The Subgradient Simplex Cutting Plane Method for Extended Locational Marginal Prices

Congcong Wang, *Student Member, IEEE*, Tengshun Peng, Peter B. Luh, *Fellow, IEEE*, Paul Gribik, and Li Zhang, *Member, IEEE*

Abstract—In current electricity markets of the USA, locational marginal prices (LMPs) are obtained from the economic dispatch process and cannot capture costs associated with commitment decisions. The extended LMPs (ELMPs) were established as the optimal Lagrangian multiplier of the dual of the unit commitment and economic dispatch problem. Commitment related costs are included and uplift payments are minimized. To obtain ELMPs, the dual problem should be solved with multiplier optimality and computational efficiency. Subgradient methods suffer from the multiplier zigzagging difficulty. Cutting plane methods encounter computational complexity issues in calculating query points. In this paper, a subgradient simplex cutting plane method is developed to obtain ELMPs. Transmission is not considered for simplicity, while key features of ELMPs are still captured. By innovatively using subgradients and simplex tableaus, query points are efficiently obtained through an adaptive three-level scheme. A query point along the subgradient is easily calculated at Level 1. As needed, Level 2 obtains Kelley's query point and Level 3 obtains the Chebyshev center, both by pivoting simplex tableaus. Numerical results show that the optimal multiplier is efficiently obtained.

Index Terms—Cutting plane methods, electricity markets, extended locational marginal prices (ELMPs), Lagrangian relaxation.

I. INTRODUCTION

I N the current wholesale electricity markets of the USA, the auction mechanism selects generation offers and their corresponding levels to minimize the total bid cost. The congestion-dependent locational marginal prices (LMPs) are then determined in the economic dispatch process with fixed unit commitment decisions. Start-up and no-load costs associated with unit commitment decisions cannot be included in LMPs, resulting in significant uplift payments [1]. To improve price signals, extended LMPs (ELMPs) were developed in [2] as the optimal Lagrangian multiplier of the dual of the unit commitment and economic dispatch (UCED) problem. The corresponding optimal dual values as a function of demand form the convex hull of the total cost function. Commitment related costs are included in ELMPs, and uplift payments are minimized. Economic significance of ELMPs as compared with that of the current LMPs was presented in [2], and the following will focus on developing an efficient algorithm to obtain the optimal multiplier as ELMPs. For simplicity but without loss of key features of ELMPs, transmission capacity constraints and ancillary services are not considered.

The mixed-integer UCED problem has been well solved by branch and cut methods, however, in the primal space. To obtain ELMPs, multiplier optimality rather than the optimality or feasibility of the primal solution is required. Traditional subgradient methods suffer from the multiplier zigzagging difficulty [3], and are thus ineffective in obtaining the optimal multiplier. Central cutting plane methods have been used to solve non-differentiable optimization problems [4], [5]. Given an initial polyhedron containing the optimal solution, non-optimal portions of the polyhedron are iteratively cut off by constructing cuts from query points [5]. A key issue is to select query points such that effective cuts can be constructed. Without knowing in advance which part to cut off, various centers deep inside the polyhedron have been investigated as query points. However, centers such as the center of gravity or the analytic center can be computationally expensive to obtain.

In this paper, a subgradient simplex cutting plane method (SSCPM) is developed to efficiently solve the dual problem. After the literature review in Section II, the UCED problem is formulated, and the dual problem is obtained by using Lagrangian relaxation in Section III. The SSCPM is developed in Section IV to solve the dual problem by innovatively using subgradients and simplex tableaus. Given an initial polyhedron and query point, cuts are constructed from the query point. In the updated polyhedron, an adaptive three-level scheme is established to obtain the next query point. Rather than calculating centers of the polyhedron through an expensive procedure, a point along the subgradient is easily obtained as the query point at Level 1. However, this point may not always be deep inside the polyhedron, resulting in ineffective cuts. In this case, Kelley's query point is calculated at Level 2. If effective cuts cannot be constructed at Kelley's point, then the Chebyshev center is obtained at Level 3, and the process repeats. By pivoting simplex tableaus, query points at Levels 2 and 3 are efficiently obtained.

Numerical results are presented in Section V. A simple two-hour example illustrates the adaptive three-level process to cut off non-optimal multipliers. A 24-hour 32-unit example based on the IEEE Reliability Test System shows that our algorithm efficiently obtains the optimal multiplier. A MISO-sized problem for the day-ahead market and one hundred Monte Carlo simulation runs then demonstrate that our method solves large problems with robust performance.

The preliminary results were presented at the IEEE Power and Energy Society General Meetings in 2009 [6] and 2010

[7]. These results are extended by improving the method, establishing the adaptive transition mechanism, enabling the resolution of larger problems, and adding large examples.

II. LITERATURE REVIEW

The dual problems of mixed-integer minimization derived from Lagrangian relaxation are concave and non-differentiable [3]. Many methods have been developed to solve them, including subgradient methods, cutting plane methods, column generation methods [8] and multiplier adjustment methods [9] as summarized in [10] and [11]. The subgradient method, originally presented in [12], iteratively updates multipliers by moving along a subgradient direction obtained after minimizing the relaxed problem [13]. This method is easy to implement and works well for many applications [14]. However, minimizing the relaxed problem can be costly, and multipliers may zigzag across facet intersections of the dual function (each facet corresponds to one solution of the relaxed problem) [15]. To overcome the zigzagging difficulty, the bundle method obtains an ascent direction from subgradients of neighboring multipliers [15], [16]. The extreme-point subgradient method developed in [17] is of this category. Obtaining such an ascent direction can be expensive. More importantly, as the problem size increases, the number of solutions increases drastically for NP hard problems, causing the size of facets to decrease and rendering the direction to ascend within a very small neighborhood. The surrogate subgradient method was developed to reduce the complexity of minimizing the relaxed problem by obtaining an approximate solution [18], [19]. The resulting direction is smooth and the zigzagging difficulty is alleviated. Its step-sizing rule, however, is a major challenge and is currently under investigation [20].

Cutting plane methods obtain the optimal multiplier by iteratively constructing cuts from query points to cut off non-optimal portions of an initial polyhedron, either in the multiplier space or in the epigraph space [4], [5]. In either space, a key issue is to choose query points. Kelly's method obtains the query point by maximizing the dual function through its iteratively updated polyhedral approximations [21]. The approximation can be poor, resulting in ineffective cuts and the method may converge slowly as shown in [22]. Central cutting plane methods obtain query points at the centers of iteratively updated polyhedrons [5]. Centers are less sensitive to the quality of approximations and "half" of the polyhedron can always be cut off. Nevertheless, centers such as the center of gravity or the analytic center can be computationally expensive to obtain. In addition, when solving piecewise linear dual problems, central cutting plane methods may be slow to identify the optimum [4], [23].

Given the query points, effective cuts can be more conveniently constructed in the epigraph space than in the multiplier space. The subgradient simplex cutting plane method will be developed in the epigraph space to obtain the optimal multiplier with improved efficiency by innovatively using subgradients and simplex tableaus.

III. PROBLEM FORMULATION

The UCED problem is formulated following the standard procedure in Section III-A to minimize the total bid cost given system demand. For simplicity, transmission capacity constraints and ancillary services are not considered. The dual problem is obtained by Lagrangian relaxation following [2] in Section III-B.

