Bid Cost Minimization Versus Payment Cost Minimization: A Game Theoretic Study of Electricity Auctions

Feng Zhao, *Member, IEEE*, Peter B. Luh, *Fellow, IEEE*, Joseph H. Yan, *Senior Member, IEEE*, Gary A. Stern, and Shi-Chung Chang, *Member, IEEE*

***Abstract*—Currently, most independent system operators in the U.S. run auctions by minimizing the total bid cost ["bid cost minimization" (BCM)], and then calculate payments based on market clearing prices. Under this setup, the payment cost could be significantly higher than the minimized bid cost. Recently, an alternative auction mechanism that minimizes the consumer payment cost ["payment cost minimization" (PCM)] has been discussed. Literature has shown that for the same set of bids, PCM leads to reduced consumer payments. However, market participants may bid differently under the two auctions, and therefore, the payment reduction may not be realized. This necessitates the study of strategic behaviors of participants. In this paper, suppliers' bidding strategies in a day-ahead energy market are investigated for both auctions by using a game theoretic framework with Nash equilibrium as the solution concept. To simplify the solution process, the originally continuous strategies are discretized to form matrix games. Discretization may cause the loss of equilibria and the creation of artificial solutions. To reduce these side effects, "approximate Nash equilibria" are introduced to recover lost equilibria, and additional strategy samples are evaluated to eliminate artificially created solutions. Games are then solved by examining supplier payoffs obtained from running auctions. Characteristics of auctions are exploited, leading to improved computational efficiency. Numerical testing results show that the PCM leads to significant payment reductions and relatively small increases of production costs as compared to BCM. Also, the "hockey-stick" bidding is more likely to occur under BCM. Finally, long-term impacts of PCM are discussed, and whether it would lower costs to consumers in the long run, including capacity payments, remains to be investigated.**

***Index Terms*—Bid cost minimization (BCM), discretization, electricity auction, hockey-stick bidding, matrix game, Nash equilibrium, payment cost minimization (PCM), production efficiency, strategic behavior.**

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F. Zhao was with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269-2157 USA. He is now with the ISO New England, Holyoke, MA 01040-2841 USA.

P. B. Luh is with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269-2157 USA.

S.-C. Chang is with the Department of Electrical Engineering, National Taiwan University, Taipei 10617, Taiwan.

J. H. Yan and G. A. Stern are with the Market Strategy and Resource Planning, Southern California Edison, Rosemead, CA 91770 USA.

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I. Introduction

In deregulated wholesale electricity markets (e.g., day-ahead energy market) in the U.S., independent system operators (ISOs) use an auction mechanism to select supply and demand bids for energy and ancillary services. A payment mechanism is then used to calculate payments for the selected bids. Currently, most ISOs adopt a "bid cost minimization" (or BCM) auction that minimizes the total bid cost for the planning horizon to select bids and their associated levels. Then a "pay at clearing price" scheme is used to calculate payments based on uniform market clearing prices (MCPs) or locational marginal prices (LMPs). Under this setup, the payments made by consumers could be significantly higher than the minimized bid cost obtained from the auction. An alternative auction mechanism that minimizes the consumer payment cost ("payment cost minimization" or PCM) has been discussed.

Disparate views were held on these two auction mechanisms. On the one hand, BCM leads to maximization of social welfare if supply bids represent production costs, an assumption that usually does not hold [1], [2]. On the other hand, some recent studies on PCM show that it leads to reductions in consumer payments as compared to BCM, given the same set of bids [3]–[10]. Market participants, however, may not bid the same under different auction mechanisms. As a result, the possible payment reductions identified in these studies may not be realized. Moreover, efficiency and long-term impact of PCM were barely addressed [5]. All these necessitate the investigation of strategic bidding behaviors. This, however, is a difficult task in view of the interplay among participants through complex auctions.

This paper investigates electricity suppliers' strategic behaviors under BCM and PCM for a day-ahead energy market. The demand is assumed fixed without considering its strategic behaviors for simplicity. In view that market participants choose strategies to maximize their own profits and each one's profit is affected by others' strategies, game theory is a natural platform for such a study. Literature review on game theoretic studies of the two auctions is presented in Section II. Games for the two auctions are then formulated in Section III. For simplicity, transmission constraints are not considered, uniform MCPs are used, and startup costs are assumed to be fully compensated. Also, single-block bids are considered, and a supplier's strategy variables are bid block prices and bid startup costs for his/her generation units. These variables are continuous in nature, resulting in complex continuous games that are difficult to solve. To simplify the solution process, strategy variables are discretized into finite numbers of values, leading to matrix games. Considering that suppliers are noncooperative, Nash equilibrium where no supplier can gain by unilateral change of his strategy is used as the solution concept, following [12]–[16].

To obtain matrix payoffs, the BCM algorithm of [22] and the PCM algorithm of [8] are used. The matrix games for the two auctions are then solved by checking Nash equilibrium conditions for each tuple of suppliers' strategies in Section IV. Comparing to the solutions of original continuous games, discretization may cause the loss of equilibria and the creation of artificial solutions. To reduce these side effects, "approximate equilibria" are introduced to recover lost equilibria, and additional strategy samples are evaluated to eliminate artificially created solutions. Moreover, characteristics of the two auctions are exploited by examining suppliers' reaction points, leading to improved computational efficiency. With examples including an IEEE reliability test system, numerical testing results presented in Section V demonstrate the effectiveness of our methods to reduce the two side effects of discretization, and that the payment reduction effect under PCM over BCM still prevail when strategic behaviors of suppliers are considered. Also, the "hockey-stick" bidding is found more likely to occur under BCM. The long-term impacts of PCM are then discussed in Section VI.

II. Literature Review

Many game theoretic studies have been conducted for BCM. Nash equilibrium is often used as the solution concept since participants are noncooperative and each maximizes his own profit. In view that bid parameters such as bid prices are continuous, many focus on continuous games [11]–[16]. An analytical approach is used in [11] to investigate price spikes in California's day-ahead energy market based on a two-supplier Nash game. Linear bid price curves without startup costs are considered, and a supplier's strategy variable is his bid quantity. It is concluded that price spikes may be a result of gaming behaviors. This analytical approach is limited to problems with analytical solutions, a luxury that practical electricity auctions do not have. In [12]–[14], an approach based on mathematical programming is used for auctions without unit commitment decisions. Since these auction models are continuous and convex, their solutions are characterized by Karush-Kuhn-Tucker (KKT) conditions. Auction games are then formed where each supplier maximizes his own profit with KKT conditions as constraints. This approach requires convexity of the auction model and cannot be extended to more realistic auctions involving discrete unit commitment decisions. In [15] and [16], a coevolutionary method—a type of genetic algorithms, is used, allowing a population of potential equilibrium solutions to evolve and does not require convexity. The convergence, however, could be slow.

In view of the difficulties for solving continuous games, various studies focus on matrix games by discretizing continuous strategy variables, so that equilibria can be searched among a finite number of strategy tuples [17]–[19]. Discretization, however, may cause side effects, i.e., equilibria obtained from a matrix game may not represent those of the original game. A two-supplier game without unit commitment decisions is studied in [17], where a supplier's bid cost is proportional to his quadratic production cost, with the proportion being the strategy variable. These variables are discretized, and mixed strategies of the resulting matrix game are characterized by a set of linear complementarity conditions. It is shown that solutions may not resemble those of the original game, and a method for tuning the discretization process is introduced to recapture continuous solutions. In [18], the earlier process is extended to a three-supplier game, and difficulties with more participants are acknowledged because of the complexity of the approach. An auction with linear bid curves and without unit commitment decisions is considered in [19], and each supplier's strategy is the ratio of his bid price over his incremental production cost. A concept of approximate Nash equilibrium is introduced to capture the possibly missing continuous equilibria due to discretization, and is based on the distance in the strategy space. Approximate solutions are obtained by examining all strategy tuples, and it is concluded that transmission congestions may enhance the suppliers' ability to exercise market power.

In spite of the aforementioned results on BCM, very limited game theoretic studies can be observed on PCM. Simple two-supplier and three-supplier matrix games with suppliers bidding low or high are analyzed in [20]. Nash solutions are compared with those of BCM, and it is concluded that there might be a tradeoff between consumer payments and production efficiency for the two auctions. These results, however, are difficult to extend to more realistic problems, considering that matrix payoffs are obtained from analytically derived auction solutions, an approach that is impractical for complex auctions with discrete unit commitment decisions and more units and hours.

Our preliminary results on the two auction games with unit commitment decisions were presented in [21]. Yet issues such as side effects of discretization, characteristics of auctions, and production efficiency were not fully addressed.

III. Problem Formulation

In this section, matrix auction games under BCM and PCM are formulated for a day-ahead energy market with given system demand. For simplicity, transmission, minimum up/down time, and ramp rate constraints are not considered, and uniform MCPs are used. Also, single-block bid curves are used and bid startup costs are fully compensated. The demand at hour $t (1 \leq t \leq 24)$ is $P^D(t)$. There are I suppliers. Supplier $i (1 \leq i \leq I)$ has K_i generation units, and for his kth unit ($1 \leq k \leq K_i$), its production cost is characterized by the incremental energy cost $c_{ik,0}$ (in dollars per megawatthour) and the startup cost $S_{ik,0}$ (in dollars). The single-block bid for this unit is composed of the minimum generation $p_{ik}^{\min}$ (in megawatts), maximum generation $p_{ik}^{\max}$ (in megawatts), bid block price c_{ik} (in dollars per megawatthour), and bid startup cost S_{ik} (in dollars). Each supplier is considered to maximize his profit by choosing bid block prices and bid startup costs for his units.[1] Let γ_i denote supplier i's strategy for his K_i number of units, i.e., $\gamma_i \equiv \{(c_{ik}, S_{ik}) | k = 1 \text{ to } K_i\}$, and γ_{-i} denote the other suppliers' strategies. Given

[1]Based on ISO-New England's historical day-ahead bid data, minimum/maximum levels of units are usually fixed, and therefore are not considered as strategic variables.

