Optimization Based Bidding Strategies in the Deregulated Market

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***Abstract*—With the deregulation of electric power systems, market participants are facing an important task of bidding energy to an Independent System Operator (ISO). This paper presents a model and a method for optimization-based bidding and self-scheduling where a utility bids part of its energy and self-schedules the rest as in New England. The model considers ISO bid selections and uncertain bidding information of other market participants. With appropriately simplified bidding and ISO models, closed-form ISO solutions are first obtained. These solutions are then plugged into the utility's bidding and self-scheduling model which is solved by using Lagrangian relaxation. Testing results show that the method effectively solves the problem with reasonable amount of CPU time.**

***Index Terms*—Bidding strategies, Lagrangian relaxation.**

I. Introduction

A. Bidding, Self-Scheduling, and Literature Review

WITH the deregulation of electrical power systems, market participants bid energy to an Independent System Operator (ISO). In the daily market, participants submit bids to the ISO who then decides energy clearing prices (ECP) and hourly generation levels of each participant over a 24-hour period. The relationship between ISO and participants is shown in Fig. 1. In regions such as New England, a utility bids part of the energy and self-schedules the rest, whereas an independent power producers (IPP) bids all its energy. This paper focuses on the daily bidding and scheduling of a utility.

For each participant, bidding strategies ideally should be selected to maximize its profit. Game theory is a natural platform to model such an environment [1]–[3]. In the literature, matrix games have been used for its simplicity, and bidding strategies are discretized, such as "bidding high," "bidding low," or "bidding medium." With discrete bidding strategies, payoff matrices are constructed by enumerating all possible combinations of strategies, and an "equilibrium" of the "bidding game" can be obtained. It is difficult to incorporate self-scheduling in the method.

Modeling and solving the bid selection process by the ISO have also been discussed. In [4], bids are selected to minimize total system cost, and the ECP is determined as the price of the highest accepted bid. In [5], a bid-clearing system in New Zealand is presented. Detailed models are used, including network, reserve, and ramp-rate constraints, and the problem is solved by using linear programming. In [6], the dynamics of pricing is considered in market power assessment. In [7], an optimal bidding strategy is proved to be a unit's true cost under the assumption that the bid of each unit does not effect the ECP. This assumption is realistic since units with significant capacities will affect ECP.

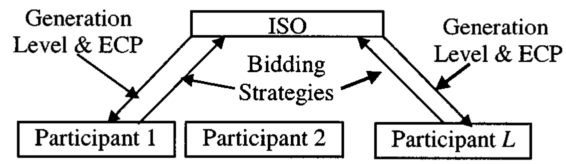

Fig. 1. The relationship between ISO and participants.

B. Overview of the Paper

The purpose of the paper is to present a model and a method for the bidding and self-scheduling problem from the viewpoint of a utility, say Participant 1. To obtain effective solutions with acceptable computation time, bids are represented as quadratic functions of power levels. For Participant 1, these parameters are to be optimized. For other participants, the parameters are assumed to be available as discrete distributions. Based on bids submitted, the ISO is to minimize the total system cost. The problem for Participant 1 is then formulated to minimize its expected cost, including generation costs and payment to the market. The detailed models are presented in Section II.

In Section III, the ISO problem is first solved with closed-form solutions. The solutions are plugged into Participant 1's model with detailed modeling of units, and the problem is solved by using Lagrangian relaxation as presented in Section IV. Besides the subproblems in traditional hydrothermal scheduling, an additional bidding subproblem is constructed to optimize bidding parameters. This subproblem is stochastic because of the uncertainty of the market, and is optimized by a gradient method. It is shown that this subproblem is inherently degenerated with an infinite number of equivalent solutions. Numerical testing presented in Section V shows that the method produces good bidding and self-scheduling results for practical problems. Compared with a "mean method," this new method reduces system cost and effectively handles uncertainties with a small increase in CPU time.

Manuscript received February 14, 2000. This work was supported in part by the National Science Foundation under Grants ECS-9726577 and a grant from Northeast Utilities Service Company.

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Publisher Item Identifier S 0885-8950(00)07732-4.