A. Unit Commitment and Economic Dispatch Problem

Consider an energy market with I units and a time horizon T. For unit i at time t, state $x_i(t)$ is the number of time periods it has been on $(+)$ or off $(-)$ since the previous on/off transition. The commitment decision $u_i(t)$ is whether unit i is up (1) or down (-1) at time t, satisfying the minimum up/down time constraints, i.e., unit i should be kept on if it is online for less than the minimum up time τ_i^u, and kept off if it is offline for less than the minimum down time τ_i^d:

$$u_i(t) = 1, \quad if \ 1 \leq x_i(t) < \tau_i^u, \tag{1}$$

$$u_i(t) = -1, \quad if \tau_i^d < x_i(t) \leq -1, \quad \forall i, \forall t. \tag{2}$$

The state $x_i(t)$ accumulates to $-\tau_i^d$ or τ_i^u if there is no start-up or shut-down; otherwise $x_i(t)$ is 1 or -1:

$$x_i(t+1) = x_i(t) + u_i(t), \ if \ x_i(t)u_i(t) > 0,$$
$$-\tau_i^d < x_i(t) < \tau_i^u, \tag{3}$$
$$x_i(t+1) = x_i(t), \ if \ x_i(t)u_i(t) > 0,$$
$$x_i(t) = -\tau_i^d \ or \ \tau_i^u, \tag{4}$$
$$x_i(t+1) = u_i(t), \ if \ x_i(t)u_i(t) < 0, \tag{5}$$
$$x_i(t), u_i(t) \in N, \ \forall i, \forall t. \tag{6}$$

Unit i should satisfy generation capacity constraints if it is online, i.e., generation $p_i(t)$ is in-between the minimal and maximal levels p_{it}^{min} and p_{it}^{max}:

$$p_{it}^{\min} \leq p_i(t) \leq p_{it}^{\max}, \quad if \ u_i(t) > 0, \tag{7}$$

$$p_i(t) = 0, \quad if \ u_i(t) < 0, \quad \forall i, \forall t. \tag{8}$$

Ramp rate constraints state that between two successive online periods, unit i cannot ramp up/down beyond ramp limit Δ_{it}:

$$-\Delta_{it} \leq p_i(t) - p_i(t-1) \leq \Delta_{it}, \ if \ x_i(t), u_i(t) > 0, \ \forall i, \forall t. \tag{9}$$

There is a special requirement if unit i is just turned on:

$$p_i(t) \leq p_{it}^{\min} + \Delta_{it}, \quad if \ x_i(t) < 0, u_i(t) > 0, \ \forall i, \forall t. \tag{10}$$

For unit i at time t, the bid cost includes the energy cost $C_{it}(p_i(t))$ as a convex and piecewise linear function of $p_i(t)$ with $C_{it}(0) = 0$ for a step offer curve, the time-invariant start-up cost S_i^{up} incurred when the unit is turned on $(x_i(t) < 0, u_i(t) > 0)$, and the no-load cost S_{it}^{NL} if the unit is online $(u_i(t) > 0)$.

In addition to individual unit constraints (1)–(10), system demand constraints require the total generation equal the system demand $p^D(t)$ for all time periods:

$$\sum_{i=1}^{I} p_i(t) = p^D(t), \quad \forall t. \tag{11}$$

The objective is to minimize the total bid cost:

$$\min_{\substack{\{u_i(t)\} \\ \{p_i(t)\}}} \sum_{t=1}^{T} \sum_{i=1}^{I} \left\{ C_{it}(p_i(t)) + S_i^{up}(x_i(t), u_i(t)) + S_{it}^{NL}(u_i(t)) \right\}. \tag{12}$$

Since the objective function and the system demand constraints which couple all units together are additive, the UCED problem is separable [3].

B. Dual Problem

The Lagrangian function is obtained by relaxing the system demand constraints using Lagrangian multipliers $\{\lambda(t)\}$:

$$L(\lambda, u, p) = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{I} \left(C_{it}\left(p_i(t)\right) + S_i^{up}\left(x_i(t), u_i(t)\right) \right. \right.$$
$$\left. \left. + S_{it}^{NL}\left(u_i(t)\right) \right) + \lambda(t)\left(p^D(t) - \sum_{i=1}^{I} p_i(t) \right) \right\}. \quad (13)$$

The relaxed problem minimizes (13) given λ, and can be decomposed into unit-level subproblems. The subproblem for each unit i, $i = 1, \ldots, I$ is

$$\min_{\{u_i(t)\},\{p_i(t)\}} L_i, \quad \text{with}$$

$$L_i \equiv \sum_{t=1}^{T} \left(C_{it}\left(p_i(t)\right) - \lambda(t)p_i(t) + S_i^{up}\left(x_i(t), u_i(t)\right) \right.$$
$$\left. + S_{it}^{NL}\left(u_i(t)\right) \right) \quad (14)$$

subject to individual unit constraints (1)–(10). Denoting $L_i^*(\lambda)$ as the minimized subproblem cost in (14), the dual problem is to obtain the multiplier that maximizes the concave and piecewise linear dual function [3]:

$$\max_{\lambda} \quad q(\lambda), \quad \text{with}$$

$$q(\lambda) \equiv \sum_{i=1}^{I} L_i^*(\lambda) + \sum_{t=1}^{T} \lambda(t)p^D(t). \quad (15)$$

Since the T multipliers relax equality demand constraints, the dual problem is an unconstrained maximization. Its epigraph form with variables $\lambda \in R^T$ and $z \in R$ is obtained as [3], [5]

$$\max_{(\lambda, z)} \quad z$$
$$\text{s.t.} \quad z \leq q(\lambda). \quad (16)$$

IV. SOLUTION METHODOLOGY

The subgradient simplex cutting plane method is developed in this section to solve the dual problem by iteratively cutting off non-optimal solutions. Each iteration contains two steps: constructing cuts from query points by using subgradients and dual values in Section IV-A, calculating query points by using subgradients or simplex tableaus through an adaptive three-level scheme in Section IV-B.

A. Cut Construction

An initial polyhedron P^0 containing the optimal solution is first determined as

$$P^0 = \{(\lambda, z) | 0 \leq \lambda_t \leq \lambda_t^{\max}, t = 1, \ldots, T; 0 \leq z \leq z^{\max}\}. \quad (17)$$

If the lower bounds are not zero, they can be converted to zero by shifting. By using heuristics and to be conservative, the upper bounds are determined based on the most expensive generation offers to avoid the exclusion of possible optimal solutions [24]. If the optimal solutions are excluded, the solution obtained will fall to the upper bounds, and these bounds are then increased
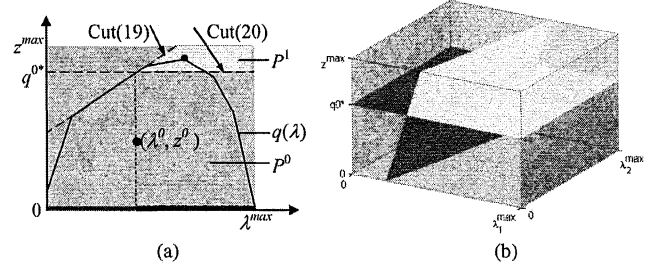

Fig. 1. Initial polyhedron P^0 [the square in (a) and the cube in (b)] is updated to the polyhedron P^1 by cuts (19) and (20) [the dashed lines in (a) and the hyperplanes in (b)].

based on heuristics. An initial query point (λ^0, z^0) is selected inside P^0, e.g., as its center.

