j(t)$: minimum water discharge of hydro unit j at time t;
$u_{hj}(t)$: discrete decision variable of hydro unit j at time t, "1" for up, "-1" for down;
$x_{hj}(t)$: state of hydro unit j at time t, denoting number of hours that the unit has been on (positive) or off (negative);
$\bar{\varsigma}_j$: minimum up time of hydro unit j, in hours;
$\underline{\varsigma}_j$: minimum down time of hydro unit j, in hours;
$\xi_j(t)$: natural inflow to the reservoir of hydro unit j at time t;
τ_j: time required for the water discharged from reservoir j to reach its direct down stream reservoir, in hours.
B: reservoir connection matrix with element $b_{ij} = 1$ if hydro unit j is a direct up stream of unit i, $b_{ij} = 0$, otherwise.

The scheduling problem can then be formulated as the following mixed integer programming problem

$$\min_{p_{ti}(t), p_{hj}(t), p_{pk}(t)} C, \text{ with}$$

$$C = \sum_{t=1}^{T} \{\sum_{i=1}^{I} [C_{ti}(p_{ti}(t)) + S_{ti}(t)] + \sum_{j=1}^{J} S_{hj}(x_{hj}(t))\}, \quad (1)$$

subject to system wide demand and reserve requirements and individual unit constraints to be described below.

System demand:

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{j=1}^{J} p_{hj}(t) + \sum_{k=1}^{K} p_{pk}(t) = P_d(t), \quad t = 1,2,\ldots.T. \quad (2)$$

Spinning reserve requirement

$$\sum_{i=1}^{I} r_{ti}(p_{ti}(t)) + \sum_{j=1}^{J} r_{hj}(p_{hj}(w_j(t))) + \sum_{k=1}^{K} r_{pk}(p_{pk}(t)) \geq P_r(t),$$
$$t = 1,2,\ldots.T. \quad (3)$$

Thermal and pumped-storage constraints

Detailed descriptions of individual constraints for thermal and pumped-storage units can be found in [20, 22].

Constraints for river catchment and hydro units

-- Water balance equation

$$V(t+1) = V(t) + \mathbf{B}w_d(t,\tau) - w(t) + \xi(t), \quad (4)$$

where $V(t)$ is the stack vector of $v_j(t)$, and $w(t)$ the stack vector of $w_j(t)$, and $w_d(t,\tau)$ the delayed water discharge to downstream reservoirs defined as:

$$w_d(t,\tau) = [w_1(t-\tau_1) \quad w_2(t-\tau_2) \quad \ldots w_J(t-\tau_J)]^{\mathrm{T}}.$$

Equation (4) requires conservation of flow among reservoirs in the river catchment. Without loss of generality, it is implicitly assumed that the time required for water to travel from a reservoir to a reservoir direct downstream is far less than the scheduling horizon.

-- Reservoir level limits:

$$\underline{V} \leq V(t) \leq \overline{V} \quad (5)$$

-- Initial and terminal reservoir levels:

$$V(0) = V^0, \quad (6a)$$

$$V(T) = V^T \quad (6b)$$

-- Operating regions:

$$\underline{w}_j(t) \leq w_j(t) \leq \overline{w}_j(t), \quad \text{if } x_{hj}(t) > 0, \quad (7a)$$

$$w_j(t) = 0, \quad \text{if } x_{hj}(t) < 0. \quad (7b)$$

Although only two operating regions, (7a) and (7b), are considered, the method developed can be directly used to solve problems with multiple operating regions or even discrete output levels caused by restricted loading bands or operating efficiency.

-- Minimum up/down time

$$u_{hj}(t) = 1 \quad \text{if } 1 \leq x_{hj}(t) \leq \overline{\varsigma}_{hj}, \quad (8a)$$

$$u_{hj}(t) = -1 \quad \text{if } -\underline{\varsigma}_j \leq x_{hj}(t) \leq -1, \quad (8b)$$

which discourage frequent start-ups and shut-downs.

