An Algorithm for Solving the Dual Problem of Hydrothermal Scheduling

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Abstract
Lagrangian relaxation has been widely used for the hydrothermal scheduling of power systems. The idea is to use Lagrangian multipliers to relax system-wide demand and reserve requirements, and decompose the problem into unit-wise subproblems that are much easier to solve. The multipliers are then updated at the high level, most commonly by using a subgradient method (SGM). Since the high level dual function is non-differentiable with many "ridges," SGM may zigzag across ridges resulting in slow convergence. This paper presents an algorithm that utilizes a recently developed "reduced complexity bundle method" (RCBM) to update the multipliers at the high level. The RCBM is a kind of "bundle method" that enjoy faster convergence compared to SGM, but has much reduced complexity as compared to a conventional bundle method. Testing results show that RCBM can find better directions, avoid zigzagging behavior, and obtain better dual and feasible solutions as compared to the SGM.

1. Introduction

Lagrangian Relaxation for Hydrothermal Scheduling

Hydrothermal scheduling is concerned with the commitment and dispatch of generating units. The objective is to minimize the total generation cost over a period of up to one week, subject to system-wide demand and reserve requirements and individual unit constraints. This mixed integer programming problem is believed to be NP-hard where the computational requirements for obtaining an optimal solution grow exponentially as the problem size increases. Lagrangian relaxation has been successful for obtaining near-optimal solutions with quantifiable quality. The basic idea is to use Lagrangian multipliers to relax system-wide demand and reserve requirements, and decompose the problem into unit-wise subproblems. The multipliers are then updated at the high level, mostly by using a subgradient method (SGM) [1, 2, 3]. Since dual solutions are usually infeasible, heuristics are then used to obtain a feasible schedule.

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Shortcomings of Subgradient-Type Methods

Given the current set of multipliers, the SGM sets the search direction to be one of the subgradients. This is computationally simple. However, since the dual function is non-differentiable with many "ridges," the SGM may cause the multipliers to zigzag across ridges resulting slow convergence [4, 5]. The zigzagging behavior is demonstrated by the SGM trajectory in Fig. 1 for the example to be presented in subsection 4.1. Although techniques have been developed to address the slow convergence issue, including the modified subgradient method [6] and adaptive step-sizing techniques [7], the basic difficulties of subgradient-type methods have not been altered.

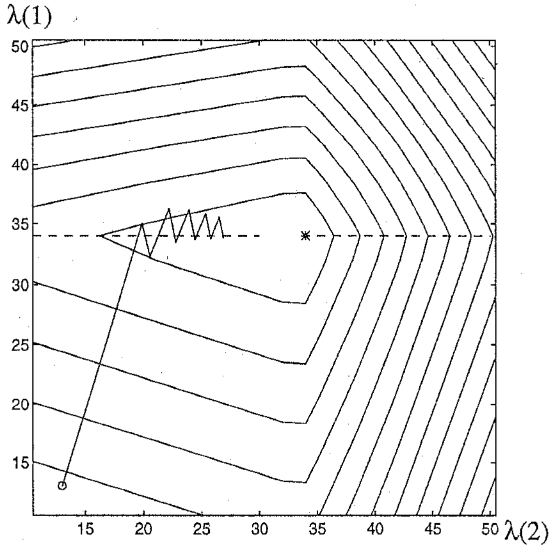
Necessity for an Efficient High Level Algorithm and the Reduced Complexity Bundle Method

The slow convergence of SGM is becoming a serious problem in view of the needs to solve larger and more complicated problems, e.g., mid-term scheduling, unit commitment with environmental and transmission capacity constraints [8, 9], power transaction problems [10], etc. For example, a large transaction with a constant MW price creates sharp ridges in the dual function, and makes the dual problem difficult to solve. An efficient high level algorithm that can generate high quality solutions in short CPU times is becoming critical to capture frequently emerging opportunities and to avoid crises in the increasingly deregulated power market.