Given the query point at iteration k, cuts are constructed by using the subgradient $g(\lambda^k)$ and the dual value $q(\lambda^k)$. To obtain $g(\lambda^k)$ and $q(\lambda^k)$, subproblems (14) are solved by dynamic programming (DP) in [25], or its enhancement which handles units with ramp rate constrains as presented in [26]. Optimal subproblem solutions are obtained in these DP processes. Otherwise, the optimal multiplier can be cut off by an inaccurate cut. After decisions on commitment $\{u_i(t)\}$ and dispatch $\{p_i(t)\}$, $i = 1, \ldots, I$, $t = 1, \ldots, T$ are obtained, the dual value $q(\lambda^k)$ is evaluated, and the subgradient $g(\lambda^k) = [g_1(\lambda^k), g_2(\lambda^k), \ldots, g_t(\lambda^k), \ldots, g_T(\lambda^k)]^T$ of the Lagrangian function (13) is obtained as the level of constraint violation:

$$g_t(\lambda^k) \equiv p^D(t) - \sum_{i=1}^{I} p_i(t), \quad t = 1, \ldots, T. \quad (18)$$

Orthogonal to the subgradient $g(\lambda^k)$, a tangent to the dual function is obtained at $(\lambda^k, q(\lambda^k))$, and the concave dual function $q(\lambda)$ lies below it. The portion of P^k above the tangent is non-optimal, and is thus cut off by the following cut:

$$z \leq q(\lambda^k) + g^T(\lambda^k)(\lambda - \lambda^k). \quad (19)$$

In addition, the largest dual value obtained thus far q^{k^*} is iteratively updated based on $q(\lambda^k)$, and serves as a lower bound on $q(\lambda^*)$. The portion of P^k lying below it is cut off by the cut:

$$q^{k^*} \leq z. \quad (20)$$

The polyhedron P^k is thus updated by (19) and (20) as

$$P^{k+1} = P^k \cap \left\{ (\lambda, z) | z \leq q(\lambda^k) + g^T(\lambda^k)(\lambda - \lambda^k), q^{k^*} \leq z \right\}. \quad (21)$$

Fig. 1 shows P^{k+1} in a two-dimensional epigraph space in (a) and in a three-dimensional epigraph space in (b).

B. Query Point Calculation

An adaptive three-level scheme is developed to obtain the next query point in the updated polyhedron P^{k+1} by using subgradients and simplex tableaus. A point along the subgradient is easily obtained at Level 1. Kelley's query point is obtained at Level 2 and the Chebyshev center is obtained at Level 3, both as needed and by pivoting simplex tableaus.

Level 1: The Subgradient Midpoint: By letting z equal $q(\lambda^k)$, (19) is equivalent to the following cut in the multiplier space:

$$q(\lambda^k) \leq q(\lambda^k) + g(\lambda^k)^T(\lambda - \lambda^k). \quad (22)$$
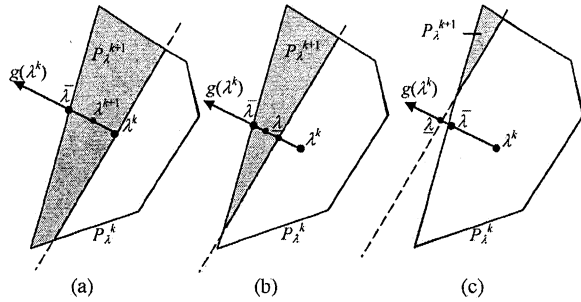
Fig. 2. Subgradient midpoint λ^{k+1} at Level 1 with $z = q^{k^*}$, i.e., on the horizontal hyperplane in the epigraph space of Fig. 1(b). Subfigures (a), (b), and (c) are for cases $q^{k^*} = q(\lambda^k)$, $q^{k^*} > q(\lambda^k)$, and $q^{k^*} \gg q(\lambda^k)$, respectively.

It is the acute angle direction and is a "neutral cut," since it passes through λ^k [5]. With λ^k on the new boundary of the updated polyhedron P_λ^{k+1} of λ, searching from λ^k along the subgradient $g(\lambda^k)$, a point λ^{k+1} in P_λ^{k+1} halfway before hitting another boundary can be easily obtained as shown in Fig. 2(a). This "subgradient midpoint" has been used as the starting point of Newton's method to calculate the analytic center of P_λ^{k+1} [27]. In fact, the point itself can be deep inside P_λ^{k+1} and will be used directly as the query point at Level 1.

The above process of obtaining the subgradient midpoint can be re-casted in the epigraph space of (λ, z) augmented by one additional degree of freedom z, however, with complications. To cut off a large non-optimal portion, z is set to be q^{k^*} in (20). Combining with (19), we have

$$q^{k^*} \leq q(\lambda^k) + g(\lambda^k)^T(\lambda - \lambda^k). \tag{23}$$

Cut (23) cuts off those multipliers whose dual values are known to be less than q^{k^*}. If $q^{k^*} = q(\lambda^k)$, then Fig. 2(a) is obtained. If $q^{k^*} > q(\lambda^k)$, then λ^k itself is cut off. If there is still a segment of the subgradient lying within P_λ^{k+1} as shown in Fig. 2(b), then λ^{k+1} can be similarly obtained halfway between $\underline{\lambda} = \lambda^k + \underline{\theta}g(\lambda^k)$ on the new boundary (23) with

$$\underline{\theta} = \frac{q^{k^*} - q(\lambda^k)}{g^T(\lambda^k)g(\lambda^k)} \tag{24}$$

and $\overline{\lambda} = \lambda^k + \overline{\theta}g(\lambda^k)$ on another boundary of P_λ^{k+1} with $\overline{\theta}$ obtained as presented in [27]. However, if q^{k^*} is substantially larger than $q(\lambda^k)$ ($\underline{\theta} \geq \overline{\theta}$) such that there is no segment of the subgradient lies within P_λ^{k+1} as shown in Fig. 2(c), then the Chebyshev center will be calculated at Level 3.

When the subgradient midpoint λ^{k+1} is obtained, the distance between λ^{k+1} and the boundary of P_λ^{k+1}:

$$d = \frac{(\overline{\theta} - \underline{\theta})}{2} \left\| g(\lambda^k) \right\| \tag{25}$$

is used to evaluate whether λ^{k+1} is deep inside P_λ^{k+1}. An approximate size r of the polyhedron P_λ^{k+1} is initially determined

by half of the minimal λ_t^{max}, $t = 1, \ldots, T$ in (17) and is updated by the latest Chebyshev radius of P^{k+1} to be obtained at Level 3. If d is small compared to r, or d keeps decreasing, then λ^{k+1} is not deep inside P_λ^{k+1} and Level 1 will be transitioned to Level 2 at the next iteration. Transition is typically made when $d < 2r$ or d keeps decreasing for 3 iterations before λ^{k+1} is far away from the "center."