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Block 1	15	60
Block 2	20	40

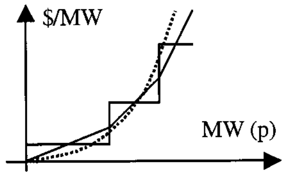

Fig. 2. An example of bids.

II. Problem Formulation

A. Representation of Bids

A bid consists of price offers and the amount of load to be satisfied by the market for each hour. Price offers specify a stack of MW levels and the corresponding prices as illustrated in Fig. 2. By integrating a staircase price offer curve, the bidding cost function is piecewise linear. The amount of load to be satisfied by the market is denoted as $p_{Ml}(t)$.

To reduce the number of parameters associated with a bid, the piece-wise linear bidding cost function is approximated by a quadratic $C_l(p_{Al}(t))$ (often done in scheduling problems [8]):

$$C_l(p_{Al}(t)) = a_l(t)p_{Al}^2(t) + b_l(t)p_{Al}(t), \qquad l = 1, 2, \cdots L, \tag{1}$$

where

l	is the participant index,
t	the hour index,
$p_{Al}(t)$	the accepted level by ISO,
$a_l(t)$ and $b_l(t)$	are nonnegative bidding parameters, and
L	the number of participants.

The $p_{Al}(t)$ is nonnegative, and is upper bounded by a maximal value, i.e.,

$$0 \le p_{Al}(t) \le \overline{p}_{Al}(t). \tag{2}$$

Participant 1 does not have exact bids of others, but has probability distributions of $\{a_l(t), b_l(t)$ and $p_{Ml}(t)\}$, $l = 2, \cdots L$, based on market information and experiences. The distributions are represented by J discrete sets of bids:

$$B^j = \{a_l^j, b_l^j, p_{Ml}^j, l = 2, \cdots L\}, \qquad j = 1, 2, \cdots J. \tag{3}$$

The probability of B^j is p^j with $\sum_{j=1}^{J} p^j = 1$. For Participants 1, $a_1(t)$, $b_1(t)$ and $p_{M1}(t)$ depend on its unit characteristics and others' bids, and are to be optimized.

B. The ISO Model

The ISO decides hourly generation levels of participants to satisfy the total submitted load at the minimum cost over 24 hours. Bids of all participants are available to the ISO, and the deterministic ISO problem is

$$J_{\text{ISO}} = \min_{p_{Al}(t)} \sum_{t=1}^{T} \sum_{l=1}^{L} C_l(P_{Al}(t)), \tag{4}$$

subject to

$$\sum_{l=1}^{L} p_{Al}(t) = \sum_{l=1}^{L} p_{Ml}(t), \tag{5}$$

where $p_{Ml}(t)$ is the amount of load that will be satisfied by the market for Participant l at hour t.

Constraints (2) should be satisfied for all participants. For simplicity of derivation, however, they are only required to be satisfied for Participant 1 but are ignored for others.

C. The Model of Participant 1

Participant 1 is to decide the generation levels of each unit and a bidding strategy to maximize its profit, or to minimize its costs while satisfying various constraints. The costs include generation costs and payment to the market, with the payment equal to ECP multiplied by $(p_{M1}(t) - p_{A1}(t))$. Only thermal units are considered to simplify presentation, however, there is no difficulty in incorporating hydro and pumped-storage units. The problem is therefore

$$\min_{p_{M1}(t),\, a_1(t),\, b_1(t),\, p_{ti}(t)} J,$$

with

$$J \equiv E \sum_{t=1}^{T} \sum_{i=1}^{I} \{C_{ti}(p_{ti}(t)) + \lambda_M^*(t)(p_{M1}(t) - p_{A1}(t))\}. \tag{6}$$

In the above, T is the number of hours, I the number thermal in units, C_{ti} the cost function of thermal unit i, $p_{ti}(t)$ the generation level of unit i at hour t, and $\lambda_M^*(t)$ the ECP at hour t. The expectation is taken with respect to uncertain bidding parameters reflected through $\lambda_M^*(t)$ and $p_{A1}(t)$.