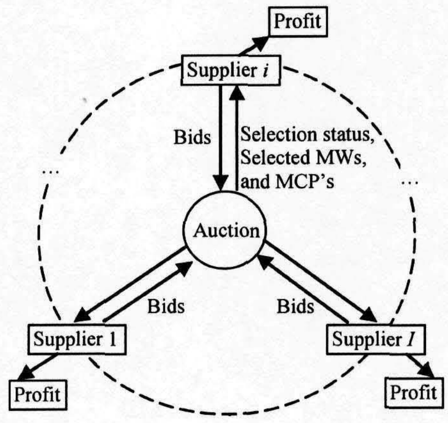

Fig. 1. Auction game among suppliers.

$\{\gamma_i\}_{i=1 \text{ to } I}$ from all suppliers, the BCM or PCM determines bid selections $\{x_{ik}(t)\}_{i,k,t}$ ("0" for "not selected" and "1" for "selected"), generation levels $\{p_{ik}(t)\}_{i,k,t}$ (in megawatthour) of selected bids, and uniform energy clearing prices $\{\text{MCP}(t)\}_t$ for each hour. At hour t, the revenue for the kth unit of supplier i is composed of the energy payment $\text{MCP}(t) \cdot p_{ik}(t)$ and the startup payment $x_{ik}(t)[1 - x_{ik}(t-1)]S_{ik}$, and the production cost is the energy cost $c_{ik,0}p_{ik}(t)$ plus the startup cost $x_{ik}(t)[1-x_{ik}(t-1)]S_{ik,0}$. The supplier's total profit π_i is therefore the total revenue minus the production cost over all hours and all his units, i.e.,

$$\pi_i(\gamma_i, \gamma_{-i}) = \sum_{k=1}^{K_i}\sum_{t=1}^{T}\{(\text{MCP}(t) - c_{ik,0})p_{ik}(t) + (1 - x_{ik}(t-1))x_{ik}(t)(S_{ik} - S_{ik,0})\}. \quad (1)$$

Since each supplier's profit is affected by others' strategies, a game is formed among all suppliers with Nash equilibrium as the solution concept.

In the following, games for the two auctions are described in Section III-A. To calculate suppliers' profits for a given tuple of strategies, the BCM algorithm of [22] is used as summarized in Section III-B and the PCM auction algorithm of [8] is used as summarized in Section III-C. Differences of the two auction algorithms are highlighted by a simple example in Section III-D.

A. Auction Games and Nash Equilibrium

With suppliers as players, games are formed under BCM and PCM. For each game, a supplier chooses a bid strategy to maximize his profit while taking into account the other suppliers' strategies. The structure of the games is depicted in Fig. 1, where an auction algorithm is used as the core to calculate suppliers' profits. Assuming that suppliers are noncooperative, Nash equilibrium where no supplier has incentive to unilaterally deviate from his strategy is used as the solution concept following [12]–[16].

Since a bid block price can take any value between zero and the price cap c^{cap} specified by the ISO, and a bid startup cost can be between zero and the maximum bid startup S^{cap},[2] strategy variables are continuous in nature. Let Γ_i be the set of continuous strategies of supplier i, i.e., $\Gamma_i = \{(c_{ik}, S_{ik})_{k=1,\dots,Ki}\}, 0 \le c_{ik} \le c^{\text{cap}}$ and $0 \le S_{ik} \le S^{\text{cap}}$. For a continuous game, a strategy tuple $\{\gamma_i^*\}_{i=1 \text{ to } I}$ is a Nash equilibrium if the following "continuous equilibrium condition" is satisfied:

$$\pi_i(\gamma_i^*, \gamma_{-i}^*) \ge \pi_i(\gamma_i, \gamma_{-i}^*) \quad \forall \gamma_i \in \Gamma_i \quad \forall i. \quad (2)$$

Continuous equilibria are difficult to obtain because the payoff function π in (1) has no explicit formula as it involves the solution of a complex mixed integer auction, and is nondifferentiable by having discrete variables $\{x_{ik}(t)\}_{i,k,t}$.

To simplify the solution process, supplier i's strategy variables for his kth unit are appropriately discretized (e.g., sampled from the continuous space at a constant interval) into a finite number of N_{ik} choices, i.e., $\{(c_{ik,n}, S_{ik,n}) | 0 \le c_{ik,n} \le c^{\text{cap}}, 0 \le S_{ik,n} \le S^{\text{cap}}, n = 1 \text{ to } N_{ik}\}$. Supplier i then has a total number of $\Pi_{k=1 \text{ to } K_i} N_{ik}$ strategies for his K_i units. Let Ω_i be the set of supplier i's strategy choices with $|\Omega_i| = \Pi_k N_{ik}$. A matrix game can then be formed with a total number of $\Pi_i |\Omega_i| = \Pi_i \Pi_k N_{ik}$ strategy tuples. Similar to (2), $\{\gamma_i^*\}_{i=1 \text{ to } I}$ (with $\gamma_i^* \in \Omega_i$) is a Nash equilibrium for the matrix game if the following "discrete equilibrium condition" is satisfied:

$$\pi_i(\gamma_i^*, \gamma_{-i}^*) \ge \pi_i(\gamma_i, \gamma_{-i}^*) \quad \forall \gamma_i \in \Omega_i \quad \forall i. \quad (3)$$

Concepts of dominance, equivalence, and interchangeability between two equilibria γ^* and γ^{**} can be established. The equilibrium γ^* dominates γ^{**} if all suppliers are no worse off and at least one is better off at γ^* as compared to γ^{**}, i.e.,

$$\pi_i(\gamma^*) \ge \pi_i(\gamma^{**}) \quad \forall i \text{ and} \quad (4)$$
$$\pi_j(\gamma^*) > \pi_j(\gamma^{**}) \quad \text{for some } j. \quad (5)$$

Two equilibria γ^* and γ^{**} are "equivalent" if the payoffs of any supplier at these two equilibria are the same, i.e.,

$$\pi_i(\gamma^*) = \pi_i(\gamma^{**}) \quad \forall i. \quad (6)$$

Finally, γ^* and γ^{**} are "interchangeable" if interchanging any supplier i's two strategies at γ^* and γ^{**} also leads to equilibria with the same payoffs, i.e.,

$$(\gamma_i^*, \gamma_{-i}^{**}) \text{ and } (\gamma_i^{**}, \gamma_{-i}^*) \text{ are also equilibria } \forall i \quad (7)$$

and they all have the same payoff tuples. These concepts will be used to analyze multiple equilibria in the solution process presented in Section IV.

As aforementioned, an auction algorithm will be used to calculate suppliers' profits. In the following, formulations and algorithms of BCM and PCM auctions are summarized.

B. BCM Auction

The BCM auction minimizes the total bid cost, i.e., the bid energy cost $c_{ik}(t)p_{ik}(t)$ plus the bid startup cost $[1 - x_{ik}(t-1)]x_{ik}(t)S_{ik}$ over all hours and all units, while

[2]Although ISOs do not announce bid startup caps, suppliers' startup bids are restricted by market mitigation rules, and this restriction is assumed to be reflected by S^{cap}.

satisfying the hourly system demand and individual bid level constraints. Mathematically, the auction problem is

$$\text{Min}_{\{x,p\}}\ J_B,\ \text{with}\ J_B \equiv \sum_{i=1}^{I}\sum_{k=1}^{K_i}\sum_{t=1}^{T}\{c_{ik}(t)p_{ik}(t) + (1-x_{ik}(t-1))x_{ik}(t)S_{ik}\} \quad (8)$$

$$\text{s.t.}\quad \sum_i\sum_k p_{ik}(t) - P^D(t) = 0 \quad \forall t \quad (9)$$

$$p_{ik}^{\min}\cdot x_{ik}(t) \le p_{ik}(t) \le p_{ik}^{\max}\cdot x_{ik}(t) \quad \forall i \quad \forall k \quad \forall t \quad (10)$$

$$x_{ik}(t) = 0 \text{ or } 1 \quad \forall i \quad \forall k \quad \forall t. \quad (11)$$

In view of problem separability, it can be effectively solved by using Lagrangian relaxation[3] or other mixed integer programming methods. The algorithm in [22] based on Lagrangian relaxation is used in this paper. After the aforementioned auction problem is solved, an economic dispatch problem considering all selected bids at each hour is used to determine the MCP. With $\psi_i^*(t)$ being the index set of supplier i's selected bids at hour t, economic dispatch for hour t is

$$\text{Min}_{\{p_{ik}\}} J^{ED}, \text{with } J^{\text{ED}} \equiv \sum_{i=1}^{I}\sum_{k\in\psi_i^*(t)}\{c_{ik}\cdot p_{ik}\} \quad (12)$$

s.t. (9)–(10) for hour t.

The MCP is then the Lagrange multiplier $\lambda(t)$ associated with the system demand constraint (9) in the earlier economic dispatch problem,[4] i.e.,

$$\text{MCP}(t) \equiv \lambda(t). \quad (13)$$

Note that the aforementioned process where MCPs are determined as a byproduct of the auction is consistent with the current practice of most ISOs (e.g., PJM and ISO-NE). Instead of using (13), the MCP for an hour is defined as the maximum price of selected bids, following [8], i.e.,

$$\text{MCP}(t) = \max_{\forall i,\ \forall k\in\psi_i^*(t)}\{c_{ik}(t)\} \quad \forall t. \quad (14)$$

From the derivations in [8, eq. (23)], it can be shown that (14) is equivalent to (13) if no unit is selected at its minimum or maximum generation limits.[5]

[3]A problem is separable and can be decomposed into individual subproblems by using Lagrangian relaxation if both the objective function and the constraints that couple the subproblems are additive in terms of subproblem variables [26]. For BCM, subproblems are formed based on individual bids after system demand constraints are relaxed. Each subproblem is solved by using dynamic programming where startup costs are modeled as state-transition costs.