Level 2: Kelley's Query Point: The polyhedron P^{k+1} in (21) is constrained by a polyhedral approximation of the piecewise linear dual function [3]:

$$\min \left\{ q(\lambda^0) + g^T(\lambda^0)(\lambda - \lambda^0), \ldots, q(\lambda^k) + g^T(\lambda^k)(\lambda - \lambda^k) \right\}. \tag{26}$$

By maximizing z over P^{k+1}, Kelley's point λ^{k+1} that maximizes this approximation is obtained for Level 2. This linear programming (LP) problem is solved by pivoting simplex tableaus. In our algorithm, simplex tableaus are used to calculate query points at Level 2 and Level 3. Although simplex methods have been well developed to solve LP problems, there are fixed numbers of constraints and the basic solutions described by the tableaus are always feasible in the pivoting processes. In the SSCPM, cuts (constraints) are iteratively added as in (21), and the basic solutions may be cut off (become infeasible). To address this, the idea is to add cuts as additional rows to the tableaus, and to restore feasibility by moving to the cuts that create the infeasibility while maintaining feasibility to existing constraints. By using the resulting tableaus, LP problems at Level 2 and Level 3 can be solved more efficiently than by directly using standard simplex methods.

To construct tableaus, the polyhedrons defined by linear inequalities are converted to the standard equality form by adding slack variables [28]. The initial tableau at iteration $k = 0$ is constructed for polyhedron P^0 in (17) with standard form $A^0x = b^0$, $x \geq 0$. A basic feasible solution x_0^0 is obtained at the origin and the initial tableau with basis B^0 is constructed as

$$(B^0)^{-1}[A^0 \mid b^0]. \tag{27}$$

Given the tableau for P^k at x_0^k, cut (19) in the standard form $a_kx = b_k$, is added as an additional row [28]:

$$\begin{bmatrix} B^k & 0 \\ a_{k,B^k} & 1 \end{bmatrix}^{-1} \begin{bmatrix} A^k & 0 \mid b^k \\ a_k & \mid b_k \end{bmatrix} \tag{28}$$

where a_{k,B^k} contains the elements in the basic columns of a_k. By using the partitioned matrix inverse formula, (28) is obtained as the tableau in (29) at the bottom of the page, which involves only basic matrix operations [28].

The tableau of a basic solution for P^{k+1} is obtained in (29). However, this solution may be infeasible, since x_0^k can be cut off by cut (19). This infeasibility is identified from (29) by the associated negative basic variable:

$$b_k - a_{k,B^k}(B^k)^{-1}b^k < 0. \tag{30}$$

To restore feasibility, the idea is to move from x_0^k to a vertex on cut (19), since x_0^k is only infeasible for cut (19). Feasibility for

$$\begin{bmatrix} (B^k)^{-1}A^k & 0 & \mid (B^k)^{-1}b^k \\ a_k - a_{k,B^k}\left[(B^k)^{-1}A^k \ 0\right] & \mid b_k - a_{k,B^k}(B^k)^{-1}b^k \end{bmatrix} \tag{29}$$
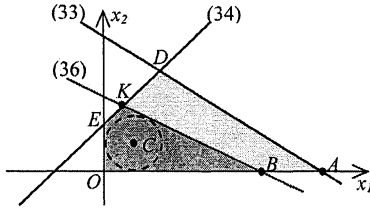
Fig. 3. Example to illustrate the usage of simplex tableaus.

TABLE I
SIMPLEX TABLEAU AT V^A FOR POLYHEDRON P^{ADEO}

	x_1	x_2	x_3	x_4	
x_1	1	4/3	1	0	4
x_4	0	7/3	1	1	5

TABLE II
SIMPLEX TABLEAU AT V^A FOR POLYHEDRON P^{BKEO}

	x_1	x_2	x_3	x_4	x_5	
x_1	1	4/3	1	0	0	4
x_4	0	7/3	1	1	0	5
x_5	0	2/3	-1^*	0	1	-1

existing constraints defining P^k is maintained along the way. A vertex on cut (19) is obtained when (30) is increased to zero by pivoting tableau (29) with the entering non-basic variable x_{j^*} selected as

$$j^* = \arg\min_j \left\{ a_k - a_{k,B^k} \left[(B^k)^{-1} A^k \quad 0 \right] \right\}. \tag{31}$$

Cut (20) can be added in a similar way, and a tableau for P^{k+1} is then obtained at a basic feasible solution x_0^{k+1} as

$$(B^{k+1})^{-1}[A^{k+1} \mid b^{k+1}]. \tag{32}$$

The tableau in (32) can be directly used as the tableau for the initial basic feasible solution of the LP problem at Level 2. Pivoting from x_0^{k+1}, the optimal vertex x^{k+1} of P^{k+1} maximizing z is obtained following the standard simplex method as presented in [28] and [29].

To illustrate the above tableau method, a simple example is used to calculate Kelley's query point. As shown in Fig. 3, polyhedron P^{BKEO} is obtained by updating polyhedron P^{ADEO}:

$$x_1 + \frac{4}{3}x_2 + x_3 = 4 \tag{33}$$

$$-x_1 + x_2 + x_4 = 1 \tag{34}$$

$$x_1, x_2 \geq 0 \tag{35}$$

with cut C^{BK}:

$$x_1 + 2x_2 + x_5 = 3. \tag{36}$$

To calculate Kelley's point for P^{BKEO} given the tableau for P^{ADEO} at vertex V^A in Table I, the tableau (29) for a basic solution is first obtained in Table II by adding cut C^{BK}. In Table II, the basic variable associated with cut C^{BK} is $-1 < 0$, and the basic solution at V^A thus is infeasible. To restore feasibility, Table II is pivoted following (31). Tableau (32) for a basic feasible solution at V^B is then obtained in Table III.

TABLE III
SIMPLEX TABLEAU AT V^B FOR POLYHEDRON P^{BKEO}

	x_1	x_2	x_3	x_4	x_5	
x_1	1	2	0	0	1	3
x_4	0	3	0	1	1	4
x_3	0	-2/3	1	0	-1	1

Table III is used as the tableau for the initial basic feasible solution of the LP problem to maximize x_2 over P^{BKEO}. Pivoting from this tableau following the standard simplex method, Kelley's point is attained at vertex $V^K = (1/3, 4/3)$.

After Kelley's query point is obtained, a large non-optimal portion can be cut off by cut (20) if polyhedral approximation (26) approximates the dual function well and q^{k^*} is significantly improved to $q(\lambda^{k+1})$. Level 2 is then transitioned back to Level 1. In addition, when (26) already contains facets that intersect at the optimal point of the piecewise linear dual function, the optimum $(\lambda^*, q(\lambda^*))$ is immediately obtained. However, it is difficult to assess beforehand whether the dual value $q(\lambda^{k+1})$ is larger than q^{k^*}. If $q(\lambda^{k+1})$ is obtained less than q^{k^*}, then Level 3 will be used at the next iteration. A special situation is at the early stage iterations when a good approximation is unlikely to be obtained and Level 1 is directly transitioned to Level 3 as needed. The number of early stage iterations depends on the dual function complexity.