-- State transitions

$$x_{hj}(t+1) = x_{hj}(t) + u_{hj}(t) \quad \text{if } x_{hj}(t)u_{hj}(t) > 0, \quad (9a)$$

$$x_{hj}(t+1) = u_{hj}(t) \quad \text{if } x_{hj}(t)u_{hj}(t) < 0. \quad (9b)$$

The water-power conversion is approximated by the following concave quadratic function

$$p_{hj}(w_j(t)) = a_j w_j^2(t) + b_j w_j(t) + c_j. \quad (10)$$

The reserve contribution of a hydro unit is calculated as the difference between generation capacity and the generation level

$$r_{hj}(w_j(t)) = \overline{p}_{hj}(w_j(t)) - p_{hj}(w_j(t)), \quad (11)$$

if the unit is up, and is zero if the unit is down.

III. Solution Methodology

III. 1 The Lagrangian Relaxation Framework

The basic idea of Lagrangian relaxation is to relax system-wide demand (2) and reserve requirements (3) by using Lagrange multipliers and to form a hierarchical optimization structure. Given the multipliers, the low level consists of individual thermal and pumped-storage units, and hydro river catchments as in [20-22]. The high level dual problem is to update the multipliers. The methods for solving thermal and pumped-storage subproblems have been presented in detail in [20, 22], and only solution methodology for the hydro subproblem is presented here.

III. 2 Solving hydro subproblem

The hydro subproblem is presented within the Lagrangian relaxation framework:

$$\min_{w_j(t)} L_h, \text{ with}$$

$$L_h = \sum_{t=1}^{T} \{-\lambda(t)\sum_{j=1}^{J} p_{hj}(w_j(t)) - \mu(t)\sum_{j=1}^{J} r_{hj}(p_{hj}(w_j(t)))$$
$$+ \sum_{j=1}^{J} S_{hj}(x_{hj}(t))\} \quad (12)$$

subject to constraints (4)-(9).

The key idea to solve the subproblem is to substitute out the water balance equation (4) and to relax the reservoir level limits of (5) and terminal reservoir level (6b) by using additional sets of Lagrangian multipliers. An intermediate level is thus created. The hydraulic coupling among reservoirs is "cut-off," and the river catchment subproblem then becomes unit-wise and stage-wise decomposable. At the low level, decomposed subproblems for individual units are efficiently solved by first optimizing a series of single variable functions, and then applying DP with a small number of states and well-structured transitions without discretizing the reservoir levels. These multipliers are updated at the intermediate level. At the convergence of the dual problem, a nonlinear network flow algorithm is then applied to generate a near optimal feasible schedule, with the discrete operating states obtained in the dual problem, and possibly adjusted by heuristics.

Problem decomposition

After relaxing the reservoir level limits (5) and terminal reservoir level (6b), the sub-Lagrangian (12) becomes:

$$L_h = L_h + \sum_{t=1}^{T-1}[\beta_1{}^{\mathrm{T}}(t)(\underline{V} - V(t)) + \beta_2{}^{\mathrm{T}}(t)\ (V(t) - \overline{V})]$$
$$+\beta_3{}^{\mathrm{T}}[V^T - V(T)], \quad (13)$$

where $\beta_1(t)$, $\beta_2(t)$ $(t = 1,2,\ldots,T\text{-}1)$ and β_3 are multipliers defined as

$$\beta_1(t) \equiv [\beta_{11}(t),\ \beta_{12}(t),\ \ldots\beta_{1J}(t)]^{\mathrm{T}},$$

$$\beta_2(t) \equiv [\beta_{21}(t),\ \beta_{22}(t),\ \ldots\beta_{2J}(t)]^{\mathrm{T}}, \quad t = 1,2\ldots T\text{-}1$$