For each hour, the load balance constraint requires that

$$\sum_{i=1}^{I} p_{ti}(t) + E(p_{M1}(t) - p_{A1}(t)) = p_d(t). \tag{7}$$

In the following, the of ISO scheduling will be solved first, followed by bidding and self-scheduling.

III. The ISO Scheduling

From ISO's point of view, its problem is deterministic. When Participant 1 solves the ISO problem, however, it only has distributions of parameters, and has to solve the ISO problem for every set of bidding parameters. Solution of the ISO problem for the jth set B^j is derived as follows. The index j is omitted as appropriate for simpler presentation.

Lagrangian multipliers $\lambda_M(t)$ are used to relax (5), and $\pi_1(t)$ and $\pi_2(t)$ to relax (2). The resulting Lagrangian is:

$$L_{\text{ISO}} = \sum_{t=1}^{T} \left\{ \sum_{l=1}^{L} C_l\left(p_{A_l}(t)\right) + \lambda_M(t) \cdot \left(\sum_{l=1}^{L} p_{Ml}(t) - \sum_{l=1}^{L} p_{Al}(t) \right) \right\} - \sum_{t=1}^{T} \pi_1(t) p_{A1}(t) - \sum_{t=1}^{T} \pi_2(t)(p_{A1}(t) - \overline{p}_{A1}(t)). \tag{8}$$

In (8), $\pi_1(t)$, $\pi_2(t)$, and $p_{A1}(t)$ satisfy

$$\pi_1(t)p_{A1}(t) = 0, \text{ and } \pi_2(t)p_{A1}(t) = 0.$$

The three cases of ISO solutions are presented below.

A. Case 1: Accepted Level in Bound ($0 < p_{A1}(t) < \overline{p}_{A1}(t)$)

With the given $\{\lambda_M(t)\}$, (8) can be decomposed subproblems. The subproblem for participant l is

$$L_l = \min_{p_{A1}(t)} \sum_{t=1}^{T} \{a_l(t)p_{Al}^2(t) + b_l(t)p_{Al}(t) - \lambda_M(t)p_{Al}(t)\}. \quad (9)$$

The solution for (9) is

$$p_{Al}^*(t) = \frac{\lambda_M(t) - b_l(t)}{2a_l(t)}. \quad (10)$$

In (10), $a_l(t)$ is assumed to be nonzero. If it is zero, a quadratic function with a small $a_l(t)$ is used to approximate the linear function following the idea of [9]. With an analytical solution for each subproblem, it is not necessary to iteratively update $\lambda_M(t)$ at the high level. Substituting (10) into (5), one obtains the ECP $\lambda_M^*(t)$:

$$\lambda_M^*(t) = \frac{\sum_{l=1}^{L}\left(\frac{b_l(t)}{a_l(t)} + 2P_{Ml}(t)\right)}{\sum_{l=1}^{L}\frac{1}{a_l(t)}.}. \quad (11)$$

Substituting (11) into (10), one obtains the accepted level for Participant 1 at hour t as

$$p_{A1}^*(t) = \frac{\sum_{l=1}^{L} 2p_{Ml}(t) + \sum_{l=2}^{L}\frac{b_l(t) - b_1(t)}{a_l(t)}}{2 + 2a_1\sum_{l=2}^{L}\frac{1}{a_l(t)}}. \quad (12)$$

The energy clearing price $\lambda_M^*(t)$ and the accepted level $p_{A1}^*(t)$ for Participant 1 are functions of $a_1(t)$, $b_1(t)$ and $p_{M1}(t)$. To simplify (11) and (12), let

$$c_0(t) \equiv 2\sum_{l=2}^{L} p_{Ml}(t) + \sum_{l=2}^{L}\frac{b_l(t)}{a_l(t)}, \quad (13)$$

and

$$c_1(t) \equiv \sum_{l=2}^{L}\frac{1}{a_l(t)}. \quad (14)$$

Then (11) and (12) can be rewritten as:

$$\lambda_M^*(t) = \frac{(c_0(t) + 2p_{M1}(t))a_1(t) + b_1(t)}{c_1(t)^* a_1(t) + 1}, \quad (15)$$

and

$$p_{A1}^*(t) = \frac{c_0(t)/2 + p_{M1}(t) - c_1(t)b_1(t)/2}{c_1(t)^* a_1 + 1}. \quad (16)$$

Participant 1 may be a buyer or a seller depending on the sign of $(p_{M1}^*(t) - p_{A1}(t))$, where $p_{M1}^*(t)$ is the solution for $p_{M1}(t)$ to be derived later.