[4]Since commitment variables are fixed in economic dispatch, startup costs are not reflected in MCPs. Consequently, they have to be separately considered as shown in (1). Various alternative pricing schemes have been studied in the recent literature; however, they are not considered here.

[5]Otherwise, the two definitions are different, i.e., a unit at its minimum generation level may set MCP under (14), but not under (13). Nevertheless, the same MCP definition (14) is used for BCM and PCM (for the latter, see the following section). Thus, in our opinion, the choice of the MCP definition should not alter the major conclusions on the comparison of the two auction methods.

TABLE I
DEMAND AND BIDS FOR THE 1-H FOUR-BID EXAMPLE

Bid i	$p_i^{\min}$ (MW)	$p_i^{\max}$ (MW)	Bid Block Price c_i ($/MWh)	Bid Startup Cost S_i ($)
1	0	45	10	8000
2	0	45	20	8000
3	0	12	100	20
4	0	40	30	2000
System Demand P^D = 100 MWh				

C. PCM Auction

The payment cost is composed of energy payment and startup payment, i.e.,

$$J_P = \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{k=1}^{K_i}\{\text{MCP}(t)p_{ik}(t) + x_{ik}(t)(1-x_{ik}(t-1))S_{ik}\}. \quad (15)$$

Since MCPs appear explicitly in the objective function, they are part of decision variables to be optimized as opposed to being byproducts as in BCM. Consequently, the MCP definition (14) needs to be operationalized by having the "MCP-bid inequality constraints" following [8], i.e.,

$$x_{ik}(t)(c_{ik}(t) - \text{MCP}(t)) \le 0 \quad \forall i \quad \forall k \quad \forall t. \quad (16)$$

The PCM auction is then to minimize (15) subject to (9)–(11) and (16).

As compared to BCM, the aforementioned problem is more difficult to solve in view of its nonseparability caused by the cross-product terms between $\text{MCP}(t)$ and individual bid variables in (15) coupled by the discrete nature of MCP in view of (16). An effective algorithm based on augmented Lagrangian relaxation and surrogate optimization[6] has been developed in [8], and is adopted in this paper.

D. Comparison of BCM and PCM Auctions

Given the same set of bids, the aforementioned two auctions could lead to significantly different results as illustrated in [9] by using a simple 1-h four-bid example with bid parameters and demand presented in Table I. It is clear that under both auctions, low-price bids 1 and 2 (representing baseload units and were "ON" initially) are both selected at their 45 MW capacity. The remaining 10 MW ($= P^D - 45 - 45$) demand gap is either met by bid 3 (initially "OFF") with a high bid block price, but a low bid startup cost, or bid 4 (initially "OFF") with a low bid block price, but a high bid startup cost. It can be seen that BCM selects bid 3 since its bid cost for the 10 MW, i.e., $100 \times 10 + 20$ [illustrated in Fig. 2(a) by the slashed and dotted area under bid 3], is lower than bid 4's cost of $30 \times 10 + 2000$. With bid 3 selected, the total bid cost is $\$2370 = 10 \times 45 + 20 \times 45 + 100 \times 10 + 20$. However, MCP is set by bid 3 at $100/MWh to pay all the selected megawatts, thus causing a total payment cost of $\$10\,020 = 100 \times (45 + 45 + 10) + 20$ [represented in

[6]The key idea of surrogate optimization is to approximately optimize the relaxed problem with respect to a particular bid one at a time. The analysis of its convergence can be observed in [25]. The algorithm has been used to solve PCM problems in [8] and [10].

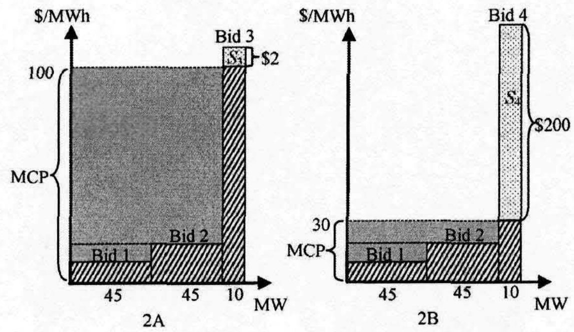

Fig. 2. Illustration of different bid selections under the two auctions.

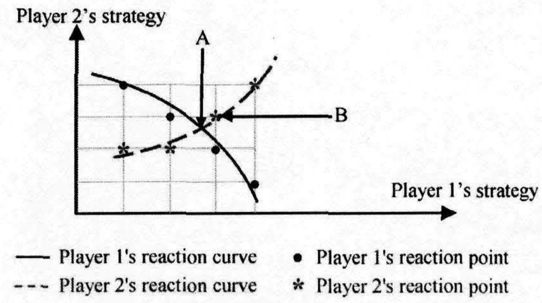

Fig. 3. Illustrative reactions of two-player continuous and discrete games.

Fig. 2(a) by the sum of slashed, dotted, and gray areas], which is significantly higher than the total bid cost of $2370.

In contrast, PCM selects bid 4, leading to a lower MCP of $30/MWh and a lower consumer payment of $\$5000 = 30 \times (45 + 45 + 10) + 2000$. It can be seen that PCM considers the impact of selecting a bid on the MCP and the total payment, as opposed to BCM that considers the total bid cost only. As a result, bids with high block prices, but low startup costs (e.g., bid 3), are less likely to be selected to fill a small demand gap under PCM, since they lead to high MCPs for all the MWs and result in high payments. However, these bids are more likely to be selected under BCM since their bid costs for a small megawatts are low.

The earlier differences are caused by the different weightings of bid prices and startup costs in their objective functions. With longer time horizons, the impact of startup costs on commitment decisions tends to be outweighed by bid prices, and the solutions to the two auctions are likely to be close. Constraints such as ramping and minimum up/down times also tend to have similar limiting effects. The reduction of consumer payments by PCM, however, could still be substantial by eliminating certain price spikes via selecting marginal units differently. It will be demonstrated in Section V that such effects still prevail (and are even reinforced) when strategic behaviors of suppliers are considered.

IV. Solution Methodology

In this section, matrix auction games resulting from discretization of continuous games are solved for BCM and PCM. In Section IV-A, possible loss of Nash equilibria and possible creation of artificial equilibria through discretization are discussed. The concept of approximate Nash equilibrium is introduced to capture the possibly missing equilibria, and additional strategies are randomly examined to eliminate artificial Nash solutions. In Section IV-B, the solution process for both BCM and PCM games is presented, where suppliers' payoffs for each cell of a matrix game are obtained by running an auction algorithm. These payoffs are then examined for Nash or approximate Nash equilibria. Dominance, equivalence, and interchangeability of solutions are discussed. In Section IV-C, characteristics of BCM and PCM are explored, thereby leading to improved computational efficiency by reducing the number of times to run auction algorithms.

A. *Effects of Discretization on Existence of Nash Equilibrium*

For simplicity of analysis, the originally continuous strategy variables are discretized into finite numbers of choices, resulting in a matrix game whose Nash solutions can be searched among a finite number of strategy tuples. The discretization, however, may cause loss or artificial creation of equilibria as discussed shortly.

1) Loss of Nash Equilibria: Since only a finite number of choices are considered in a matrix game, strategies at a continuous Nash equilibrium are likely to be missed after discretization. The matrix game may thus fail to capture continuous Nash equilibria, thus leading to the false conclusion of the nonexistence of a solution for the original problem. To illustrate this, consider a generic two-player continuous game shown in Fig. 3. Player 1's reaction curve is the trajectory of his best reaction to player 2's strategies, and is depicted by the solid curve. Player 2's reaction curve is depicted by the dash curve.[7] The intersection of these two reaction curves, i.e., point A, is a Nash equilibrium since each player is at his best reaction to the other's strategy. Now suppose that each player's strategy is discretized into four values. Then a reaction point of player 1 represents the best strategy among his four choices in reaction to a choice of player 2, and these points are depicted by "dots" in the figure. Player 2's reaction points are depicted by "stars." An intersection of these two sets of reaction points would be a Nash equilibrium for the discrete game. However, it can be seen that there is no such intersection, indicating the absence of Nash equilibrium for this discrete game. A continuous Nash equilibrium could thus be missed after discretization.[8]

To capture a possibly missing Nash solution, the standard discrete equilibrium condition (3) is loosened by ε to yield an "approximate Nash equilibrium" $\{\gamma_i^*\}_i$, $\gamma_i^* \in \Omega_i$, i.e.,

$$\pi_i(\gamma_i^*, \gamma_{-i}^*) \geq \pi_i(\gamma_i, \gamma_{-i}^*) - \varepsilon \quad \forall \gamma_i \in \Omega_i \quad \forall i. \tag{17}$$

Intuitively, no supplier $i (1 \leq i \leq I)$ can increase his profit by more than $\varepsilon (> 0)$ by unilaterally deviating from γ_i^*. This approximation parameter ε can be viewed as the amount of

[7]These reaction curves do not resemble those of the auction games considered in this paper, and are used here for illustration purpose only.