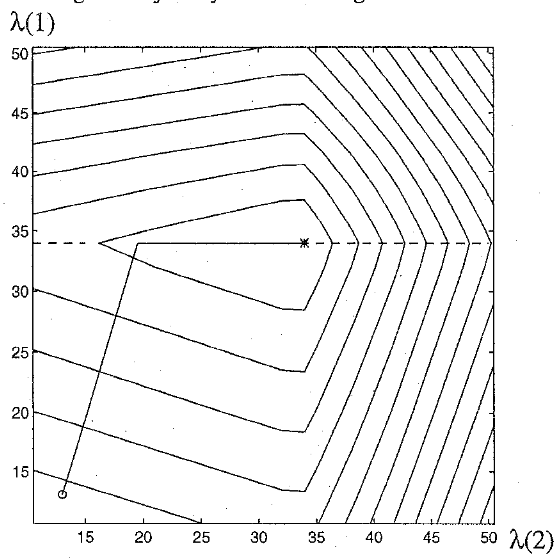
The bundle method overcomes the slow convergence of subgradient-type methods for solving non-differentiable optimization problems [11]. However, finding a search direction becomes computationally intensive as the problem size increases. The recently developed Reduced Complexity Bundle Method (RCBM) maintains the convergence property while reducing the complexity of bundle method [12]. The method can thus be applied to problems of practical sizes such as the dual problem of hydrothermal scheduling.

Compared to SGM, RCBM adopts the subgradient direction only when it is good. Otherwise, it accumulates subgradiets in a "bundle," and finds a trial direction based on bundle elements. If the trial direction is "good," the method moves along the direction; otherwise, the bundle is updated, and the next trial direction is found. The trajectories generated by

RCBM for the same example of subsection 4.1 is presented in Fig. 2. The zigzagging behavior of SGM has been eliminated. In the following, problem formulation and solution methodology of a hydrothermal scheduling problem are presented in Section 2. The RCBM and its implementation for solving the dual problem are presented in Section 3. Results from extensive testing are presented in Section 4.



"o": initial point; "*": optimal point;
λ(i) the multiplier with respect to system demand at hour i ($/MW)
Fig. 1. Trajectory from the Subgradient Method



"o": initial point; "*": optimal point;
λ(i): the multiplier with respect to system demand at hour i ($/MW)
Fig. 2 Trajectory generated by RCBM

2. Problem Formulation and Solution Methodology

2.1 Problem Formulation

Consider a power system with I thermal units, J hydro units, and K pumped-storage units. The objective is to minimize the total generation cost. This minimization is subject to system-wide demand and reserve requirements, and individual unit constraints.

The **objective function** to be minimized is the sum of thermal generation costs $C_{ti}(p_{ti}(t))$ and start up costs $S_{ti}(t)$, i.e.,

$$\min_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} J, \text{ with } J = \sum_{t=1}^{T} \{ \sum_{i=1}^{I} [C_{ti}(p_{ti}(t)) + S_{ti}(t)] \}. \tag{2.1}$$

In the above, $p_{ti}(t)$ is the generation of thermal unit i at time t, $w_{hj}(t)$ the water released of hydro unit j, $w_{pk}(t)$ the water released of pumped-storage unit k (negative for pumping), and T is the time horizon (e.g., T = 168 hours).

The **system demand constraints** require that the sum of all thermal generation $p_{ti}(t)$, hydro generation $p_{hj}(w_{hj}(t))$, and pumped-storage generation $p_{pk}(w_{pk}(t))$ (negative for pumping) should equal the system demand $P_d(t)$ at each hour, i.e.,

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{j=1}^{J} p_{hj}(w_{hj}(t)) + \sum_{k=1}^{K} p_{pk}(w_{pk}(t)) = P_d(t), \quad t=1, ..., T, \tag{2.2}$$

where $p_{hj}(w_{hj}(t))$ and $p_{pk}(w_{pk}(t))$ are water-power conversion functions for hydro unit j and pumped-storage unit k, respectively.