B. Case 2: Zero Accepted Level ($p_{A1}(t) = 0$)

In this case, the bidding prices of Participant 1 are high, causing $p_{A1}(t) = 0$. The derivation is similar to Case 1 with

$$\lambda_M^*(t) = \frac{2p_{M1}(t) + c_0(t)}{c_1(t)}, \quad (17)$$

and

$$p_{A1}^*(t) = 0. \quad (18)$$

C. Case 3: Maximum Accepted Level ($p_{A1}(t) = \overline{p}_{A1}(t)$)

In this case, Participant 1's bidding price is low, resulting in $p_{A1}(t) = \overline{p}_{A1}(t)$ and the following energy clearing prices

$$\lambda_M^*(t) = \frac{\sum_{l=2}^{L}\left(\frac{b_l(t)}{a_l(t)} + 2p_{Ml}(t)\right)}{\sum_{l=2}^{L}\frac{1}{a_l(t)}}. \quad (19)$$

The derivation is similar to Case 1 with ECP determined by other participants' bids.

IV. Bidding Strategy and Self-Scheduling

For Participant 1's problem, using multipliers $\lambda_1(t)$ to relax demand constraints (7) the Lagrangian can be written as

$$L = E\sum_{t=1}^{T}\sum_{i=1}^{I}[C_{ti}(p_{ti}(t)) + \lambda_M^*(t)(p_{M1}(t) - p_{A1}^*(t))] + \sum_{t=1}^{T}\lambda_1(t)\left[p_d(t) - \sum_{i=1}^{I}p_{ti}(t) - E(p_{M1}(t) - p_{A1}^*(t))\right]. \quad (20)$$

The RHS of (20) is separable for a given $\{\lambda_1(t)\}$, and can be decomposed into individual thermal unit subproblems and a bidding subproblem. A two-level algorithm is developed, where at the low level, individual subproblems are solved, and at the high level, $\{\lambda_1(t)\}$ is updated.

A. Solutions of Thermal Subproblems

A **Thermal Subproblem** is

$$\min_{p_{ti}(t)} L_{ti}, \text{ with } L_{ti} = \min_{p_{ti}(t)} \sum_{t=1}^{T}\{C_{ti}(p_{ti}(t)) - \lambda_1(t)p_{ti}(t)\}.$$

This minimization is subject to individual unit constraints. With $\{\lambda_1(t)\}$ given, the subproblem is deterministic, and can be solved by using the method presented in [10].

B. The Solution of the Bidding Subproblem

The bidding subproblem is

$$\min_{p_{M1}(t),\, a_1(t),\, b-1(t)} L_B,$$

with

$$L_B \equiv E\sum_{t=1}^{T}\{\lambda_M^*(t)[p_{M1}(t) - p_{A1}^*(t)] - \lambda_1(t)\lfloor p_{M1}(t) - p_{A1}^*(t)\rfloor\}. \quad (21)$$

In (21), $\lambda_M^*(t)$ and $p_{A1}^*(t)$ are obtained from ISO scheduling as presented in Section III, and depend on bids of participants. This bidding subproblem is therefore stochastic. In the following, the deterministic version of the subproblem will be solved first. The stochastic version can be similarly solved except that the expectation of LB is optimized. Following the derivations of Section III, three cases will be considered, i.e.,

$$0 < p_{A1}(t) < \overline{p}_{A1}(t),\ p_{A1}(t) = 0,\ \text{and}\ p_{A1}(t) = \overline{p}_{A1}(t).$$

Case 1—Accepted Level in Bound ($0 < p_{A1}(t) < \overline{p}_{A1}(t)$)*:* The degeneracy of the bidding subproblem (21) will be analyzed first, and then a numerical method to obtain a solution is presented.