and

$$\beta_3 \equiv [\beta_{31},\ \beta_{32},\ \ldots\beta_{3J}]^{\mathrm{T}},$$

The water balance equation (4) is substituted out to obtain

$$V(t) = V(0) + \sum_{n=1}^{t} \mathbf{B}w_d(n,\tau) - \sum_{n=1}^{t} w(n) + \sum_{n=1}^{t} \xi(n) \quad (14)$$

where terms such as $w_j(n-\tau_j)$ for $n-\tau_j \le 0$ in (14) are water discharges from the previous scheduling cycle, and are considered as given. By substituting (14) into (13), the Lagrangian can be rewritten as

$$L_h = \sum_{t=1}^{T-1}[\beta_1^{\mathrm{T}}(t)\underline{V} - \beta_2^{\mathrm{T}}(t)\overline{V}] + \beta_3^{\mathrm{T}}[V^T - V^0 - \sum_{n=1}^{T}\xi(n)]$$
$$+ \sum_{t=1}^{T-1}\{(\beta_2^{\mathrm{T}}(t) - \beta_1^{\mathrm{T}}(t))[V^0 + \sum_{n=1}^{t}\xi(n)]\} + \sum_{j=1}^{J} L_{hj}, \quad (15)$$

where

$$L_{hj} \equiv \sum_{t=1}^{T}[-\lambda(t)p_{hj}(w_j(t)) - \mu(t)r_{hj}(w_j(t))]$$
$$- \sum_{t=\tau_j+1}^{T} w_j(t-\tau_j)b_j^{\mathrm{T}}\beta_3 + \sum_{t=1}^{T} w_j(t)e_j^{\mathrm{T}}\beta_3$$
$$+ \sum_{t=1}^{T-1}\{[\sum_{n=1}^{t} w_j(n-\tau_j)]b_j^{\mathrm{T}}(\beta_2(t) - \beta_1(t))\}$$
$$+ \sum_{t=1}^{T-1}\{[\sum_{n=1}^{t} w_j(n)]e_j^{\mathrm{T}}(\beta_1(t) - \beta_2(t))\} + \sum_{t=1}^{T} S_{hj}(x_{hj}(t)), \quad (16)$$

e_j is a unit vector, i.e., its jth element is one and all the other elements are zeros; b_j is the jth column vector of connection matrix **B**. With multipliers λ, μ, β_1, β_2 and β_3 given, the sub-Lagrangian in (15) is unit-wise decomposable.

Suppose that reservoir with index j_d is directly downstream to reservoir j, then the subproblem for unit j is described as

$$\min_{w_j(t), u_{hj}(t)} L_{hj}, \text{ with } L_{hj} = \sum_{t=1}^{T}[h_j(w_j(t)) + S_{hj}(x_{hj}(t))]$$

where

$$h_j(w_j(t)) \equiv -\lambda(t)p_{hj}(w_j(t)) - \mu(t)r_{hj}(p_{hj}(w_j(t)))$$
$$+\beta_{3j}w_j(t) + \{\sum_{n=t}^{T-1}[\beta_{1j}(n) - \beta_{2j}(n)]\}w_j(t)$$
$$-\beta_{3j_d}w_j(t) + \{\sum_{n=t+\tau_j}^{T-1}[\beta_{2j_d}(n) - \beta_{1j_d}(n)]\}w_j(t),$$
$$t = 1,2\ldots T-1, \quad (17a)$$

and

$$h_j(w_j(T)) \equiv -\lambda(T)p_{hj}(w_j(T)) - \mu(T)r_{hj}(p_{hj}(w_j(T)))$$
$$+\beta_{3j}w_j(T), \quad (17b)$$

subject to its individual constraints (5-9). Note that (17a) and (17b) contain multipliers associated with unit j and its direct downstream unit only. A hydro unit is thus coordinated with other units within the same river catchment by the multipliers associated with its direct downstream unit.