[8]The absence of discrete Nash equilibrium could also be caused by the nonexistence of continuous equilibria. The differentiation of these two causes is difficult, except for some small examples that can be fully analyzed.

profit that suppliers are willing to forfeit to form an equilibrium. This approximation concept is consistent with the standard Nash equilibrium concept since both are based on "profit," and (17) degenerates to the standard equilibrium condition (3) if ε is zero. Also, an approximate solution under a smaller ε is a solution under a larger ε. Note that this concept is different from [19], where approximation is based on "distance" in the strategy space.

2) Artificial Creation of Equilibria: While discretization may cause the loss of continuous Nash equilibria, it may also create artificial solutions. These solutions satisfy the finite number of discrete equilibrium conditions (3) or (17), but not the infinite number of continuous conditions (2). Therefore, they are not true equilibria, but "artificial" ones to the originally continuous game. For a candidate solution, "artificial equilibrium test" is conducted where additional strategies are sampled from suppliers' continuous strategy space. If the solution satisfies (3) or (17) for these additional samples, then it is considered as an equilibrium; otherwise, it is artificial, and therefore, eliminated.

B. Solving Matrix Games

Consider first the BCM matrix game. Matrix payoffs are supplier profits and obtained by running the auction algorithm of [22] for all strategy tuples in the increasing order of bid prices. Propositions to improve computational efficiency by reducing the number of auction runs will be presented in Section IV-C. After all payoffs are obtained, strategy tuples are examined by using condition (17) with $\varepsilon = 0$ to find Nash equilibria. Tuples that satisfy this condition are further examined by using the artificial equilibrium test, and those passing the test are Nash equilibria. If no equilibrium is obtained, then approximate equilibria are sought. To find them, the approximation parameter ε for each strategy tuple $\{\bar{\gamma}_i\}$ is first calculated based on (17), i.e.,

$$\varepsilon_{\{\bar{\gamma}_i\}} = \max_i \max_{\gamma_i \in \Omega_i} [\pi_i(\gamma_i, \bar{\gamma}_{-i}) - \pi_i(\bar{\gamma}_i, \bar{\gamma}_{-i})]. \tag{18}$$

To find the approximate equilibria that require the smallest ε, strategy tuples associated with these ε's are sequentially examined in the ascending order of ε by using artificial equilibrium tests. The examination terminates when part or all of the tuples for an ε pass the artificial equilibrium test or when ε exceeds a predetermined limit $\varepsilon_{\max}$. The earlier procedure may lead to zero, unique, or multiple equilibria or approximate equilibria. For multiple solutions, dominance, equivalence, and interchangeability are then examined. Nondominated, nonequivalent, and noninterchangeable solutions may exist, and can be analyzed by using the mixed strategy concept. However, this is beyond the scope of this paper. The complete solution process is depicted in Fig. 4, where the aforementioned steps are grouped into two parts: obtaining matrix payoffs on the left side and solving equilibria or approximate equilibria on the right.

The aforementioned process for BCM game also applies to PCM game with the auction algorithm of [8] replacing that of [22].

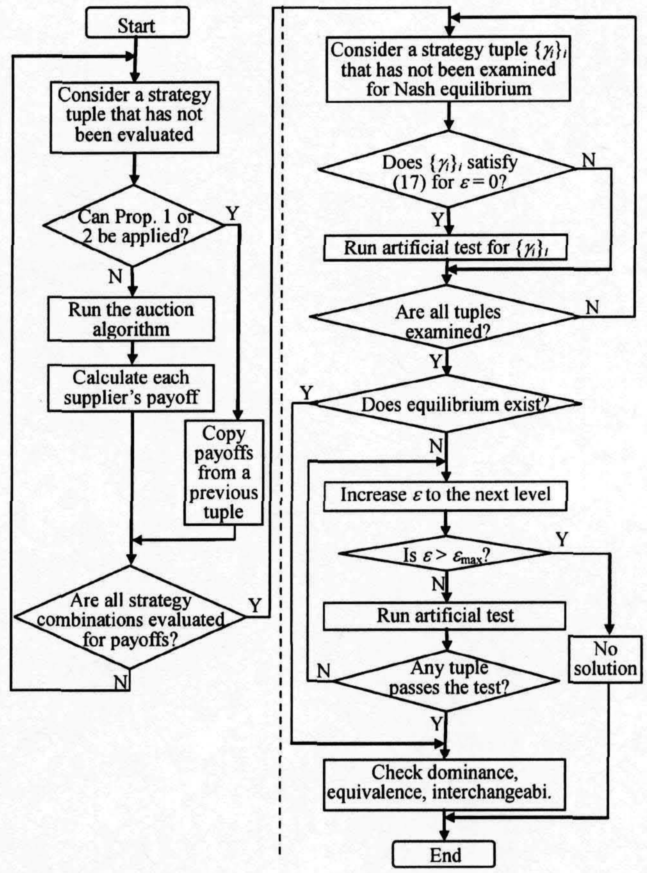

Fig. 4. Flowchart for solving matrix games.

C. Reducing the Number of Times for Running Auctions

Matrix payoffs can be obtained by running an auction algorithm for each strategy tuple. This could be time-consuming since an auction itself is a complex optimization problem and the number of strategy tuples could be large.[9] To improve computational efficiency, auction characteristics are explored shortly to reduce the number of auction runs.

For both auctions, it is intuitively clear that if a bid is not selected for any hour, then increasing its bid block price or bid startup cost while keeping other bids unchanged will not change the auction results. Consequently, the auction results for a strategy tuple can be used for other tuples with increased bid block prices or bid startup costs for nonselected units, thus reducing the number of auction runs. The earlier insight leads to the following Proposition 1 for BCM and Proposition 2 for PCM.

Proposition 1: If a bid is not selected at any hour under the BCM auction (8)–(11), then increasing the bid's block price or startup cost while keeping other bids unchanged will not change the auction solution.

Proof: Assume that bid m is not selected for any hour, i.e., $x^*_m(t) = 0$ and $p^*_m(t) = 0 \ \forall t$. With the increase of bid m's bid block price by $\Delta c_m (\geq 0)$ and its bid startup cost by $\Delta S_m (\geq 0)$,

[9]Our testing experience confirms that the time of computing matrix payoffs dominates the time of solving matrix games.

it will be demonstrated shortly that the BCM solution remains unchanged. To show this, the impact of Δc_m and ΔS_m on the optimal bid cost J_B^* is analyzed as follows:

$$\begin{aligned} J_B^*(c_m + \Delta c_m, S_m + \Delta S_m) &= \min_{\{x,p\}} {}_{\text{s.t. (9)–(11)}} J_B(c_m + \Delta c_m, S_m + \Delta S_m) \\ &= \min_{\{x,p\}} {}_{\text{s.t. (9)–(11)}} \Big\{ J_B(c_m, S_m) \\ &\quad + \sum_t [p_m(t)\Delta c_m + (1 - x_m(t-1))x_m(t)\Delta S_m] \Big\} \\ &\geq \min_{\{x,p\}} {}_{\text{s.t. (9)–(11)}} J_B(c_m, S_m) \\ &= J_B^*(c_m, S_m) \end{aligned} \quad (19)$$

where the inequality is a result of nonnegativity of the additional cost term $\Sigma\{p_m(t)\Delta c_m + [1 - x_m(t-1)]x_m(t)\Delta S_m\}$. It can be easily seen that, for the original auction solution with $x_m^*(t) = 0$ and $p_m^*(t) = 0$ for all t, this additional term is zero, and therefore, the inequality in (19) is actually an equality. As a result, the minimal bid cost $J_B^*(c_m + \Delta c_m, S_m + \Delta S_m)$ is obtained at the original solution, implying that the original solution is also a solution to the auction problem with Δc_m and ΔS_m.

The earlier conclusion for BCM also applies to PCM, as described shortly.

Proposition 2: If a bid is not selected at any hour under PCM (9)–(11) and (15)–(16), then increasing the bid's block price or startup cost while keeping other bids unchanged will not change the auction solution.

Proof: Following the notation of Proposition 1, it is demonstrated that Δc_m and ΔS_m will not change the PCM solution. To show this, first observe that Δc_m affects constraint (16) by changing it to

$$x_m(t)[c_m + \Delta c_m - \text{MCP}(t)] \leq 0 \ \forall t. \quad (20)$$

Clearly, (20) implies (16). Consequently

$$\begin{aligned} J_P^*(c_m + \Delta c_m, S_m + \Delta S_m) &= \min_{\{x,p\}} {}_{\text{s.t. (9)–(11),(20)}} J_P(c_m + \Delta c_m, S_m + \Delta S_m) \\ &\geq \min_{\{x,p\}} {}_{\text{s.t. (9)–(11),(16)}} J_P(c_m + \Delta c_m, S_m + \Delta S_m) \\ &\geq \min_{\{x,p\}} {}_{\text{s.t. (9)–(11),(16)}} J_P(c_m, S_m) \\ &= J_P^*(c_m, S_m) \end{aligned} \quad (21)$$

where the first inequality is due to the fact that the feasible region with (20) is a subset of the region with (16), and the second inequality follows the arguments similar to those of Proposition 1. Furthermore, the original solution with $x_m^*(t) = 0$ and $p_m^*(t) = 0 \ \forall t$ is also feasible for the problem with Δc_m and ΔS_m. Therefore, the inequalities in (21) are equalities and the original solution is an optimal solution for the auction problem with Δc_m and ΔS_m.