Level 3: The Chebyshev Center: As reviewed in Section II, centers lie deep inside a polyhedron and are good query points to construct effective cuts. Among various centers, the Chebyshev center is determined by an LP problem that can be solved by pivoting simplex tableaus derived from (32), and is therefore selected as the query point for Level 3. For polyhedron P^{k+1} described by linear inequalities $\{\bar{a}_i \bar{x} \leq b_i, i = 1, \ldots, m, \bar{x} = (\lambda, z) \geq 0\}$ in (21), the Chebyshev center $\bar{x}^{k+1}$ is a point in P^{k+1} that maximizes the smallest distance r (the Chebyshev radius) from that point to the boundaries, i.e., it solves the LP problem [30]:

$$\max \quad r \tag{37}$$

$$\text{s.t.} \quad \bar{a}_i \bar{x} + \|\bar{a}_i\| r \leq b_i, \ i = 1, \ldots, m \tag{38}$$

$$-\bar{x}_j + r \leq 0, \ j = 1, \ldots, T+1 \tag{39}$$

$$\bar{x}, r \geq 0. \tag{40}$$

To solve this LP problem, it is noted that with x_0^{k+1} from (32) and $r = 0$, $(x_0^{k+1}, 0)$ is a basic feasible solution to (37). The corresponding simplex tableau is constructed by adding to (32) an additional column associated with the new variable r. With coefficients for the new variable given by

$$A_r = [\|\bar{a}_1\|, \ldots, \|\bar{a}_m\|]^T \tag{41}$$

constraints (38) are now captured by the following tableau:

$$(B^{k+1})^{-1}[A^{k+1} | A_r | b^{k+1}]. \tag{42}$$

The additional $(T+1)$ constraints (39) are then added to (42) as additional rows similarly as in (29). The tableau of the initial basic feasible solution thus is obtained and the LP problem is solved by pivoting following the standard simplex method.

To illustrate this, the previous example in Fig. 3 is used to calculate the Chebyshev center of polyhedron P^{BKEO}. To solve

TABLE IV
SIMPLEX TABLEAU WITH THE ADDITIONAL COLUMN ASSOCIATED WITH r

	x_1	x_2	x_3	x_4	x_5	r	
x_1	1	2	0	0	1	$\sqrt{5}$	3
x_4	0	3	0	1	1	$\sqrt{2}+\sqrt{5}$	4
x_3	0	-2/3	1	0	-1	$5/3-\sqrt{5}$	1
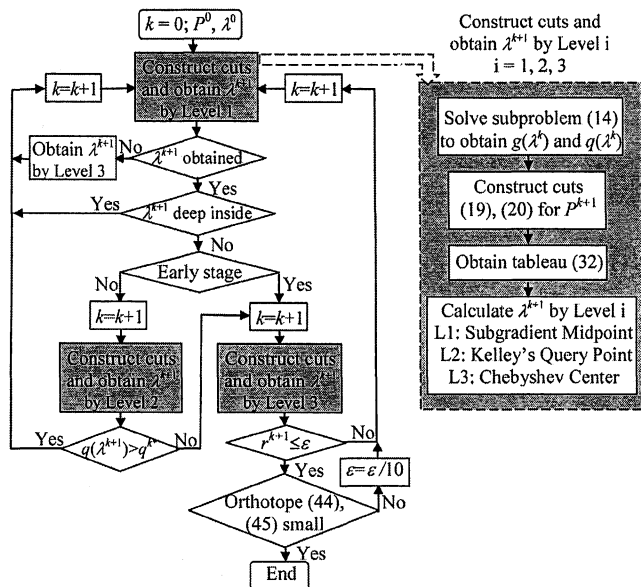
Fig. 4. Flowchart of the SSCPM.

TABLE V
SUPPLY OFFERS FOR EXAMPLE I

Unit	P^{min} MW	P^{max} MW	Block1 Cost $/MWh	Block1 Size MW	Block2 Cost $/MWh	Block2 Size MW	Start-up $	No-load $
1	20	210	65	100	110	200	3800	0
2	20	200	40	100	90	100	3000	600
3	50	190	25	100	85	100	1100	800

the LP problem (37) with the new variable r and coefficients $A_r = [5/3; \sqrt{2}; \sqrt{5}]$, tableau (42) is first obtained in Table IV by adding to Table III an additional column.

The constraints (39) are then added to Table IV as additional rows, and the tableau for an initial basic feasible solution is obtained. Following the standard simplex method, the Chebyshev center is attained at point $C = (0.57, 0.57)$ and the Chebyshev radius is 0.57.

After the Chebyshev center $\bar{x}^{k+1}$ is obtained, effective cuts are constructed since $\bar{x}^{k+1}$ is deep inside P^{k+1}, and Level 3 is then transitioned back to Level 1. As a by-product of solving the LP problem (37), the Chebyshev radius r^{k+1} provides an approximate size of P^{k+1} and has been used to determine the adaptive transition at Level 1. It is also used in the stopping criteria below.

The SSCPM stops when non-optimal portions of P^{k+1} are cut off and the distance from λ^{k+1} to the optimum λ^* is sufficiently small. The Chebyshev radius r^{k+1} is the radius of the maximum-volume ball inscribed P^{k+1}, and is used as a rough measurement:

$$r^{k+1} \leq \varepsilon \qquad (43)$$

where ε is the stopping threshold [31]. The ball inscribed P^{k+1}, however, may not guarantee the solution optimality, and a circumscribed solid of P^{k+1} is then calculated. As compared with circumscribed solids such as the ellipsoid, the orthotope is comparatively easy to obtain by solving $2(T+1)$ LP problems:

$$\min_{\bar{x}\in P^{k+1}} \bar{x}_j - \bar{x}_j^{k+1}, \quad j = 1,\ldots,T+1 \qquad (44)$$

and

$$\max_{\bar{x}\in P^{k+1}} \bar{x}_j - \bar{x}_j^{k+1}, \quad j = 1,\ldots,T+1. \qquad (45)$$

If the circumscribed orthotope is sufficiently small, then the solution is close enough to the optimum. Otherwise, the stopping threshold ε is reduced to cut off the remaining non-optimal solutions.

There is a special case that the dual function has a flat facet at $z = q(\lambda^*)$. The dual problem thus has infinite optimal solutions and they form this flat facet. In this case, the Chebyshev radius r^{k+1} is still small to satisfy (43), since in the epigraph space it can be determined by the distance from the Chebyshev center to the boundaries on z. If the circumscribed orthotope is sufficiently flat, then the solution is also close enough to the optimum. Such multi-solution case occurs under extremely rare conditions in real markets. Optimal solutions, however, are equivalent form the optimization point of view, and are valid LMPs when LMPs are calculated [32]. Accordingly, the optimal solution obtained by the SSCPM among infinite optimal solutions can be used as ELMPs.

Other stopping criteria have also been used when solving the dual problem such as a fixed number of iterations in [33] or a small duality gap in [24]. However, these criteria may not be sufficient to obtain the multiplier optimality. In addition, searching for feasible primal solutions is needed to obtain the duality gap. The flow chart of the overall algorithm is presented in Fig. 4.

V. NUMERICAL RESULTS

The SSCPM was implemented in Matlab7.01 on an Intel Core 2 Duo CPU T9300 2.50-GHz Dell M6300 laptop. Three examples are presented. Example 1 is a simple two-hour problem to illustrate the adaptive three-level process. Example 2 is a 24-hour 32-unit problem based on the IEEE Reliability Test System, and is used to show the efficiency of the SSCPM. Example 3 is a MISO-sized problem for the day-ahead market to show the capability of solving large problems, and 100 Monte Carlo simulation runs to demonstrate the robust performance of the SSCPM. Complete testing data and results for Examples 1 and 2 are at http://www.engr.uconn.edu/msl.