Solving an individual unit subproblem

To model the discrete dynamics, the concept of "operating state" is introduced following what was used for thermal units with minimum up/down constraints ([22]). A state for a hydro unit is defined as the number of hours that the unit has been up (positive) or down (negative). Since the unit can be kept on or shut down after it has been up for $\bar{\varsigma}_j$ hours, the number of up states needed is the minimum up time. Similarly the number of down states is the minimum down time. By combining the above analysis, the state transition diagram can be constructed as in Fig. 1, where each node represents a state, and start-up and shut-down costs are associated with edges.

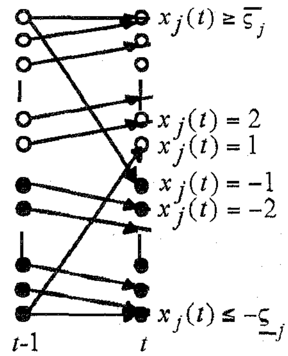

Fig. 1 The state transition diagram

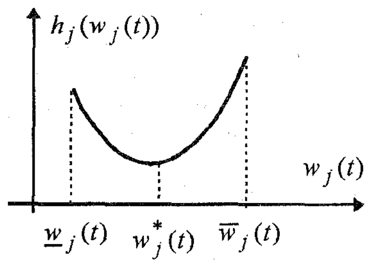

Fig. 2 Function $h_j(w_j(t))$

Based on the water-power conversion function $p_{hj}(w_j(t))$ and reserve contribution $r_{hj}(p_{hj}(w_j(t)))$, the stage-wise cost function $h_j(w_j(t))$ in (17) is depicted in Fig. 2. The optimal water discharge at time t for a particular operating region can then be obtained by

$$w_j^*(t) = \operatorname{argmin} h_j(w_j(t)), \quad (18)$$

subject to the range of the operating region.

After the optimal generation level for each operating region has been obtained for each hour, the associated cost function $h_j(w_j(t))$ can be calculated. Based on the state transition diagram of Fig. 1, dynamic programming can then be applied to optimize states across hours as in [20, 22]. Similarly, states can be extended to commit and dispatch the aggregated hydro units as in [18]. The optimal water discharge over the entire scheduling horizon can thus be obtained without discretizing reservoir levels.

III. 3 Updating the multipliers

A modified subgradient method with adaptive step sizing is applied to update the multipliers $\beta_1, \beta_2, \beta_3$ associated with reservoir level limits at the intermediate level, and multipliers λ, μ associated with system demand

and reserve requirements at the high level. This method has been described in detail in [20-22]. The subgradients for $\beta_1, \beta_2, \beta_3$ are

$$g_{\beta_1(t)} = \underline{V} - V(t), \quad (19)$$

$$g_{\beta_2(t)} = V(t) - \overline{V}, \quad (20)$$

and

$$g_{\beta_3} = V^T - V(T), \quad (21)$$

respectively. It should be noted that these multipliers are needed for the hydro subproblem only.

III. 4 Obtaining Feasible Solutions

The subproblem solutions obtained from Lagrangian relaxation are generally infeasible, i.e., the relaxed constraints (2, 3, 5, 6b) may not always be satisfied. To obtain a good feasible solution, a feasible hydro schedule is first obtained. With the operating states obtained in the dual solution fixed, the network flow algorithm ([8-12]) can usually generate feasible hydro schedules. However, in some cases, too many idle hours of a unit may cause forced spillage, and too many up hours may result in no feasible schedule even with minimum generation. In these cases, a heuristic method is applied to modify operating states. The adjustment is accomplished from up- to down-stream reservoirs as in [16]. The number of states to be changed is estimated according to the total mount of "spillage or under-minimum-level water" with the given operating regions. The cost increase of changing a state is calculated based on the costs asscociated with states in (17a) and (17b) to provide quantitative information ([23]). For example, if changing from a down state to an up state is necessary, the hour with the minimum cost increase while satisfying the minimum down time will be selected. The network flow algorithm is then applied to generate a feasible schedule satisfying all continuous hydro constraints (4-7a) including the terminal reservoir levels with the adjusted states.