Reserve requirements states that the sum of reserve contributions of thermal units $r_{ti}(p_{ti}(t))$, hydro units $r_{hj}(p_{hj}(t))$, and pumped-storage units $r_{hp}(p_{pk}(t))$ should be greater than or equal to the reserve required $P_r(t)$ at each hour, i.e.,

$$\sum_{i=1}^{I} r_{ti}(p_{ti}(t)) + \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) + \sum_{k=1}^{K} r_{pk}(p_{pk}(t)) \geq P_r(t), \quad t=1, ..., T. \tag{2.3}$$

Individual thermal unit constraints include capacity and minimum generation, minimum up/down times, ramp rate, minimum generation for the first and last hour, and must-run and must-not-run. Individual hydro unit constraints include capacity and minimum generation, and available hydro energy. Individual pumped-storage unit constraints include pond level dynamics and limit constraints, and generation and pumping level constraints. The detailed descriptions have been presented in [2], [13], and [14], respectively.

2.2 Solution Methodology

2.2.1 The Relaxed Problem

Relaxing system-wide demand (2.2) and reserve requirements (2.3) by using Lagrange multipliers λ and μ, respectively, the following relaxed problem is formed:

$$\min_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} L, \text{ with } L \equiv \sum_{t=1}^{T}\{\sum_{i=1}^{I}[C_{ti}(p_{ti}(t)) + S_i(t)]\}$$
$$+\sum_{t=1}^{T}\lambda(t)\left[P_d(t) - \sum_{i=1}^{I}p_{ti}(t) - \sum_{j=1}^{J}p_{hj}(w_{hj}(t)) - \sum_{k=1}^{K}p_{pk}(w_{pk}(t))\right]$$
$$+\sum_{t=1}^{T}\mu(t)\left[P_r(t) - \sum_{i=1}^{I}r_{ti}(p_{ti}(t)) - \sum_{j=1}^{J}r_{hj}(p_{hj}(t)) - \sum_{k=1}^{K}r_{pk}(p_{pk}(t))\right]. \quad (2.4)$$

2.2.2 Solving Subproblems

After re-grouping relevant terms, individual thermal, hydro, and pumped-storage sub-problems are formed, one for each unit. The resolution of these subproblems can be found in [2], [13] and [14], respectively.

2.2.3 The High Level Dual Problem

The dual problem is to update multipliers to maximize the dual cost $\phi(\lambda, \mu)$, i.e.,

$$\max_{\lambda,\mu\ge 0} \phi(\lambda,\mu), \text{ with}$$
$$\phi(\lambda,\mu) \equiv \min_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} L, \quad (2.5)$$

where L is defined in (2.4). The dual problem will be solved by the RCBM to be presented in subsection 3.3.

2.2.4 Obtaining A Feasible Solution

The solutions of subproblems are usually associated with an infeasible schedule, i.e., system demand and reserve requirements may not be satisfied. Heuristic methods as presented in [2, 13 and 14] are used to adjust subproblem solutions to obtain a feasible schedule. Heuristics are performed at the end of each high level iteration after all subproblems are solved, except for the first few high level iterations. The feasible solution with the lowest cost is recorded as the scheduling decisions.

3. The Reduced Complexity Bundle Method

3.1 The Bundle Method (BM)

The bundle method emerged recently as a promising approach for maximizing non-smooth concave functions (e.g., [11]). It employs a concept called the "ε-subdifferential," defined as

$$\partial_\varepsilon\phi(x) = \left\{g \in R^n | \phi(y) \le \phi(x) + (g, y - x) + \varepsilon, \forall y \in R^n\right\}, \quad (3.1)$$

which is an extension of the "subdifferential" concept. Elements in $\partial_\varepsilon\phi$ are called ε-subgradients. Correspondingly, the ε-directional derivative along direction d at x is defined as

$$\phi'_\varepsilon(x,d) \equiv \sup_{t>0}\frac{\phi(x+td) - \phi(x) - \varepsilon}{t}. \quad (3.2)$$

It is shown that $\phi'_\varepsilon(x,d) = \inf_{g\in\partial_\varepsilon\phi(x)} g'd$ [15, p. 102]. From (3.2), if a direction can be found such that $\phi'_\varepsilon(x,d) > 0$, then the dual cost can be increased by at least ε. The bundle method thus sets the direction to be the one that maximizes the directional derivative, i.e.,

$$d^* = \arg\left\{\max_{\|d\|=1}\phi'_\varepsilon(x,d)\right\} = \arg\left\{\max_{\|d\|=1}\inf_{g\in\partial\phi_\varepsilon(x)} g'd\right\}$$
$$= \arg\left\{\inf_{g\in\partial_\varepsilon\phi(x)}\max_{\|d\|=1} g'd\right\} = \arg\left\{\inf_{g\in\partial_\varepsilon\phi(x)}\|g\|\right\}. \quad (3.3)$$