TABLE II
DEMAND AND PRODUCTION COST INFORMATION FOR EXAMPLE 1

Supplier i	$P^{\min}$ (MW)	$P^{\max}$ (MW)	Incremental Energy Cost $c_{i,0}$($/MWh)	Startup Cost $S_{i,0}$ ($)
1	5	45	50	1500
2	5	30	90	120

The previous two propositions are used in the solution process shown in Fig. 4 to reduce the number of auction runs, leading to an improved computational efficiency.[10]

V. Numerical Testing Results

The solution process shown in Fig. 4 for matrix games has been implemented in C++ and run on a Pentium-4 2.79-GHz PC with 512 MB of memory. In the following, four examples are presented. Example 1 examines two side effects of discretization on the existence of Nash equilibrium (i.e., loss of Nash equilibrium and creation of artificial equilibrium) by using a simple example, and demonstrates the effectiveness of our methods to reduce these side effects. Example 2 with two suppliers, three units, and 1 h compares bidding behaviors, payment costs, and production efficiencies under the two auctions, and demonstrates the significant reduction of consumer payment under PCM at a relatively small cost of production efficiency. Example 3 with four suppliers, eight units, and 24 h demonstrates the effectiveness of the two propositions to reduce computational time, and highlights the hockey-stick bidding behaviors under BCM. Example 4 then demonstrates the robustness of our algorithm based on a modified IEEE 24-bus reliability test system ignoring transmission by using Monte Carlo simulations for randomly selected load profiles and production costs. Complete testing data are available at http://www.engr.uconn.edu/msl/.

Example 1: Consider a two-supplier 1-h BCM example with each supplier having one unit. System demand is 40 MW. Production cost information of the units is presented in Table II. In addition, the energy bid price cap c^{cap} is \$200/MWh and the bid startup cost cap S^{cap} is \$3000. The maximum approximation parameter $\varepsilon_{\max}$ is \$200, and both units are "OFF" at hour 0.

In this example, supplier 1 alone can meet the 40 MW demand, while supplier 2 with 30 MW capacity cannot. As a result, supplier 1 is always selected to provide at least 10 MW and compensated for his bid startup cost. Supplier 1's bid startup cost is therefore always considered to be at the maximum S^{cap}, and his only decision variable is the bid block price c_1. Supplier

[10]Still, scalability is a concern for large problems. However, since the main purpose of this paper is to analyze the differences of BCM and PCM auction games as opposed to simulating an actual market, smaller but representative examples should be sufficient in serving this purpose. Nevertheless, it would be interesting to extend our approach to large problems in the future. One way is to aggregate similar participants into groups. Yet this may distort the competitive relations among them (consider an extreme case where all participants have similar portfolio of units and are aggregated into a single group). Another approach is to model a tractable number of participants for the game while fixing other participants' bids at their true costs. The underlying assumption is that only a few participants have gaming power while others can be considered as price-takers. The two approaches can also be combined. There will be a tradeoff between model granularity and computational efficiency.

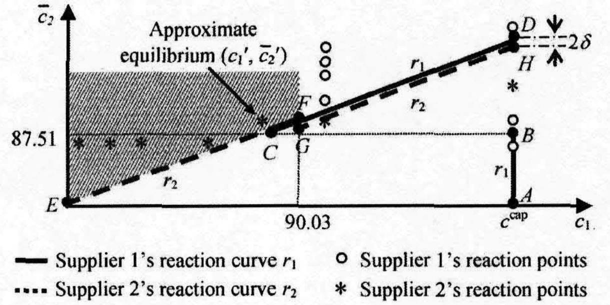

Fig. 5. Reaction curves and reaction points of Example 1 case 1.

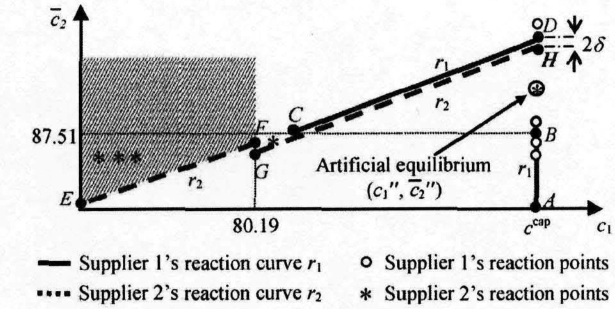

Fig. 6. Reaction curves and reaction points of Example 1 case 2.

2 has to compete with supplier 1 for the remaining 30 MW demand. Correspondingly, whether supplier 2 can win the 30 MW or not is solely dependent on his bid cost for the 30 MW. Supplier 2's decision can therefore be represented by his average bid cost $\bar{c}_2$ at $p_{2\,\max}$ $(= 30$ MW), i.e., $\bar{c}_2 \equiv c_2 + S_2/p_{2\,\max}$. In the following, loss of Nash equilibrium (case 1) and creation of artificial equilibrium (case 2) caused by discretization are illustrated. For each case, the continuous game is first analyzed based on suppliers' reaction curves, the matrix game is then solved to show the side effect of discretization, and finally, the method presented in Section IV-A is applied to reduce the side effect.

Case 1) Loss of Nash equilibrium: For the continuous game, let $c_1 \in [0, c^{\text{cap}}]$ and $\bar{c}_2 \in [0, c^{\text{cap}} + S^{\text{cap}}/p_{2\,\max}]$. As mentioned in Section IV-A, a Nash equilibrium for the continuous game is an intersection of suppliers' reaction curves, which are derived shortly.

Supplier 1's reaction curve: To obtain supplier 1's reaction curve r_1, observe that supplier 1 must provide at least 10 MW, and the remaining 30 MW $(= 40 - 10)$ goes to the supplier with a lower bid cost. Thus, supplier 1 has two options of reaction to a given $\bar{c}_2$. one is to bid high (i.e., $c_1 \times 30 > \bar{c}_2 \times 30$ or $c_1 > \bar{c}_2$) to only provide the guaranteed 10 MW and set MCP at the energy bid price cap, i.e., $c_1 = c^{\text{cap}}$, with a profit of $(c^{\text{cap}} - c_{1,0}) \times 10$ (excluding the secured profit obtained from S^{cap}). The second is to bid low to win the remaining 30 MW (i.e., $c_1 < \bar{c}_2$) while setting MCP as high as possible to maximize his profit, i.e., $c_1 = \bar{c}_2 - \delta$ (δ is a small positive number) with a profit of $(\bar{c}_2 - \delta - c_{1,0}) \times 40$. By comparing the profits under these two options, it can be concluded that if supplier 2 bids low, i.e., $(\bar{c}_2 - \delta - c_{1,0}) \times 40 < (c^{\text{cap}} - c_{1,0}) \times 10)$, or $\bar{c}_2 < (c^{\text{cap}} + 3c_{1,0})/4 + \delta$ (=87.51 with $\delta = 0.01$), then supplier 1 would not compete for the remaining 30 MW, but rather set high MCP with $c_1 = c^{\text{cap}}$. This part of reaction curve r_1 is depicted in Fig. 5 by the line segment between $A(c^{\text{cap}}, 0)$ and $B(c^{\text{cap}}, 87.51)$ for $\bar{c}_2 < 87.51$. Otherwise, supplier 1 would choose to win the remaining 30 MW by bidding $c_1 = \bar{c}_2 - \delta$ as depicted by the line segment between C (87.5, 87.51) and $D(c^{\text{cap}} - \delta, c^{\text{cap}})$ for $\bar{c}_2 \geq 87.51$.

Supplier 2's reaction curve: Similar to the aforementioned analysis, supplier 2 has two options for his reaction to a given c_1: one is to bid any value above c_1 (i.e., $\bar{c}_2 > c_1$), and in this case, supplier 2 will not be selected with zero profit. The second option is to bid low to win the 30 MW (i.e., $\bar{c}_2 < c_1$), but as high as possible to maximize his profit, i.e., $\bar{c}_2 = c_1 - \delta$ with a profit of $(c_1 - \delta - c_{2,0}) \times 30 + S_2 - S_{2,0}$. By comparing these two options, it can be concluded that if supplier 1 bids low (i.e., $(c_1 - \delta - c_{2,0}) \times 30 + S_2 - S_{2,0} < 0$, or $c_1 < 90.02 + \delta$), then supplier 2 would not compete for the 30 MW. This part of reaction curve r_2 is, in fact, a region—the shaded region above and including the line segment between E (0, 0.01) and F (90.03, 90.04) for $c_1 < 90.02 + \delta$ (=90.03 with $\delta = 0.01$) shown in Fig. 5. Otherwise, supplier 2 would bid $\bar{c}_2 = c_1 - \delta$ to win the 30 MW, as depicted by the line segment between G (90.03, 90.02) and $H(c^{\text{cap}}, c^{\text{cap}} - \delta)$ for $c_1 \geq 90.03$. It can be seen that the two reaction curves share the line segment *CF*. Thus, any point on *CF* is a Nash equilibrium where supplier 1 provides the entire demand and sets MCP. Point F with the highest c_1 (or the highest MCP) dominates other equilibria, and is taken as the solution.

For the matrix game, let supplier 1's strategy $c_1 \in [0, c^{\text{cap}}]$ be discretized at 0.9, 1.0, 1.1, 1.5, 1.7, and 1.9 times of his incremental energy cost $c_{1,0}$, plus an additional strategy at c^{cap}. Also, let supplier 2's $\bar{c}_2 \in [0, c^{\text{cap}} + S^{\text{cap}}/p_{2\,\max}]$ be discretized at the same discretization factors of his average production cost $\bar{c}_{2,0} (= c_{2,0} + S_{2,0}/p_{2\,\max})$, plus an additional strategy at $(c^{\text{cap}} + S^{\text{cap}}/p_{2\,\max})$. As mentioned in Section IV-A, a Nash equilibrium for the discrete game is an intersection of supplier 1' reaction points (depicting his best discrete choices in response to supplier 2's choices) and supplier 2's reaction points. In Fig. 5, supplier 1's reaction points are depicted by circles and supplier 2's reaction points are depicted by stars. It can be seen that there is no intersection between these two sets of points, implying that the earlier continuous equilibrium at point F has been lost. To capture this missing equilibrium, the approximate equilibrium concept presented in Section IV-A is used, and testing result shows that with $\varepsilon = 100$, a unique approximate equilibrium is obtained at $(c_1{\prime}, \bar{c}_2^{\prime}) = (1.7c_{1,0}, \bar{c}_{2,0}) = (85, 90.4)$.