With the feasible hydro schedule fixed, the thermal and pumped-storage units are adjusted to meet the system demand and reserve requirements as in [20, 22].

IV. Implementation and Testing Results

The algorithm is implemented in FORTRAN on HP 715/33 workstations. Numerical testing is performed using modified billing data sets from Northeast Utility Service Company (NU). There are about 70 thermal units, 1 large pumped-storage unit, and 7 hydro units belonging to one river catchment as shown in Fig. 3. The system features have been presented in [20]. Two sets of data were selected from the NU billing database and modified to form a cascaded river catchment.

Testing results are summarized in Table 1. Cases 2 and 4 are obtained by using the method developed in this paper (Lagrangian relaxation + network flow or LNF), and are compared with Cases 1 and 3 obtained by using a pure network flow algorithm (NF). As stated in Section I, the NF algorithm cannot handle hydro units' minimum down time (MDT), and the minimum water discharge has to be set to 0. It can be seen that the costs of the LNF are no more than 0.7% of the corresponding NF costs, which in fact are lower bounds to optimal costs in view of the absence of MDT and

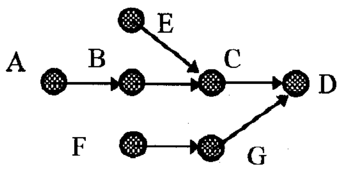

Fig. 3 Reservoirs in the river catchment

Table 1 Testing Results

Date set	Jan. W3, 1991		Oct. W2, 1991	
Case	1	2	3	4
MDT(hr)*	1, 1, 1,1	6,4,2,4	1, 1, 1,1	6,4,2,4
Cost($)	4,654,993	4,663,640	4,441,612	4,450,913
IT	47	54	45	65
CPU(s)	460	502	426	540

* MDT for Reservoirs A, D, F and G.
IT: number of high level iterations

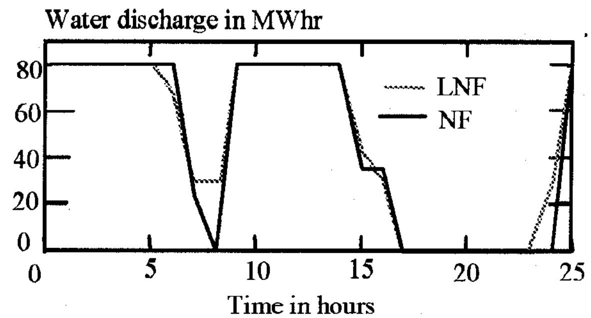

Fig. 4 Water discharge of reservoir A

zero minimum water discharge. The schedules obtained by LNF are therefore near optimal. The CPU times are in the range of a few minutes on a low-end workstation, efficient for daily scheduling.

The generation levels of Reservoir A in Case 1 and Case 2 are presented in Fig. 4. For a better view, only one day is shown. Violations of MDT and minimum discharge by using the NF algorithm are observed at hour 7 and hour 8. It can also be observed that the schedule produced by LNF algorithm does satisfy the MDT and minimum discharge constraints.

V. CONCLUSIONS

An optimization-based algorithm has been presented for scheduling hydro units with cascaded reservoirs and discrete hydro constraints within the Lagrangian relaxation framework. The algorithm can systematically deal with discontinuous operating regions and discrete operating states without discretizing reservoir levels. Numerical testing results based on a practical system show that the algorithm is computationally efficient, and near optimal schedules are obtained.

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VII. REFERENCES

[1] A. Cohen and V. Sherkat, "Optimization-Based Methods for Operations Scheduling," *Proceedings of IEEE*, Vol. 75, No. 12, 1987, pp. 1574-1591.