This d* is therefore the ε-subgradient with the smallest norm. The availability of only one subgradient at each point within the Lagrangian relaxation framework, however, implies the unavailability of the complete $\partial_\varepsilon\phi$. Bundle method accumulates ε-subgradients of the current iterate in a bundle B_b, and approximate $\partial_\varepsilon\phi$ by the convex hull P_b of bundle elements:

$$P_b = \left\{g | g = \sum_{i=1}^{b}\gamma_i g_i, g_i \in B_b, 0 \le \gamma_i \le 1, \sum_{i=1}^{b}\gamma_i = 1\right\}, \quad (3.4)$$

where g_i is the i-th element of the bundle with a total of b elements. A trial direction is then obtained as the element in P_b that has the smallest norm. This direction can be obtained by quadratic programming, however, the process can be computationally intensive as the size of the bundle increases.

3.2 Reduced Complexity Bundle Method (RCBM)

The minimum-norm direction obtained by BM can be viewed as the projection of the origin onto the convex hull P_b. The RCBM sets the direction to be the orthogonal projection of the origin onto the *affine manifold* $A(P_b)$ containing P_b, i.e.,

$$A(P_b) = \left\{x | x = \sum_{i=1}^{b}\gamma_i g_i, g_i \in B_b, \gamma_i \in R, \sum_{i=1}^{b}\gamma_i = 1\right\}. \quad (3.5)$$

The orthogonal projection of the origin onto $A(P_b)$ is equivalent to the projection of any bundle element onto the subspace normal to $A(P_b)$, and can be efficiently performed by using Cholesky decomposition [12]. This direction maintains global convergence of the traditional bundle method, but is obtained with much reduced computational requirements than that required by quadratic programming. The method can thus be applied to problems of practical sizes such as maximizing the dual function of the power system scheduling problem.

3.3 Implementing RCBM for the Dual Problem

Since dual function $\phi(\lambda, \mu)$ is concave according to [16, p. 423], RCBM can be used to maximize it. The subgradient g of $\phi(\lambda, \mu)$ is a 2T by 1 vector consisting of g_λ and g_μ, where g_λ and g_μ are the subgradient of ϕ with respect to λ and μ, respectively. The t-th element of g_λ is

$$g_\lambda(t) = p_d(t) - \sum_{i=1}^{I} p_{ti}(t) - \sum_{j=1}^{J} p_{hj}(t) - \sum_{k=1}^{K} p_{tk}(t), \quad (3.6)$$

and the t-th element of g_μ is

$$g_\mu(t) = p_r(t) - \sum_{i=1}^{I} r_{ti}\left(p_{ti}(t)\right) - \sum_{j=1}^{J} r_{hj}\left(p_{hj}(t)\right) - \sum_{k=1}^{K} r_{pk}\left(p_{pk}(t)\right). \quad (3.7)$$

At each high level iteration, RCBM first uses the subgradient as a trial direction. Line search is then performed, with only the following two possible outcomes [11, p. 213]:

1. The dual function can be increased by a pre-specified value ε, i.e.,

$$\phi(\lambda + td, \mu + td) \geq \phi(\lambda, \mu) + \varepsilon; \quad (3.8)$$

where t is the step size. In this case, the next point will be set to (λ + td, μ + td).

2. If the dual function cannot be increased by ε, the line search will find a subgradient belonging to the ε-subdifferential of φ(λ, μ). This ε-subgradient is then added to the bundle, and a new trial direction is formed with the augmented bundle. The process then repeats till condition (3.8) is satisfied. The RCBM can now be summarized as follows.

Step 1: [Initialize Multipliers.] Initialize λ according to a priority-list commitment and dispatch. Initialize μ to zero. Solve individual subproblems as presented in subsection 2.2.2 and calculate the dual function φ(λ, μ) and subgradient $g_1 = [g_\lambda, g_\mu]$. Set iteration number iter = 1.