Example 1: Consider a two-hour problem with system demand $P^D = [320; 450]$. Three supply offers are specified in Table V, with each identical over the two hours.

The initial polyhedron P^0 in (17) is selected as $\{(\lambda, z)|0 \leq \lambda_t \leq 150, t = 1,2; 0 \leq z \leq 10^5\}$, and the initial multiplier λ^0 is selected as (50, 50).

The problem is solved by using the SSCPM in the epigraph space through the following adaptive three-level process. The updated polyhedron P^{k+1} is shown in Fig. 5 for iterations $k = 0, 2, 6$ and 7. At these iterations, the process to cut off λ with $z = q^{k^*}$ is shown in Fig. 6. In subfigure (a), cuts are constructed from λ^0 to obtain the updated polyhedron P^1. By using Level 1, λ^1 is obtained and the process repeats, e.g., at iteration 2 as shown in subfigure (b). Level 1 is transitioned to Level 2 at iteration 4. However, the dual value $q(\lambda^4) = 51\,100$ is less than
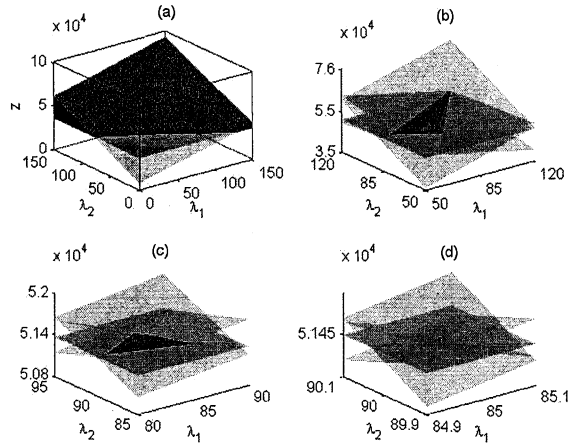
Fig. 5. Updated polyhedron P^{k+1} for Example 1. In each subfigure, the horizontal plane is cut (20) and other planes are cuts (19). As P^{k+1} is updated, some cuts become redundant and they are not plotted to clearly present P^{k+1}. (a) $k = 0$. (b) $k = 2$. (c) $k = 6$. (d) $k = 7$.
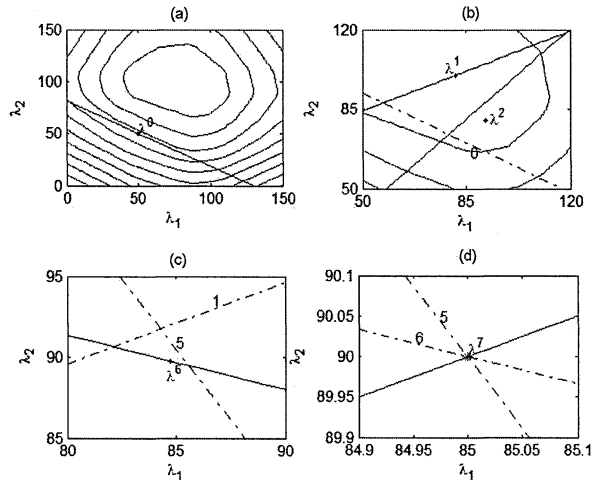

Fig. 6. Process to cut off λ with $z = q^{k^*}$ for Example 1. The curves are level curves. In each subfigure, the solid line is the cut of iteration k with z in (19) set to be q^{k^*}. When q^{k^*} is increased to q^{i^*} at a later iteration $i > k$, the cut of iteration k is depicted by a dashed line labeled by k. (a) $k = 0$. (b) $k = 2$. (c) $k = 6$. (d) $k = 7$.

$q^{3^*} = 51\,339$, and Level 2 is transitioned to Level 3 to obtain λ^5. At iteration 6 as shown in subfigure (c), λ^6 is obtained by using Level 1. Since λ^6 is not deep inside, Level 2 is used to obtain λ^7 in subfigure (d). At iteration 8, Level 1 fails to obtain λ^8. Level 3 thus is used and obtains the Chebyshev radius as zero, indicating that $\lambda^7 = (85, 90)$ is the optimal multiplier.

The trajectory of multipliers in the above process of the SSCPM is compared with that of subgradient method (SM):

$$\lambda^{k+1} = \lambda^k + s^k g(\lambda^k). \tag{46}$$

The step size s^k is given by

$$s^k = \alpha^k \frac{\overline{q} - q(\lambda^k)}{\|g(\lambda^k)\|^2}, \quad 0 < \alpha^k < 2 \tag{47}$$

where $\overline{q}$ is an estimate of the optimal dual value and is obtained by solving the primal problem (12). Among different ways to
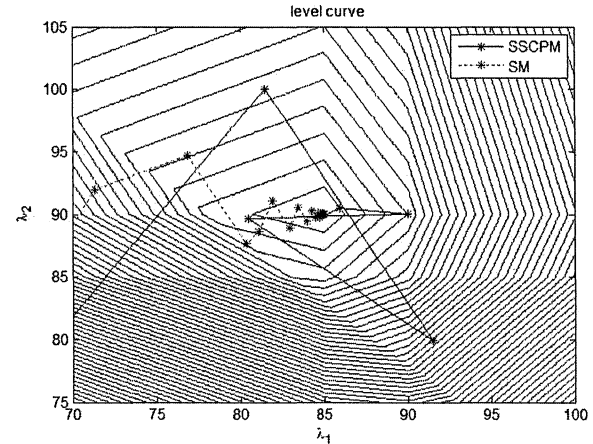

Fig. 7. Trajectories of multipliers by using the SSCPM and the SM for Example 1.

TABLE VI
GENERATION OFFERS OF EXAMPLE 2

Unit type	No. of Units	P^{min} MW	P^{max} MW	Incremental cost \$/MWh	Start-up \$
U12	5	3	12	67.15	247
U20	4	4	20	65.9	32.5
U50	6	0	50	15	0
U76	4	10	76	12.16	727.1
U100	3	100	100	56.60	1625
U155	4	20	155	10.42	317.2
U197	3	20	197	57.42	2880
U350	1	35	350	10.85	2336
U400	2	40	400	43.03	6000

select parameter α^k such as those in [11] and [34], α^k is set to be 1 for simplicity as recommended by [3]. As can be seen in Fig. 7, the SM suffers from the zigzagging difficulty, and the zigzagging is not observed in the SSCPM. By adaptively transitioning among the three levels, the zigzagging difficulty is overcome in the SSCPM.

Example 2: Consider a 24-hour 32-unit problem based on the IEEE Reliability Test System of 1996 [35]. The system includes generating units, hourly system load of a year, and a transmission network. Since transmission is not considered in our work here, only generating units and hourly system load are used. Generation offers as presented in Table VI are obtained from production costs following [36]. All units are assumed initially off for simplicity. The peak-load day within the year is selected. To capture a few key features of ELMPs, these data are slightly modified.