[2] J. J. Shaw and D. P. Bertsekas, "Optimal Scheduling of Large Hydrothermal Power Systems," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-104, 1985, pp. 286-293.

[3] L. A. F. M. Ferreira, T. Anderson, C. F. Imparato, T. E. Miller, C. K. Pang, A. Svoboda, A. F. Vojdani, "Short-Term Resource Scheduling in Multi-Area Hydrothermal Power Systems," *Electric Power & Energy Systems*, Vol. 11, No. 3, 1989, pp. 200-212.

[4] A. Renaud, "Daily Generation Management at Electricite de France: From Planning Towards Real Time," *IEEE Transaction on Automatic Control*, Vol. 38, No. 7, 1993, pp. 1080-1093.

[5] S. J. Wang, S. M. Shahidehpour, D. S. Kirschen, S. Mokhtari, and G. D. Irisarri "Short-Term Generation Scheduling with Transmission Constraints Using Augmented Lagrangian Relaxation," *IEEE Transactions on Power Systems*, Vol. 10, No. 3, Aug. 1995, pp. 1294-1301.

[6] A. I. Cohen and S. H. Wan, "An Algorithm for Scheduling a Large Pumped Storage Plant," *IEEE Transaction on Power Apparatus and Systems*, Vol. PAS-104, No. 8, Aug. 1985, pp. 2099-2104.

[7] W. J. Trott and W. Yeh, "Optimization of Multiple Reservoir Systems," *Journal of Hydraulics Division (ASCE)*, Oct. 1973.

[8] H. Brannlund, J. A. Bubenko, D. Sjelvgren and N Anderson, "Optimal Short Term Operation Planning of a Large Hydro-Thermal Power System Based On a Nonlinear Network Flow Concept," *IEEE Transactions on Power Systems*, Vol. 1, No. 4, 1986, pp. 75-82.

[9] R. E. Rosenthal, "A Nonlinear Network Flow Algorithm for Maximization of Benefits in a Hydroelectric Power System," *Operation Research*, Vol. 29, No. 4, pp. 763-786, July 1981.

[10] C. Li, P. Jap and D. Streiffert, "Implementation of Network Flow Programming to the Hydrothermal Coordination in an Energy Management System," *IEEE Transactions on Power Systems*, Vol. 8, No. 3, Aug. 1993, pp. 1045-1053.

[11] J. L. Kennington and R. V. Helgason, *Algorithms for Network Programming*, John Wiley & Son, 1980.

[12] H. Habibollahzadeh, D. Frances and U. Sui, "A New Generation Scheduling Problem at Ontario Hydro," *IEEE Transactions on Power Systems*, Vol. 5, No. 1, Feb. 1991, pp. 65-73.

[13] D. Sjelvgren and T. S. Dillion, "Optimal Operations Planning in a Large Scale Hydro-Thermal Power System," *IEEE Transaction on Power Apparatus and Systems*, Vol. PAS-102, No. 11, Nov. 1983.

[14] M. Christoforidis, B. Awobamise, R. J. Frowd and F. A. Rahimi, "Short-Term Hydro Generation and Interchange Contract Scheduling for Swiss Rail," *Proceedings of the 1995 IEEE Power Industry Computer Applications Conference*, Salt Lake City, UT, USA, 1995, pp. 143-149.

[15] P. Chen and H. Chang, "Genetic Aided Scheduling of Hydraulically Coupled Plants In Hydro-Thermal Coordination," *IEEE Transactions on Power Systems*, Vol. 11, No. 2, May 1996, pp. 975-981.

[16] Slobodan Ruzic, N. Rajakovic, "A Flexible Approach to Short-Term Hydrothermal Coordination Part I & Part II," *IEEE Transactions on Power Systems*, Vol. 11, No. 3, Aug. 1996, pp. 1564-1578.