Step 2: [Initialize the Bundle.] Initialize the Bundle to B_b = { g_1 }. Set the number of bundle elements b = 1.

Step 3: [Set Direction.] If b = 1, set search direction d to be the subgradient direction; else, select d by projecting an arbitrary bundle element onto the subspace normal to $A(P_b)$.

Step 4: [Test Convergence.] If $\|d\| \leq \delta$ (δ is a pre-specified stopping criterion), stop.

Step 5: [Perform Line Search and Run Heuristics.] Perform line search to find a step size t. Calculate the dual function φ(λ+td, μ+td) and the subgradients $g_{b+1} = [g_{\lambda+td}, g_{\mu+td}]$. If iter is greater than a pre-specified number, run heuristics and store the best feasible solution. If condition (3.8) is satisfied, go to Step 6; else go to Step 7.

Step 6: [Move to the Next Iterate.] Set λ = λ + td, μ = μ + td, $g_1 = g_{b+1}$, iter = iter + 1, then go to Step 2.

Step 7: [Augment the Bundle.] Add g_{b+1} to the bundle B, and set b = b + 1, then goes to Step 3.

4. Numerical Testing Results

The RCBM was implemented in C++ and the resolution of individual subproblems and heuristics were implemented in FORTRAN on a SUN Ultra Station 1.

4.1 Testing of a Small Example

A system with two thermal units is to be scheduled over two hours following [5]. Only system demand, unit minimum generation and capacity are considered. The minimum generation of unit i is denoted as $\underline{p}_{ti}$ and the capacity $\overline{p}_{ti}$. The thermal cost $C_{t1}(p_{t1}(t))$ of the first unit is piece-wise linear with four equal size blocks, and $q_{t1}(m)$ denotes the incremental cost of block m. The second unit has a single block. The parameters are listed below.

Parameters of the Two Units System

Thermal unit 1 parameters:

$\underline{p}_{t1} = 40\text{MW}, \overline{p}_{t1} = 120\text{MW}, C_{t1}(\underline{p}_{t1}(t)) = \$1188/\text{MW},$

$q_{t1}(1) = \$32/\text{MW}, q_{t1}(2) = \$35.2/\text{MW},$

$q_{t1}(3) = \$38.4/\text{MW}, q_{t1}(4) = \$41.6\text{MW}.$

Thermal unit 2 parameters:

$\underline{p}_{t2} = 40\text{MW}, \overline{p}_{t2} = 200\text{MW},$

$C_{t2}(\underline{p}_{t2}(t)) = \$1360/\text{MW}, q_{t2}(1) = \$34/\text{MW}.$

System demand at each hour:

$p_d(1) = 160.0\text{MW}, p_d(2) = 105.0\text{MW}.$

The polyhedron dual function φ(λ(1), λ(2)) and its contour are plotted in Fig. 3. There are two ridges, one at λ(1) = 34, and the other at λ(2) = 34. The initial multipliers are set to λ = (13, 13), and the optimal dual solution is λ^* = (34, 34) with ϕ^* = $8586.

The trajectory generated by SGM has been presented in Fig. 1. It can be seen that the trajectory zigzags across an ridge resulting in slow convergence. The trajectory generated by RCBM has been presented in Fig. 2. Initially, RCBM takes the subgradient direction to move to the point λ = (33.98, 19.56) by line search, which is near the ridge. At this point, the subgradient is not a good direction. Two subgradients, one at each side of the ridge, then forms the bundle, resulting a direction parallel to the ridge as shown in Fig. 4. Along this direction, RCBM moves to the point λ = (33.98, 34.05), and the algorithm stops in two high level iterations. Testing results are summarized in Table 1.