Case 2) Creation of artificial equilibria: To illustrate the creation of artificial equilibria via discretization, the parameters of case 1 are modified as follows. Supplier 2's $c_{2,0}$ is changed from 90 to 80, and the discretization factors are changed to 0.9, 1.0, 1.1, and 1.7 plus cap. With analysis similar to case 1, the reaction curves for the continuous game and reaction points for the discrete game are obtained as shown in Fig. 6. Since there is no intersection between the two reaction curves, there is no Nash equilibrium for the continuous game. However, an artificially created equilibrium exists with $\varepsilon = 0$ at $(c_1'', \bar{c}_2'') = (c^{\text{cap}}, 142.8)$ as the intersection of the two sets of reaction points for the matrix game. This is caused by the loosened equilibrium condition (3) that examines only five strategies for each supplier, as compared

TABLE III
PRODUCTION COST INFORMATION FOR EXAMPLE 2

Supplier i	Unit k	$p^{\min}$ (MW)	$p^{\max}$ (MW)	Incremental Energy Cost $c_{ik,0}$ ($/MWh)	Startup Cost $S_{ik,0}$ ($)
1	1	0	30	32	0
	2	2	15	100	10
2	1	3	45	40	800

TABLE IV
SUPPLIER STRATEGIES FOR EXAMPLE 2

Supplier i	Unit k	Low ("L") (c^L, S^L)	Middle ("M") (c^M, S^M)	High ("H") (c^H, S^H)
1	1	(32, 0)	(48, 0)	(64, 0)
	2	(100, 10)	(150, 15)	(200, 20)
2	1	(40, 800)	(60, 1200)	(80, 1600)

TABLE V
RESULTS OF EXAMPLE 2

	P^D	25 MW	33 MW	70 MW
Bid Cost Min.	ε	0	0	0
	Equilibrium	(HX, X)*	(L/M-H, X)	(XX, H)
	MCP ($/MWh)	64	200	80
	Selected MWs	25–0–0	30–3–0	30–0–40
Payment Cost Min.	ε	0	106	0
	Equilibrium	(HX, X)	(L/M-X, M)	(XX, H)
	MCP ($/MWh)	64	60	80
	Selected MWs	25–0–0	30–0–3	30–0–40

* "X" could be "L," "M," or "H"

TABLE VI
COSTS WITH P^D = 33 MW FOR EXAMPLE 2

	Bid Cost Min.	Payment Cost Min.
Total Payment ($)	6620	3180
Total Bid Cost ($)	1580	2820
Total Prod. Cost ($)	1210	1820
Total Supplier Profit ($)	5410	1360

to the continuous equilibrium condition (2) that examines the entire continuous domain. To prevent this artificially created solution, additional strategies sampled uniformly from $[0, c^{\text{cap}}]$ for supplier 1 are used to further examine condition (3). It is found that given supplier 2's strategy $\bar{c}_2 = 142.8$, supplier 1 would deviate from $c_1 = c^{\text{cap}}$ (with a profit $1500) for a sampled strategy $c_1 = 120$ (with a profit of $2800). As a result, $(c_1'', \bar{c}_2'')$ does not satisfy (3) under $\varepsilon_{\max} = 200 (< 2800-1500)$, and therefore is an artificial solution and eliminated.

Example 2: Consider a two-supplier 1-h example with supplier 1 having two units and supplier 2 having one unit. Production cost information for the three units is presented in Table III. For a continuous game, each unit's bid block price c_{ik} is assumed to be in between 1.0 and 2.0 times of the corresponding incremental energy cost (i.e., $c_{ik,0} \in [c_{ik,0}, 2c_{ik,0}]$), and the bid startup cost S_{ik} is assumed to be within the range $[S_{ik,0}, 2S_{ik,0}]$. For a matrix game, three discrete choices are considered for each unit, i.e., (c^L, S^L), (c^M, S^M), and (c^H, S^H), respectively, representing 1, 1.5, and 2 times of $(c_{ik,0}, S_{ik,0})$, as presented in Table IV. Supplier 1 thus has nine (3 × 3) choices, and supplier 2 has three. The maximum approximation parameter $\varepsilon_{\max}$ is $200, and all units are "OFF" at hour 0.

Matrix games under the two auctions are solved. For each auction, three demand levels are considered: 25 MW representing low demand, where supplier 1's unit 1 (U_{11}) or supplier 2's unit 1 (U_{21}) alone is able to meet the demand; 33 MW representing medium demand, where U_{21} is able to meet the demand; and 70 MW representing high demand, where no unit alone is able to meet the demand.

Bid cost minimization:

Case 1) P^D = 25 MW: Nash equilibria (with $\varepsilon = 0$) are obtained at (HX, X), as presented in Table V with "X" representing "Do not care" (could be "L," "M," or "H"). It can be examined that these equilibria are equivalent and interchangeable by using the concepts discussed in Section III-A. At the equilibria, supplier 1 bids "H" on U_{11} because the unit has a bid cost of $1600 $(= 64 \times P^D + 0)$, lower than "L" of the competing U_{21} (with a bid cost of $1800 $= 40 \times P^D + 800$) for the demand P^D. The unit U_{11} therefore takes the market, while U_{12} and U_{21} are not selected.

Case 2) P^D = 33 MW: Equivalent and interchangeable Nash equilibria are obtained at (L/M-H, X) with $\varepsilon = 0$. To interpret this result, it can be verified that supplier 1 guarantees the selection of his U_{11} at its capacity 30 MW by bidding "L" or "M." To satisfy the remaining small gap of 3 MW, supplier 1's "H" on U_{12} (with high bid block price, but low startup cost) will win over supplier 2's "L" on U_{21} (with low bid block price and high startup cost) as a result of the characteristics of BCM discussed in Section III-D. Therefore, supplier 1 bids "H" on U_{12} and takes the remaining 3 MW regardless of supplier 2's bid.

Case 3) P^D = 70 MW: Equivalent and interchangeable Nash equilibria are obtained at (XX, H) with $\varepsilon = 0$. Supplier 2 must be selected to satisfy the high demand, and therefore bids "H" on U_{21} to set a high MCP. For supplier 1, his U_{11} must also be selected, but does not set MCP since any strategy for this unit has a block price lower than that of "H" on U_{21}. Also, supplier 1's U_{12} will not be selected since any bid on U_{12} has a block price higher than the bids on U_{11} and U_{21}. Therefore, supplier 1 bids "X" on his two units.

Payment cost minimization:

Case 1) P^D = 25 MW: The competition is between U_{11} and U_{21}, and the winner has the entire market and his bid price sets MCP. It turns out that the Nash solutions are obtained at (HX, X), same as those for BCM.

Case 2) P^D = 33 MW: Approximate Nash equilibria are obtained at (L/M-X, M) with $\varepsilon = 106$, indicating that being willing to comprise $106, no supplier has an incentive to deviate from his strategy. These approximate solutions are equivalent and interchangeable. It can be verified that supplier 1 guarantees the selection of his U_{11} at its capacity 30 MW by bidding "L" or "M." For the remaining 3 MW, supply 2's "M" on U_{21} guarantees its selection against supplier 1's any bid "X" on U_{12}. This is because even "L" on U_{12} will have a high block price that would set a high MCP for the entire demand, as discussed in Section III-D. Observe that the equilibria are different from those of BCM. To further compare the two auctions, costs at the two equilibria are presented in Table VI. It can be seen that compared with BCM, PCM yields a lower MCP. Also, payment

TABLE VII
PRODUCTION COST PARAMETERS IN EXAMPLE 3

Supplier i	Unit k	P_{ik}^{min} (MW)	P_{ik}^{max} (MW)	Inc. Energy Cost $c_{ik,0}$ ($/MWh)	Startup Cost $S_{ik,0}$ ($/Start)
1	1	12	125	15	3000
	2	5	25	45	1600
	3	2	8	82	50
2	1	15	147	20	2500
	2	6	30	42	1800
	3	2	10	85	25
3	1	3	10	75	100
4	2	2	5	92	20

TABLE VIII
24-H SYSTEM DEMAND FOR EXAMPLE 3

Hour	P^D (MW)	Hour	P^D (MW)	Hour	P^D (MW)
1	207.36	9	281.9	17	311.0
2	194.4	10	307.8	18	311.0
3	187.9	11	320.8	19	301.3
4	181.44	12	324.0	20	298.1
5	181.44	13	320.8	21	298.1
6	187.9	14	324.0	22	301.3
7	207.4	15	324.0	23	281.9
8	246.2	16	314.3	24	233.3

cost reduction $3440 (=6620 − 3180) is achieved at a relatively small increase $610 (=1820 − 1210) of production cost.

Case 3) $P^D = 70$ *MW:* Following an analysis similar to the aforementioned, it can be shown that the Nash solutions are the same as those for BCM.

Finally, it can be seen from Table V that suppliers generally do not bid at their production costs (represented by "L" in this example) under either of the two auctions, implying that none of the two mechanisms leads to the truthful revelation of suppliers' production costs.