The initial polyhedron is selected as $\{(\lambda, z) | 10 \leq \lambda_t \leq 70, t = 1, \ldots, 24; 0 \leq z \leq 10^7\}$, and λ^0 is selected as its center. Performance of the SSCPM is compared in Table VII with that of each level alone implemented by disabling transitions to other levels within the SSCPM. Simplex tableaus are thus used in Level 2 alone and Level 3 alone, but not in Level 1 alone. Also, since neither Level 1 alone nor Level 2 alone has the Chebyshev radius, their stopping criteria is $|\lambda^{k+1} - \lambda^k| \leq 10^{-4}$. Otherwise, (43) is used with the stopping threshold $\varepsilon = 10^{-4}$. Level 1 alone can also stop if it cannot obtain λ^{k+1} as explained in Fig. 2(c). After Level 1 alone or Level 2 alone stops, the Chebyshev radius is calculated to complete the comparison in Table VII.

TABLE VII
PERFORMANCE COMPARISON FOR EXAMPLE 2

	Iteration	CPU time (s)	Dual value ($)	ChebyRad
Level 1 alone	26	< 0.1	1365910.45	6.37
Level 2 alone	297	7.2	1367213.04	1.31×10^{-11}
Level 3 alone	154	1.5	1367213.00	3.73×10^{-5}
SSCPM	118	0.8	1367213.04	3.75×10^{-12}

As presented in Table VII, query points are obtained quickly for Level 1 alone, but the final dual value is less than those of other methods since λ^{k+1} may not be deep inside P_λ^{k+1}, or the method may not be able to obtain λ^{k+1}, leading to premature algorithm termination. With Chebyshev radius 6.37, P^{k+1} still contains significant non-optimal portions. Level 2 alone obtains the optimum when facets that intersect at the optimum are obtained in approximation (26), but converges slowly since (26) may not always be good especially at the early stage iterations. Level 3 alone uses fewer iterations than Level 2 alone because the Chebyshev center is always deep inside P_λ^{k+1}. However, it may be slow to identify the optimum. The SSCPM, with 64 Level 1 iterations, 7 Level 2 iterations and 47 Level 3 iterations, uses less CPU time than Level 3 alone by adaptively taking advantages of all the three levels.

It is difficult to compare the SSCPM with methods reviewed in Section II because of different problems tested, stopping criteria used, and implementation environments. Nevertheless, the analytic center cutting plane method reported in [24] solved a 32-unit commitment problem by using 41 iterations in 55 s on a Pentium 200-MHz computer, and stopped when the duality gap is less than 1%. With this loose stopping criterion, the multiplier obtained may still be away from the optimum.

We also used the commercial software CPLEX 12.2 to solve the dual problem on the same Dell M6300 laptop to compare the performance with that of the SSCPM. In CPLEX, the mix-integer problems to minimize (13) given λ are solved by branch and cut methods, and multipliers are updated by the subgradient method as described in (46) and (47). The solutions of both methods are compared with the optimal multiplier λ^* obtained by tightening the stopping criteria of the SSCPM as $\varepsilon = 10^{-10}$. The multiplier obtained by the SSCPM using 0.8 s in Table VII is identical to λ^*. The multiplier obtained by CPLEX using similar amount of CPU time is away from the optimum as shown in Fig. 8. By using more CPU time, e.g., 10 s, the multiplier obtained by CPLEX is still not optimal. When the Lagrangian dual problem is solved in CPLEX by the subgradient method, multiplier optimality is not guaranteed or may not be obtained efficiently as reviewed in Section II [11].

With the optimal multiplier obtained by the SSCPM, ELMPs are compared with LMPs in Fig. 9. ELMPs can be higher than LMPs, e.g., at $t = 1$ and 12, due to the incorporation of start-up cost, and they can be lower than LMPs, e.g., at $t = 9$ and 23, since offline units can set ELMPs. For most hours, however, ELMPs are equal to LMPs. In real day-ahead market especially with virtual demand and supply, ELMPs can be closer to LMPs [37]. Since the purpose of this paper is to obtain ELMPs efficiently, comprehensive comparison is referred to [2] and [37].

Example 3: A MISO-sized problem for the day-ahead market is solved by using the SSCPM. This problem has 873 units and is subject to all the constraints of Section III. In addition, some units must be on or off at certain hours, and this requirement is handled within the DP processes after appropriate modifications. For this large problem, the stopping threshold ε in (43)
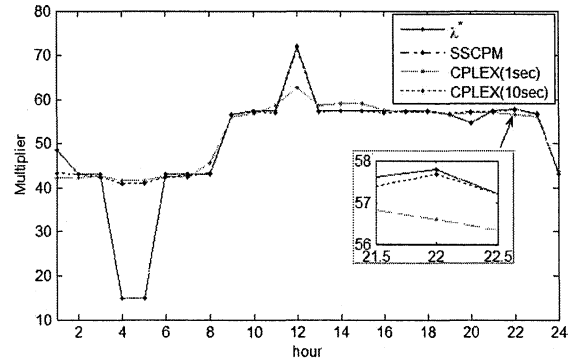

Fig. 8. Comparison of the multiplier obtained by the SSCPM and the multipliers obtained by CPLEX for Example 2.
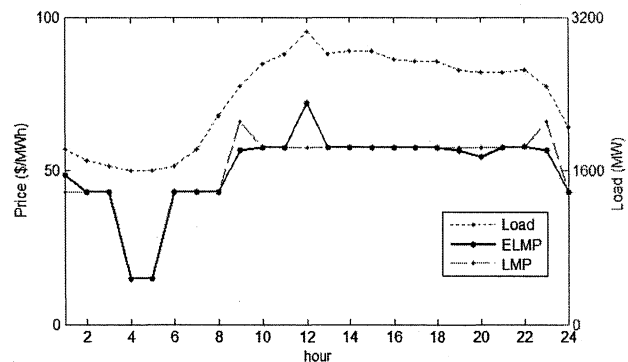

Fig. 9. Comparison of ELMPs and LMPs for Example 2.

TABLE VIII
DISTRIBUTION OF ITERATIONS AND THE TOTAL CPU TIME FOR EXAMPLE 3

Subproblems	Query point calculation			Others
97.44%	Level 1	Level 2	Level 3	0.31%
	0.03%	0.54%	1.68%	
	218 iter.	35 iter.	236 iter.	

is set to be 0.01. To avoid an over-sized initial polyhedron P^0 in (17), a simple single-hour dual problem is solved for each hour t, $t = 1, \ldots, 24$, assuming that all units are initially off. The resulting multipliers are appropriately increased and used as upper bounds in (17). If the upper bounds are too small, the solution will fall to these upper bounds, and they are further increased based on heuristics. Lower bounds are set to be 0, and λ^0 is selected as the center of P^0.

The 24 single-hour dual problems for the initialization are solved by using 0.11 s. Each single-hour problem is solved fast because there is only one multiplier and it is easily updated. Also, subproblems of the single-hour problem are solved without using sophisticated DP to manage complicated inter-temporal constraints, and loose stopping criteria are used. Computational complexity of the 24-hour problem increases drastically and the overall dual problem is solved by using 489 iterations in 778 s. Table VIII shows the distribution of these iterations and time over key segments of the algorithm.