Table 1. Summary of SGM and RCBM results

	Iterations	Function Evaluations	Dual Cost
SGM	10	10	$8438
RCBM	2	10	$8578

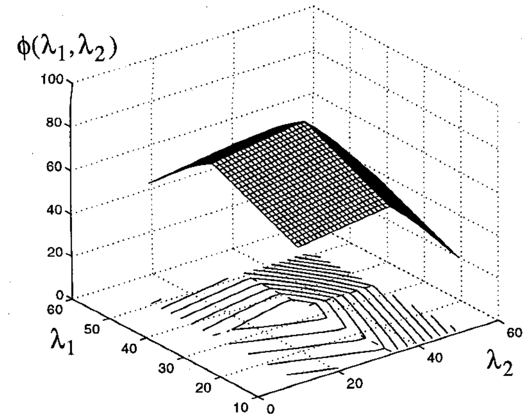
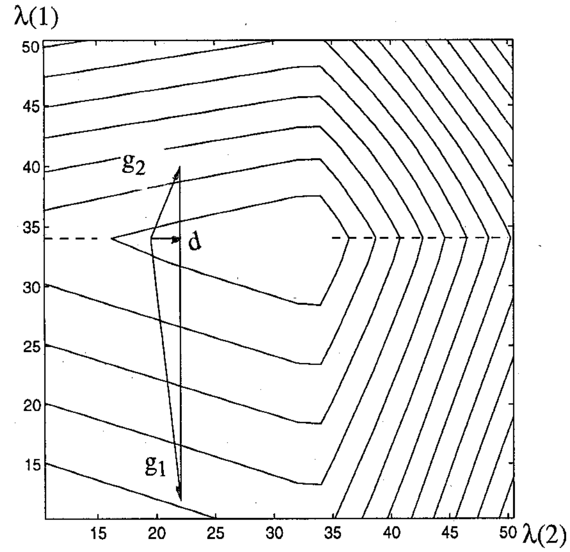


Fig. 3. The dual function and its contour



g_1 and g_2 are two ε-subgradients.

Fig. 4 Direction obtained by RCBM

4.2 Testing Results for Northeast Utilities Data Sets

In this testing, data sets from Northeast Utilities Service Company are used to compare the performance of RCBM vs. SGM in updating the multipliers. The methods for solving subproblems and the heuristics are kept identical. There are 65 thermal units, 7 hydro units, 1 large pumped-storage unit, and 2-10 schedulable contracts. The scheduling horizon is either 7 days (168 hours) or 8 days (192 hours). Detailed system characteristics can be found in [14]. Many practical considerations are included, and all the billing rules of New England Power Pool are satisfied. Results for the six 1994 data sets are summarized in Tables 2 and 3.

Table 2: Comparison of Dual and Feasible Costs (Dollars)

Cases	D-SG	D-RCBM	F-SG	F-RCBM
09, W4	6892871	6894451	7051315	7041643
12, W1	926913	929729	1013046	993998
12, W4	893546	897273	920413	912828
03, W1	6496216	6497346	6631023	6606153
09, W2	734617	735831	753888	751671
10, W1	10784121	10800353	11017553	11017501

D-SG: dual cost of SGM; D-RCBM: dual cost of RCBM; F-SG: feasible cost of SGM; F-RCBM: feasible cost of RCBM.

TABLE 3: Comparison of CPU Time (Seconds)

Cases	Iter.-SG	Iter.-RCBM	Func.E.-SG	Func.E-RCBM	CPU-SG	CPU-RCBM
09,W4	61	20	61	60	108	100
12,W1	71	22	71	75	44	50
12,W4	64	18	64	60	38	35
03,W1	36	14	36	40	71	74
09,W2	91	25	91	80	45	35
10,W1	41	15	41	45	83	90

Iter.: number of high level iterations; Func.E.: number of function evaluations.

Although RCBM needs more function evaluations in coming up with a direction, the directions are generally good -- resulting in significant reductions of the numbers of high level iterations. With very similar CPU times, RCBM can obtain higher dual costs. Higher dual costs generally correspond to lower feasible costs, resulting in better overall performance (although exceptions do exist as indicated by October Week 1 data). The average dual cost is increased by $4,450 per case, and the average feasible cost is decreased by $10,600 per case.