Solution Degeneracy Analysis: From the ISO load balance constraints (5), the net energy exchange between Participant 1 and the market is

$$P_{M1}(t) - p_{A1}(t) = \sum_{l=2}^{L} [p_{Al}(t) - p_{Ml}(t)]. \quad (22)$$

By substituting (22) into (21), L_B can be rewritten as

$$L_B = \sum_{t=1}^{T} \left\{ [\lambda_M^*(t) - \lambda_1(t)] \sum_{l=2}^{L} [p_{A1}(t) - p_{Ml}(t)] \right\}. \quad (23)$$

By substituting (10), (13) and (14) into (23), L_B becomes

$$L_B = \sum_{t=1}^{T} \tfrac{1}{2} \{ c_1(t) \lambda_M^{*2}(t) - [c_1(t)\lambda_1(t) + c_0(t)] \lambda_M^* + \lambda_1(t) c_0(t) \} \}$$

It is separable in time. To obtain its minimum, the partial derivatives with respect to $a_1(t)$, $b_1(t)$ and $p_{M1}(t)$ are set to zeros, i.e.,

$$\frac{\partial L_B(t)}{\partial a_1(t)} = \frac{\partial L_B(t)}{\partial \lambda_M^*(t)} \frac{\partial \lambda_M^*(t)}{\partial a_1(t)} = 0, \quad (24)$$

$$\frac{\partial L_B(t)}{\partial b_1(t)} = \frac{\partial L_B(t)}{\partial \lambda_M^*(t)} \frac{\partial \lambda_M^*(t)}{\partial b_1(t)} = 0, \quad (25)$$

$$\frac{\partial L_B(t)}{\partial P_{M1}(t)} = \frac{\partial L_B(t)}{\partial \lambda_M^*(t)} \frac{\partial \lambda_M^*(t)}{\partial P_{M1}(t)} = 0, \quad (26)$$

where

$$\frac{\partial \lambda_M^*(t)}{\partial a_1(t)} = \frac{c_0(t) + 2p_{M1}(t) - c_1(t)b_1(t)}{[c_1(t)a_1(t) + 1]^2},$$

$$\frac{\partial \lambda_M^*(t)}{\partial b_1(t)} = \frac{1}{c_1(t)a_1(t) + 1},$$

and

$$\frac{\partial \lambda_M^*(t)}{\partial p_{M1}(t)} = \frac{2a_1(t)}{c_1(t)a_1(t) + 1}.$$

It is clear that $(\partial \lambda_M^*(t)/\partial b_1(t))$ cannot be zero, therefore, the simultaneous equations (24)–(26) degenerate to

$$\frac{\partial L_B(t)}{\partial \lambda_M(t)} = c_1(t)\lambda_M^{*2}(t) - [c_1(t)\lambda_M^*(t) + c_0(t)] = 0. \quad (27)$$

With three variables and one constraint (27) for each t, the bidding subproblem has an infinite number of solutions. The degeneracy can be seen from Fig. 3 with the case of two participants. Suppose that Line 2 is the bidding price curve of Participant 2, and Line 1 is an optimal bidding strategy of Participant 1 with optimal a_1, b_1, p_{A1}, p_{M1} and λ_M. Another bid of Participant 1 is an equivalent solution if it satisfies $0 < b_1' < \lambda_M$ and $p_{M1}' = p_{M1} - (p_{A1} - p_{A1}')$ because it results in the same energy clearing price λ_M and the net energy exchange $(p_{M1} - p_{A1})$.

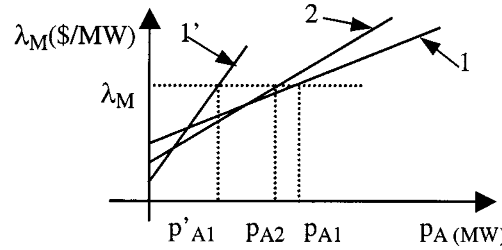

Fig. 3. Degeneracy of the deterministic bidding subproblem.