Example 3: Consider a four-supplier eight-unit 24-h example. Production cost information of the units is presented in Table VII, with suppliers 1 and 2 each having three units, and suppliers 3 and 4 each having one unit. Supplier 1's unit 1 (U_{11}) and supplier 2's unit 1 (U_{21}) are baseload units with low incremental energy costs, but high startup costs; U_{13}, U_{23}, U_{31}, and U_{41} are small peaking units with high incremental energy costs, but low startup costs; and U_{12} and U_{22} are cycling units with incremental energy costs and startup costs in between those of baseload and peaking units. For a continuous game, each unit's bid block price c_{ik} is assumed to be in between 1.0 and 2.0 times of the corresponding incremental energy cost (i.e., $c_{ik} \in [c_{ik,0}, 2c_{ik,0}]$), and the bid startup cost S_{ik} is assumed to be within the range of $[S_{ik,0}, 2S_{ik,0}]$. For a matrix game, three discrete choices are considered for each unit, i.e., "L," "M," and "H," respectively, representing 1, 1.5, and 2 times of $(c_{ik,0}, S_{ik,0})$. Thus, suppliers 1 and 2 each has 27 [3 × 3 × 3] strategies, and suppliers 3 and 4 each has 3 strategies. The hourly system demand presented in Table VIII is based on the load curve of [23] with a rescaled peak load of 324 MW (representing 90% of the total generation capacity 360 MW). The maximum approximation parameter ε_{max} is $1000, and all units except the baseload ones are assumed "OFF" at hour 0.

Solutions for the two auction games are summarized in Table IX. Consider BCM first. The approximate solution is obtained at (L-H-H, H-H-H, L-L) with $\varepsilon = 158$. Supplier 1 bids "L" on his baseload U_{11} to have it fully selected for all hours while bidding "H" on his two other units to induce high MCPs. Supplier 1's "L-H-H" (i.e., with bid block prices of $15, $90, and $164/MWh) on his three units exemplifies the "hockey-stick" bidding behavior that has been observed in current BCM auctions. Supplier 2 bids in a similar manner for his three units except for "H" on his baseload U_{21} to set high MCPs during off-peak hours 1–8. Unlike suppliers 1 and 2's

TABLE IX
EQUILIBRIUM SOLUTIONS OF TWO AUCTIONS FOR EXAMPLE 3

Supplier i	Unit k	Bid Cost Min. with $\varepsilon = 158$	Payment Cost Min. with $\varepsilon = 775$
1	1	L	L
	2	H	H
	3	H	L
2	1	H	H
	2	H	H
	3	H	M
3	1	L	L
4	2	L	M

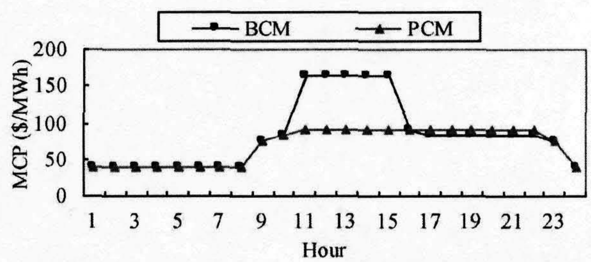

Fig. 7. Hourly MCPs under the two auctions for Example 3.

"H" on their peaking units, suppliers 3 and 4 bid "L" on their only peaking units since hockey-stick bidding cannot be applied to a supplier with a single unit. It can be seen that suppliers may not bid at their production costs, which are represented "L" in this example.

Now, consider PCM solution obtained at $\varepsilon = 775$ in Table IX. It can be seen that suppliers generally bid lower than the case under BCM. In particular, supplier 1 bids "L" on his peaking unit U_{13}, indicating no hockey-stick bidding for this supplier. This is because high-price bids on peaking units would have little chance to be selected under PCM in view of their major effects on lifting MCPs and payment cost. It can also be seen that suppliers may not bid at their production costs. To further illustrate the differences between the two auctions, 24 hourly MCPs are depicted in Fig. 7. It can be seen that for peak hours, MCPs under PCM are much lower than those under BCM, consistent with the discussion in Section III.

TABLE X
COMPARISON OF COSTS AND TIMES FOR EXAMPLE 3

	Bid Cost Min.	Payment Cost Min.
Total Consumer Payment	$ 591,537	$ 485,563
Total Supplier Profit	$ 456,282	$ 346,683
Total Prod. Cost	$ 135,255	$ 138,880
CPU Time with Prop. 1&2	22 min	53 min
CPU Time w/o Prop. 1&2	27 min	74 min

TABLE XI
PRODUCTION COST INFORMATION FOR EXAMPLE 4

Suppl.	Unit Group	N. of Units	Type	P^{min} MW	P^{max} MW	Heat-rate (Btu/kwh)	Startup Heat (Mbtu)
1	1	4	Coal	10	76	13311	596
	2	3	#6 Oil	15	100	8089	250
	3	4	#2 Oil	4	20	9859	5
2	1	4	Coal	20	155	9381	260
	2	5	#6 Oil	3	12	10178	38
3	1	2	Nuclear	40	400	9438	40000
	2	6	Hydro	0	50	*	0
4	1	1	Coal	35	350	9768	1915
	2	3	#6 Oil	20	197	8348	443

* Heat-rate not applicable for hydro units. In this example, the incremental energy cost for these units is fixed at $15/MWh.

The costs and CPU times under the two auctions are presented in Table X. Observe that the total consumer payment under PCM is $105 974 lower than that of BCM, representing 17.9% reduction in payment. This is a direct result of smaller MCPs under PCM during peak hours, as shown in Fig. 7. Also, observe that the production cost under PCM is slightly increased by $3625 (as compared to the payment reduction), indicating a relatively small loss of production efficiency. The total profit of suppliers is reduced by $109 599. This may lead to the concern of suppliers' ability to recover their capital costs. This issue could be addressed in a broader context by considering capacity markets, long-term contracts, etc. More will be discussed in Section VI. The CPU time for the PCM game is larger than that for the BCM game as expected.[11] Also, note that $\varepsilon_{\max}(=1000)$ is relatively small as compared to the costs and profits under the two auctions, implying an appropriate choice of $\varepsilon_{\max}$. To test the effectiveness of two propositions in Section IV-C, CPU times without implementing these propositions are also presented in the table. It can be seen that by implementing the propositions, the CPU times are reduced by 17% for BCM and 15% for PCM, demonstrating the effectiveness of the two propositions.[12]

Example 4: Consider a four-supplier, 32-unit, 24-h example based on the IEEE reliability test system [23] with transmission ignored. Each supplier has various groups of identical units with known heat rates (Btu/kWh) and startup heat inputs (MBtu), as presented in Table XI. For baseload units (coal or nuclear), a

[11]For the algorithm depicted in Fig. 4, the time for solving auction problems dominates the time for solving matrix games with payoff tuples given. Also, a PCM auction is usually more time-consuming to solve than a BCM auction in view of its inherent problem complexity (see [8] and [10] for details). Therefore, the overcall CPU time for the PCM game is longer than that of the BCM game.

[12]The number of strategy tuples determines the overall CPU time. Since this number is combinatorial, the CPU time increases exponentially with respect to the number of participants and discretization levels even with the implementation of the two propositions. Nevertheless, the examples presented in this section might be sufficient to illustrate our ideas.

TABLE XII
MEAN AND STANDARD DEVIATION OF FUEL PRICES FOR EXAMPLE 4

Fuel Type	Mean Price ($/Mbtu)	Stdev ($/Mbtu)
Coal	1.78	0.2
#2 Oil	10.0	2.0
#6 Oil	8.5	1.5
Nuclear	1.5	0

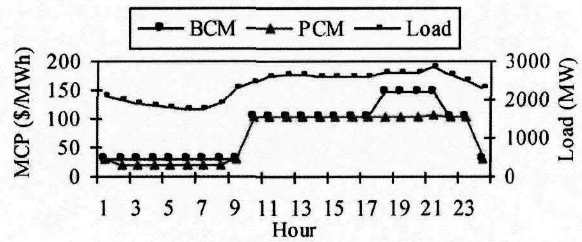

Fig. 8. Hourly MCPs under the two auctions for Example 4 case 1).

TABLE XIII
SIMULATION RESULTS FOR EXAMPLE 4

Case ID	Bid Cost Min. Consumer Payment ($)	Bid Cost Min. CPU Time (s)	Payment Cost Min. Consumer Payment ($)	Payment Cost Min. CPU Time (s)
1	4,896,344	535	4,320,833	1183
2	6,421,916	609	5,550,077	1406
3	1,007,678	595	828,837	1283
4	4,109,803	555	3,789,011	1212
5	909,762	541	668,940	1958
6	4,154,383	592	3,842,511	1241
7	2,635,261	577	2,179,108	1259
8	3,714,846	633	2,889,906	1249
9	3,223,347	638	2,655,805	1252
10	3,390,256	652	2,830,586	1255

supplier's strategy choices on the pair of bid block price c and bid startup cost S are 1.0 and 1.2 times of the corresponding production cost, i.e., $(c, S) = (c_0, S_0)$ or $(1.2c_0, 1.2S_0)$. For other units, a wider range of 0.8, 1.2, and 1.5 times are used since these units are likely to set the clearing prices. For simplicity, a supplier is assumed to apply the same strategy to all identical units in the same group, and all units except the baseload ones are "OFF" at hour 0. The approximation parameter $\varepsilon_{\max}$ is $3000. Ten cases were tested. For each case, the 24-h system demand is taken from a load profile of a day arbitrarily selected from the load year defined in [23], and unit production costs are randomly generated based on the fixed heat rates and startup heat inputs presented in Table XI and the Gaussian random fuel prices presented in Table XII.

The consumer payments under the two auctions for the ten cases are presented in Table XIII.[13] It can be seen that, for each case, the consumer payment for PCM is less than that for BCM. The average payment reduction for the ten cases is $490 798 (15.8% of the average payment cost under BCM), indicating significant cost savings for consumers. As a result, significant payment reductions achieved for a given set of bids, as reported in previous papers [3]–[10], still prevail within the game context. This is probably a result of not selecting small peaking

[13]Note that $\varepsilon_{\max}(=3000)$ is relatively small as compared to the costs in the table, implying an appropriate choice of its value.