[17] C. Li, E. Hsu, A. Svoboda, C. Tseng and R. Johnson, "Hydro Unit Commitment in Hydro-Thermal Optimization," 1996 IEEE/PES Summer Meeting Denver, CO, July 1996, 96 SM 497-8.

[18] O. Nilsson and D. Sjelvgren, "Mixed-Integer Programming Applied to Short-Term Planning of a Hydro-Thermal System," *Proceedings of the 1995 IEEE Power Industry Computer Applications Conference*, Salt Lake City, UT, USA, 1995, pp. 158-163.

[19] L. A. F. M. Ferreira, "Short-Term Scheduling of a Pumped Storage Plant, "*IEE Proceedings*, Part C, Vol. 139, No. 6, 1992, pp. 521-528.

[20] X. Guan, P. B. Luh, H. Yan, P. M. Rogan, "Optimization-Based Scheduling of Hydrothermal Power Systems with Pumped-Storage Units," *IEEE Transactions on Power Systems*, Vol. 9, No. 2, May 1994, pp. 1023-1031.

[21] X. Guan, P. B. Luh and L. Zhang, "Nonlinear Approximation Method in Lagrangian Relaxation-Based Algorithms for Hydrothermal Scheduling," *IEEE Transactions on Power Systems*, Vol. 10, No. 2, May 1995, pp. 772-778.

[22] X. Guan, P. Luh, H. Yan, and J. A. Amalfi, "A Optimization-Based Method for Unit Commitment," *Electric Power & Energy Systems*, Vol. 14, No. 1, 1992, pp. 9-17.

[23] J. Shaw, "A Direct Method for Security-Constrained Unit Commitment," *IEEE Transactions on Power Systems*, Vol. 10, No. 3, Aug. 1995, pp. 1329-1342.

Xiaohong Guan (S'89-M'94-SM'95) was born in Sichuan, P. R. China. He received his B.S. and M.S. degrees in Control Engineering from Tsinghua University, Beijing, P. R. China in 1982 and 1985, and Ph.D. degree in Electrical and Systems Engineering from the University of Connecticut in 1993, respectively. He was a consulting engineer with Pacific Gas and Electric Company from 1993 to 1995. He is with Xi'an Jiaotong University from 1985 to 1988 and since 1995, and currently he is a professor and associate director of the Institute of Systems Engineering, and his research interests include scheduling of power and manufacturing systems and optimization of large scale systems.

Ernan Ni was born in Suzhou, Jiangsu, P. R. China, in 1969. He received his B.S. in Mechtronics from Nanjing University of Science and Technology, Nanjing, in 1991 and M.S. degree in Industry Automation from Harbin Institute of Technology, in 1994. Now he is a Ph.D. candidate at the Institute of Systems Engineering, Xi'an Jiaotong University.

Renhou Li was born in Zhejiang, P. R. China. He graduated from Jiaotong University in 1956. Since then he has been with Xi'an Jiaotong University and currently is a professor of the Institute of Systems Engineering and the director of the H. S. Tsien Library. His research interests include intelligent control, theory of large scale systems and fuzzy control. Prof. Li is the chair of Shaanxi Provincial Association of Automation.

Peter B. Luh (S'77-M'80-SM'91-F'95) received his B.S. degree in Electrical Engineering from National Taiwan University, Taipei, Taiwan, Republic of China, in 1973, the M.S. degree in Aeronautics and Astronautics Engineering from MIT, Cambridge, Massachusetts, in 1977, and the Ph.D. degree in Applied Mathematics from Harvard University, Cambridge, Massachusetts, in 1980. Since 1980, he has been with the University of Connecticut, and currently is a Professor in the Department of Electrical and Systems Engineering. His major research interests include schedule generation and reconfiguration for manufacturing systems and power systems. Dr. Luh is an Editor of the *IEEE Transactions on Robotics and Automation*, was an Associate Editor of the *IEEE Transactions on Automatic Control*.