Obtaining a Solution: Having shown the degeneracy of (21) above, it is straightforward to obtain one of its solutions. Substituting $\lambda_M^*(t)$ in (15) and $p_{A1}^*(t)$ in (16) into (21), the subproblem cost at hour t can be written as

$$L_B(t) = \left\{ \left(\frac{c_0(t)a_1(t) + 2p_{M1}(t)a_1(t) + b_1(t)}{c_1(t)a_1(t) + 1} - \lambda_1(t) \right) \times \left(p_{M1}(t) - \frac{c_0(t)/2 + p_{M1}(t) - c_1(t)b_1(t)/2}{c_1(t)a_1(t) + 1} \right) \right\}. \quad (28)$$

To minimize $L_B(t)$, any two of its three decision variables $a_1(t)$, $b_1(t)$, are $p_{M1}(t)$ are fixed first, and the third one is optimized by a gradient method.

Case2—Zero Accepted Level ($p_{A1}(t) = 0$)*:* In this case, the accepted level for Participant 1 is zero, therefore the bidding subproblem cost is obtained by substituting (17) and (18) into (21):

$$L_B(t) = \frac{2}{c_1(t)} p_{M1}^2(t) + \left[\frac{c_0(t)}{c_1(t)} - \lambda_1(t) \right] P_{M1}(t). \quad (29)$$

Minimizing (29), the solution is

$$p_{M1}^*(t) = [\lambda_1(t)c_1(t) - c_0(t)]/4. \quad (30)$$

Participant 1 purchases $p_{M1}^*(t)$ from the market at the energy clearing price of the hour.

Case 3—Maximum Accepted Level ($p_{A1}(t) = \overline{p}_{A1}(t)$)*:* In this case, Participant 1's accepted level $p_{A1}(t)$ reaches the maximum, and the ECP is determined by bids of others. A solution to this bidding subproblem is to set $a_1(t)$ equal to 0, and $b_1(t)$ equal to Participant 1's marginal cost without bidding. This leads to $p_{A1}(t) = \overline{p}_{A1}(t)$ if Participant 1's generation cost is low.

The Stochastic Bidding Subproblem and Solution: Now consider the stochastic version. Following (21) and (28) the subproblem is changed to

$$\min E[L_B(t)] = \sum_{j=1}^{J} L_B^j(t) p^j. \quad (31)$$

In the above, $L_B^j(t)$ is similar to $L_B(t)$ in (28), and $E[L_B(t)]$ is a function of $a_1(t)$, $b_1(t)$ and $p_{M1}(t)$. With a derivation similar to that for the deterministic case, it can be shown that (25) and (26) degenerate to one as can be seen from Fig. 4. Suppose that Participant 2 has two possible bidding prices, Line 2L (bidding at low prices) and 2H (bidding at high prices), and Line 1 is an optimal bidding strategy for Participant 1. Another bid of

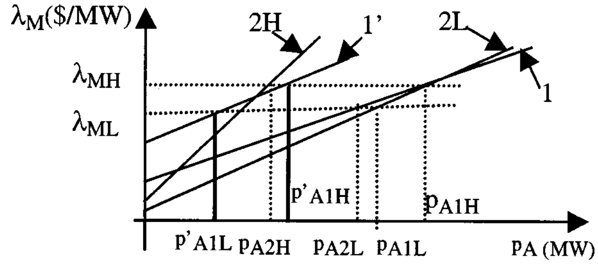

Fig. 4. Degeneracy of the stochastic bidding subproblem.

Participant 1 is an equivalent solution if it satisfies $0 < b_1' < \lambda_{ML}$, $a_1' = a_1$ and $p_{M1}' = p_{M1} - (p_{A1L} - p_{A1L}')$. The difference between this and the deterministic case is that a_1' is required to be equal to a_1 so that Line 1' is parallel to Line 1, and p_{M1}' also satisfies $p_{M1}' = p_{M1} - (p_{A1H} - p_{A1H}')$ to result in the same expected net energy exchange.

In solving the subproblem, b_1 is fixed, and p_{M1} and a_1 are optimized using the gradient method.