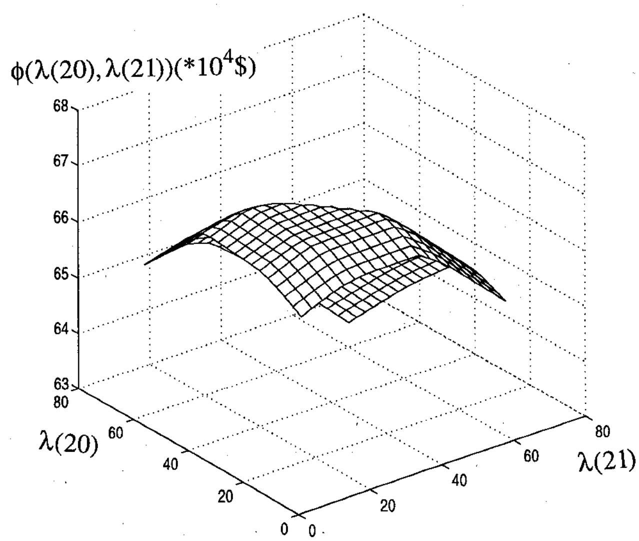


Fig.5 Dual Function Shape for Case 09, w2

To investigate the rationale behind the above RCBM performance improvement, the dual function for September Week 2 data set is plotted with respect to demand multipliers of hours 20 and 21 in Figure 5. It can be seen that the dual function is non-differentiable with many ridges, consistent with what was presented in Figure 3.

In another testing, one data set in 1996 is used as a base case. The load is first increased by 25MW for all the hours, and then by 100MW to generate two modified cases. These data sets have many transactions each with a single constant MW price block, making the dual problem difficult to solve. The feasible cost and duality gap obtained by SGM and RCBM are listed in Table 4. The CPU time are quite similar and are thus omitted. It can be seen that RCBM can constantly generate better results than what obtained by SGM.

Table 4: Feasible Cost and Gap for Base and Modified Cases

Cases	F-SG	F-RCBM	Gap-SG	Gap-RCBM
Base	7596556	7563498	2.49%	1.97%
+25MW	7730941	7665835	2.82%	1.85%
+100MW	8088178	7991332	3.01%	1.73%

5. Conclusion

This paper presents the application of RCBM for solving hydrothermal scheduling problems within the Lagrangian relaxation framework. Rationales for the performance improvement are intuitively explained and illustrated by examples. Testing results support the intuitive argument, and show that RCBM can avoid the zigzagging behavior of subgradient methods, obtain higher dual cost, and result in better overall performance.

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Discussion

N. Jiménez Redondo, A. Conejo (Universidad de Málaga, Málaga, Spain): We wish to commend the authors for their contribution in proposing a new algorithm to solve the dual problem of the short-term hydrothermal scheduling problem.

The method proposed to solve the dual problem is based on the dual form of the bundle method presented in [11]. It is called Reduced Complexity Bundle Method (RCBM).

We would like the authors to comment on the following issues:

1. In order to select the direction of ascent, the dual form of the bundle method presented in [11] solves a quadratic programming problem. The RCBM method proposed in the paper avoids the need to solve a quadratic problem by defining an affine manifold, called $A(P_b)$, containing the convex hull of the bundle, and projecting a bundle element onto the subspace normal to $A(P_b)$. The RCBM method is very briefly explained. Therefore, we would appreciate if the authors could give some more details on their algorithm. Particularly, they could explain how they select the bundle element to be projected, how they perform the projection of the bundle element, and how they guarantee the convergence of the method.
2. Once the direction of ascent is chosen, a line search has to be performed in order to select the stepsize. As pointed out in [11] the line search process is not easy and can be computationally costly (because it have to be performed at every iteration). Few parameters have to be carefully tuned up so that the process behaves efficiently. It would be very helpful if the authors comment on the line search algorithm they have implemented, and on the tune up of the parameters they have used.
3. As pointed out in [11], the key element to achieve efficiency on the dual form bundle method is the appropiate selection of the parameter ε (equation (3.8)). The selection of this parameter is not easy. How have the authors chosen this parameter?. Have they used a constant value or have they used a varying parameter in each iteration as suggested in [11]?. Is the selection of ε problem dependent?.
4. The method introduced seems to be close to the subgradient method. However, primal forms of bundle methods ([11],[D1]) are based on the definition of a model of the dual function ($\phi(\lambda,\mu)$). This model gets closer to the actual dual function as the number of iterations increases. The use of such a model guarantees the convergence of the primal forms of bundle methods and also avoids oscillations. This is not in general the case for dual forms of bundle methods. The authors use a very small size example where the subgradient method oscillates and the RCBM method does not. Can the authors guarantee the non-oscillatory behavior and the convergence of their method for large-scale case studies?. In order to analyze the stability of the RCBM method in a realistic case study, a plot of the dual function with respect to the iteration index would be useful. It can be compared with a plot of a subgradient method in order to prove the advantages of the RCBM method with respect to the subgradient method. Could the authors provide such plots?.
5. Have the authors worked on any primal form of bundle method such as the one presented in [D1]?. Could they compare the solution achieved with their method to that obtained by applying a primal form of bundle method?