TABLE XIV
CPU TIMES FOR EXAMPLE 4

CPU Time	Bid Cost Min.	Payment Cost Min.
Mean (sec.)	593	1330
Stdev (sec.)	41s	228.3

units with very high bid block prices as marginal units under PCM. To illustrate this, hourly MCPs for case 1 are depicted in Fig. 8. It can be seen that during hours 18–22 with high demand, MCPs under BCM are much higher than those under PCM, implying that bids with high block prices are selected to set MCP. Means and standard deviations of CPU times for the two auctions are presented in Table XIV. The mean time for PCM is larger than that for BCM as expected. Also, the standard deviations of CPU times are not large, indicating the robustness of the solution process of Fig. 4.

VI. LONG-TERM IMPACTS OF PCM

While the aforementioned sections focused on short-term market behaviors, long-term effects of auction mechanisms could not be ignored. While a full study of such issues require another paper, in this section, we shall share some of our ideas.

1) *Generation cost recovery*: Generators have two major types of costs: capital investment and operational costs. While the operational cost recovery can be addressed by short-term markets through make-whole payment,[14] capital cost recovery can be partially resolved by introducing additional long-term markets, e.g., forward capacity markets such as those in PJM and ISO-NE. If generators receive less profit from PCM as compared to BCM, then long-term markets might need to be appropriately designed for a fair return on the capital investment. It is possible that PCM may lead to an increase in long-term capacity payments while reducing short-term payments. If in the long run, generators must cover all their costs to enter and remain in the market, it is possible that total long-run costs to consumers will not differ between PCM and BCM. Further investigation is needed.
2) *Plant construction and fuel consumption*: As illustrated by the small 1-h example in Section III, PCM tends to favor a low-price high-startup-cost unit over a high-price low-startup-cost unit in meeting a small incremental demand. Consequently, it is possible that under PCM, fewer high-price peaking units will be built and the consumption of corresponding types of fuel will be reduced. However, these effects might be limited in practical situations with longer commitment periods and with the consideration of ramp rate and minimum up/down time constraints. The reasons are as follows. The differences in BCM and PCM arise from the different weightings of a unit's startup cost and bid price, as illustrated in Fig. 2. With longer commitment periods, the impact of startup costs on commitment decisions tends to be outweighed by bid prices, and PCM solutions may be closer to BCM solutions. Also, constraints such as ramping and minimum up/down times tend to have similar limiting effects. As a result, for a practical model, PCM solutions might not drastically deviate from BCM solutions, except for the deselecting of a few small high-price bids to reduce consumer payment costs. Therefore, it is possible that the long-term impact of PCM on plant construction plans and fuel mixes would be limited. The reduction of consumer payments, nevertheless, could still be substantial by eliminating certain price spikes via different selections of marginal units, as illustrated in Figs. 7 and 8. These need to be verified by studying long-term models including repeated day-ahead markets and capital investment models.
3) *Efficiency and investment signal*: While PCM tends to be less efficient than BCM, the difference might be limited under a practical auction model, as discussed before. If this is the case, then the impacts of efficiency loss and investment signals would be limited. These will need to be verified by studying long-term models.
4) *Efficient consumption of electricity*: To study this issue, price-sensitive demand bids have to be considered. These demand bids can be viewed as supply bids with negative prices and negative quantities, but without startup costs. As illustrated by the small 1-h example in Section III, startup costs are the main source of difference between BCM and PCM. In view that demand bids do not have startup costs, their presence might be expected not to have a major impact on the results. Consequently, the conclusion of this paper may still be valid with the consideration of demand bids, i.e., PCM may still tend to reduce the consumer payment with a relatively small loss of efficiency. Further investigation would be needed to confirm whether this is the case.

[14]Make-whole payment is the compensation paid to generators to cover their bid costs during a commitment period (e.g., 24 h). It is adopted by many ISO markets, and is also known as uplift payment.

VII. CONCLUSION

Two electricity auction mechanisms are discussed: BCM and PCM. Literature has shown that, for the same set of bids, PCM leads to the reduction of consumer payment as compared to BCM. This result, however, may not hold as suppliers could bid differently under the two auctions. This paper investigates suppliers' strategic behaviors in a simplified day-ahead energy market under the two auctions. Matrix Nash games are formed by discretizing the originally continuous strategy variables to simplify the solution process. The side effects of discretization and the methods to reduce them are presented. It is found that with strategic bidding, PCM leads to significant reductions in consumer payment at a relatively small loss of production efficiency. This is probably because the key differentiating feature of the two auctions, as previously reported, still prevails in the game context, i.e., bids with high bid block prices and low startup costs are less likely to be selected under PCM than the case under BCM. Moreover, the "hockey-stick" bidding behavior is found more likely to occur under BCM. Possible long-term impacts of PCM are also discussed. Whether PCM would lower costs to consumers in the long run, however, needs to be further investigated because capacity payments might have to be increased with the reduction in energy payments.

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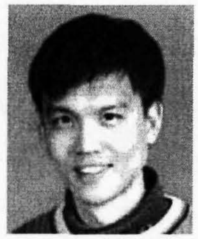

Feng Zhao (M'08) received the B.S. degree in automatic control from Shanghai Jiao Tong University, Shanghai, China, in 1998, the M.S. degree in control theory and control engineering from Tsinghua University, Beijing, China, in 2001, and the Ph.D. degree in electrical engineering from the University of Connecticut, Storrs, in 2008.

Currently, he is a Senior Analyst at the ISO New England, Holyoke, MA. His research interests include mathematical optimization, power system operations, and economics of electricity markets.

Peter B. Luh (M'80–SM'91–F'95) received the B.S. degree in electrical engineering from the National Taiwan University, Taipei, Taiwan, the M.S. degree in aeronautics and astronautics from the Massachusetts Institute of Technology, Cambridge, and the Ph.D. degree in applied mathematics from Harvard University, Cambridge, MA.

Since 1980, he has been with the University of Connecticut, Storrs, where he is currently the SNET Professor of Communications and Information Technologies. He is also a member of the Chair Professors Group in the Center for Intelligent and Networked Systems, Department of Automation, Tsinghua University, Beijing, China. His current interests include design of auction methods for electricity markets; electricity load and price forecasting with demand management; optimized resource management and coordination for sustainable, green, and safe buildings; planning, scheduling, and coordination of design, manufacturing, and service activities; decision making under uncertain, distributed, or antagonistic environments; and mathematical optimization for large-scale problems.

Prof. Luh is the Vice President for Publication Activities for the IEEE Robotics and Automation Society. He is an Associate Editor of *Institute of Industrial Engineers Transactions on Design and Manufacturing*, *Discrete Event Dynamic Systems*, and the *Acta Automatica Sinica*. He was the Founding Editor-in-Chief of the IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING (2003–2007), and the Editor-in-Chief of the IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION (1999–2003).

Joseph H. Yan (M'02–SM'06) received the Ph.D. degree in electrical and systems engineering from the University of Connecticut, Storrs.

He is currently the Manager of Market Analysis in the Market Strategy and Resource Planning, Southern California Edison, Rosemead. He was involved in areas of the electricity system operations, wholesale energy market analysis for both regulated and nonregulated affiliates, and market monitoring and market design for ten years in California. His research interest includes operation research, optimization, unit commitment/scheduling and transaction evaluation, and optimal simultaneous auction in deregulated ISO/RTO markets.

Gary A. Stern received the Ph.D. degree in economics from the University of California, San Diego.

He is currently the Director of the Market Strategy and Resource Planning, Southern California Edison Company, Rosemead. He reports to the Senior Vice President of the Power Production Business Unit, and manages a division responsible for resource planning, capacity and energy market design, and monitoring the wholesale electricity market in California. He is leading the development of the five-year corporate strategic plan. He is also currently leading a broad internal team in developing strategies to implement California's recent Greenhouse Gas legislation. He manages the design and the development of a capacity market for California, partnering with various other stakeholders, as well as overseeing the California ISO's electricity markets.

Shi-Chung Chang (M'87) received the B.S.E.E. degree from the National Taiwan University, Taipei, Taiwan, in 1979, and the M.S. and Ph.D. degrees in electrical and systems engineering from the University of Connecticut, Storrs, in 1983 and 1986 respectively.

From 1979 to 1981, he was an Ensign in the Chinese Navy, Taiwan. During 1985, he was a technical intern with Pacific Gas and Electric Company, San Francisco, CA. During 1987, he was a member of the Technical Staff, Decision Systems Section, ALPHATECH, Inc., Burlington, MA. Since 1988, he has been with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, where he was promoted to Professor in 1994. During 2001–2002, he was the Dean of Student Affairs and a Professor of electrical engineering, National Chi Nan University, Pu-Li, Taiwan. He was a Visiting Scholar at the Electrical and Computer Engineering Department, University of Connecticut, during his sabbatical leave in the 2003–2004 and 2006–2007 academic years. Besides the Electrical Engineering Department, he is now jointly appointed by the Graduate Institute of Industrial Engineering and the Graduate Institute of Communication Engineering, National Taiwan University. His research interests include optimization theory and algorithms, production scheduling and control, network management, Internet economics, and distributed decision making. He has been a Principal Investigator and Consultant to many industry and government funded projects. He has published more than 130 technical papers.

Dr. Chang is a member of Eta Kappa Nu and Phi Kappa Phi. He was the recipient of the 1996 Award of Outstanding Achievements in University-Industry Collaboration from the Ministry of Education for his pioneering research collaborations with Taiwan semiconductor industry on production scheduling and control.