C. Update of Multipliers at the High Level

Multipliers are updated to maximize dual function $\phi(\lambda_1)$:

$$\operatorname*{Max}_{\lambda_1 \geq 0} \phi(\lambda_1), \text{ with } \phi(\lambda_1) \equiv \operatorname*{Min}_{p_{M1}(t),\, a_1(t),\, b_1(t),\, p_{ti}(t)} L, \quad (32)$$

where L is defined in (20). With a given set of subproblem solutions obtained at the low level, this is a deterministic problem. The subgradient of $\phi(\lambda_1)$ is a $T \times 1$ vector $g_{\lambda 1}$, and the tth element is

$$g_{\lambda_1}(t) = p_d(t) - \sum_{i=1}^{I} p_{ti}(t) - (p_{M1}(t) - p_{A1}^*(t)). \quad (33)$$

The dual problem is usually solved by a subgradient method [11]–[13], and is solved in this paper by the bundle trust region method (BTRM) presented in [14], [15] to improve the convergence. BTRM is a kind of bundle method that accumulates subgradients obtained thus far in a bundle, and use a convex combination of these subgradients to find a search direction. Obtaining the convex combination coefficients involves quadratic programming that is recursively solved in BTRM to reduce time requirements.

V. Numerical Results

The method has been implemented in C++ based on our hydrothermal scheduling code [8]–[10]. A data set provided by Northeast Utilities (NU) is used to demonstrate the capabilities of the method in handling various market situations. To simplify testing, all other market participants are aggregated as Participant 2 with three possible bidding strategies, bidding low (L), bidding medium (M), and bidding high (H) each with the same probability 1/3. The High value for $a_2(t)$ is 0.09, Medium 0.05, and Low 0.01. Participant 2's $b_2(t)$ and $p_{M2}(t)$ are varied to represent different market situations. With Case 1 as the base, four additional cases are created where $b_2(t)$ as a percentage of Participant 1's original marginal cost (without bidding) and $p_{M2}(t)$ as a percentage of Participant 1's original load are listed in Table I.

TABLE I
BIDDING PARAMETERS OF PARTICIPANT 2

	$b_2(t)$ (%)			$p_{M2}(t)$ (%)		
Case	L	M	H	L	M	H
1	80	100	120	20	30	40
2	80	100	120	10	30	50
3	60	100	140	20	30	40
4	100	120	140	20	30	40
5	20	40	60	20	30	40

$b_2(t)$ (%): $b_2(t)$ as a percentage of Participant 1's marginal cost without bidding;
$p_{M2}(t)$ (%): $p_{M2}(t)$ as a Percentage of Participant 1's Load.

TABLE II
COST COMPARISON OF MEAN METHOD AND STOCHASTIC METHOD

Case	Mean Meth ($)	Stoc Meth ($)	Savings (%)
1	106,169	105,759	0.39%
2	100,930	100,365	0.56%
3	102,420	101,431	0.97%
4	103,076	102,682	0.38%
5	105,334	104,800	0.51%

The method is compared with the "mean method" which considers Participant 2's bidding model as deterministic with each parameter set to its mean value. Comparison of testing results for Participant 1 based on 100 simulation runs is presented in Table II. Case 2 represents a volatile market with a large variance on $p_{M2}(t)$, and the saving of the stochastic method over the mean method is increased as compared with the base case. Case 3 also represents a volatile market with large variance on $b_2(t)$, and the saving is increased as compared with the base case. Cases 2 and 3 therefore show that the method works better than the mean method in volatile markets. Case 4 represents a more expensive market with the mean value of $b_2(t)$ increased by 20%, and the saving over the mean method is 0.38%. Case 5 represents a cheap market with the mean value decreased by 40%, and the saving is 0.51%. Cases 4 and 5 therefore show that the method works well in both expensive and cheap market situations.

The average CPU time for the mean method is 70 seconds, and that of the stochastic method is about 95 seconds. The CPU time requirements are close because both methods solve the bidding subproblem using a gradient method, and there is only one stochastic bidding subproblem.

VI. Conclusions

Built on an existing hydrothermal scheduling approach, an innovative model and an efficient Lagrangian relaxation-based method are presented to solve the bidding and self-scheduling problem. Numerical testing shows that the method effectively handles various market situations.

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