Once again, we would like to congratulate the authors for their paper.

D1] F. Pellegrino, A. Renaud and T. Socroun. *Bundle and Augmented Lagrangian Methods for Short-Term Unit Commitment.* Proceedings of the 12th Power Systems Computation Conference, PSCC'96, pp.730-739. Dreden, Germany, August 1996.

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Peter B. Luh, Daoyuan Zhang (University of Connecticut, Storrs, CT 06269, USA) and **Robert N. Tomastik** (United Technologies Research Center East Hartford, CT 06108, USA):

We appreciate Professors N. Redondo and A. Conejo for their interests in our paper and comments. The questions will be answered in the order they appeared.

1. The ε-subgradient with the smallest norm is an ascent direction. Because of the unavailability of the complete ε-subgradient set, the convex hull of the subgradients in the "Bundle" is used to approximate it. The direction is the projection of the origin onto the convex hull. This projection involves quadratic programming (QP) which may be time-consuming with the standard QP method. In the Reduced Complexity Bundle Method (RCBM), an orthogonal projection is used. Any element in the "Bundle" can be selected for the orthogonal projection and the results remain the same. From Figure 4, it can be seen that whether g_1 or g_2 is used for projection, d is the same. The RCBM and its convergence are established in

detail in [12]. Suppose that there are b affine independent subgradients g_1, g_2 ,... and g_b in the bundle, the formula for the orthogonal projection is given below:

$$d = g_1 - S_b^T \sigma,$$

$$\sigma = (S_b S_b^T)^{-1} S_b g_1,$$

$$S_b = \begin{bmatrix} (g_1 - g_2)^T \\ (g_1 - g_3)^T \\ \dots \\ (g_1 - g_b)^T \end{bmatrix}^T,$$

where the matrix inverse is obtained by Cholesky decomposition.

2. The line search in [11, p. 213] is implemented in the RCBM to find a point at which the subgradient can be added to the bundle, or the dual function value can be increased by ε. Lemma 2.2.3 in [11] plays an important role for the satisfactory performance of the line search. According to the lemma, a convex combination of two subgradients may be added to the bundle though neither of them is legible.

3. It is true that the most important parameter affecting the performance of the RCBM is ε which also has a major impact on the line search process. In our implementation, ε is adjusted according to the difference of the feasible cost (an upper bound of the dual cost) and the dual cost. A percentage of this difference is used as ε.

4. It is not mentioned how the quadratic programming problem (9) is solved in [11]. If the inequality constraints [11, p. 733] are relaxed by Lagrangian multipliers, the derived dual form will be equivalent to the primal form (9). There is no reason to say that the dual form of the bundle method does not guarantee convergence while the primal form does.

   Better results have been obtained by the application of RCBM to the production of Northeast Utilities compared with the results obtained by a subgradient method. If RCBM is strictly implemented, the dual function value can be increased by ε in each iteration as proved in [12]. However, for power system scheduling problem, a safe guard checking is adopted – when the bundle size exceeds the preset maximum, the algorithm will move to the best point found in this iteration. The checking is necessary for obtaining satisfactory scheduling with reasonable CPU time.

5. We have not worked on using the primal form of bundle method for hydrothermal scheduling. The dimension of QP primal form is usually much larger than that of the dual form.

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