As can be seen, a major portion of the CPU time is on solving subproblems, and query points are efficiently calculated because of effective uses of the three levels. As shown in Fig. 10(a), as more cuts are obtained to describe updated polyhedrons over
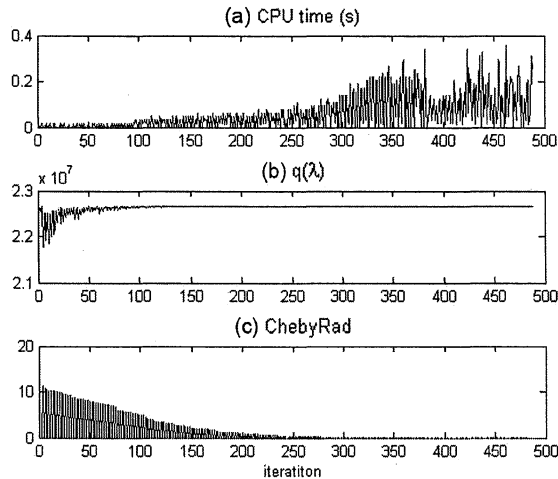
Fig. 10. (a) CPU time per iteration for query point calculation. (b) Dual value. (c) Chebyshev radius (of Level 3 iterations) for Example 3.

Fig. 11. Simulation results for Example 3.

TABLE IX
PERFORMANCE SUMMARY FOR EXAMPLE 3

	Minimum	Maximum	Average	Stdev
Iteration	324	492	390	33
CPU time (s)	507.9	791.6	618	56
ChebyRad (10^{-3})	4.6	10	9.1	1.0

iterations, the CPU time to calculate the query point at each iteration increases moderately because of the innovative usage of subgradients and simplex tableaus. By adaptively transitioning among the three levels, query points are of good quality to construct effective cuts as indicated by $q(\lambda)$ in Fig. 10(b) and the Chebyshev radius in Fig. 10(c) over iterations.

To evaluate the robustness of the SSCPM, 100 Monte Carlo simulation runs are performed using the above problem as the base case. Incremental energy offers, start-up and no-load offers are obtained by perturbing the base case with Gaussian distributions with standard deviations selected as 10% of the base case values. Load at each hour is obtained by perturbing the base case with a uniform distribution with interval length selected as 10% of the base case value, and is truncated as needed to preserve the load pattern. These 100 dual problems are solved with the same initialization process and stopping criterion as those in the base case.

The 100 simulation runs are re-ordered according to the number of iterations used to solve the dual problem. These numbers of iterations, total CPU times and final Cheyshev radiuses are depicted in Fig. 11. The curve for the number of iterations across the 100 runs is relatively flat, indicating robust performance. The curve for the CPU time is of a similar pattern, and shows that as the number of iterations increases, the CPU time increases moderately. Final Cheyshev radiuses seem to be unrelated to the number of iterations or the CPU time, but are all less than 0.01 when the algorithm stops. As can be seen from Table IX, their standard deviations are not large, indicating the robustness of the SSCPM.

VI. CONCLUSION

Extended LMPs incorporate commitment-related costs and minimize uplift payments. To obtain them, the SSCPM is developed in this paper to solve the dual of the UCED problem. Different from previous results to solve dual problems, multiplier optimality as opposed to the optimality of the primal cost is required. By innovatively using subgradients and simplex tableaus, an adaptive three-level scheme is developed to efficiently obtain query points so that non-optimal multipliers are iteratively cut off. MISO-sized cases for the day-ahead market without transmission or ancillary services are efficiently solved with robust performance. If transmission and ancillary services
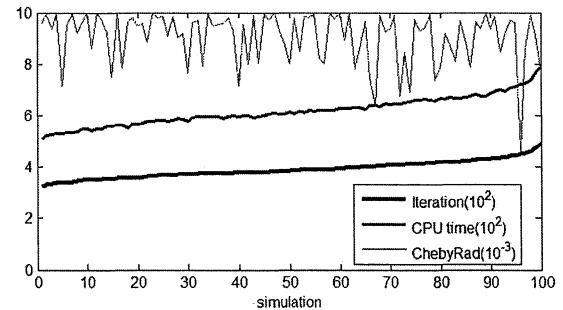
are considered based on a DC power flow network, additional multipliers will be introduced to relax each constraint at each time. Problem solution framework can be maintained while the problem complexity will increase significantly due to the increased dimensionality of decision space and the complicated coupling of energy and reserve products. The SSCPM is good for Lagrangian relaxation-based approaches when multiplier optimality is essential.

REFERENCES

[1] C. Ruiz, A. J. Conejo, and S. A. Gabriel, "Pricing non-convexities in an electricity pool," *IEEE Trans. Power Syst.*, to be published.

[2] P. R. Gribik, W. W. Hogan, and S. L. Pope, Market-Clearing Electricity Prices and Energy Uplift, white paper, Dec. 31, 2007. [Online]. Available: http://ksghome. harvard.edu/~whogan/ Gribik_Hogan_Pope_Price_Uplift_123107.pdf.

[3] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA, USA: Athena Scientific, 2003.

[4] J. L. Goffin and J. P. Vial, "Convex nondifferentiable optimization: A survey focused on the analytic center cutting plane method," *Optimiz. Meth. Softw.*, vol. 17, pp. 805–867, 2002.

[5] S. Boyd and L. Vandenberghe, Localization and Cutting-Plane Methods Stanford University, Lecture notes, Jan. 2007. [Online]. Available: http://www. stanford.edu/class/ee364b/notes/localization_methods_notes.pdf.

[6] C. Wang, P. B. Luh, P. Gribick, L. Zhang, and T. Peng, "A Subgradient-based cutting plane method to calculate convex hull prices," in *Proc. IEEE Power and Energy Society General Meeting*, Calgary, AB, Canada, Jul. 2009.

[7] C. Wang, P. B. Luh, P. Gribik, L. Zhang, and T. Peng, "The subgradient-simplex based cutting plane method for convex hull pricing," in *Proc. IEEE Power and Energy Society General Meeting*, Minneapolis, MN, USA, Jul. 2010.

[8] M. L. Fisher, "Optimal solution of scheduling problems using Lagrange multipliers: Part I," *Oper. Res.*, no. 21, pp. 1114–1127, 1973.

[9] M. Held and R. M. Karp, "The traveling salesman problem and minimum spanning trees: Part II," *Lecture notes Study*, no. 1, pp. 6–25, 1971.

[10] A. M. Geoffrion, "Lagrangian relaxation for integer programming," *Lecture notes Study*, no. 2, pp. 82–114, 1974.

[11] M. L. Fisher, "The Lagrangian relaxation method for solving integer programming problems," *Manage. Sci.*, vol. 50, no. 12, pp. 1861–1871, Dec. 2004.

[12] N. Z. Shor, "On the structure of algorithm for the numerical solution of optimal planning and design problems," Ph.D. dissertation, Cybernetic Inst. Acad. Sci., Kiev, USSR, 1964.

[13] B. T. Polyak, "Minimization of unsmooth functionals," *USSR Computat. Math and Math Phys.*, vol. 9, pp. 14–29, 1969.

[14] M. Held, "Validation of subgradient optimization," *Math. Program.*, vol. 6, pp. 62–88, 1974.

[15] J. B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms I and II.* Berlin, Germany: Springer-Verlag, 1993.

[16] R. N. Tomastik, P. B. Luh, and G. Liu, "Scheduling flexible manufacturing system for apparel production," *IEEE Trans. Robot. Autom.*, vol. 12, no. 5, pp. 789–799, Oct. 1996.

[17] G. Wang, U. V. Shanbhag, T. Zheng, E. Litvinov, and S. P. Meyn, "An extreme-point subgradient method for convex hull pricing in energy and reserve market," in *Proc. IEEE Power and Energy Society General Meeting*, Detroit, MI, USA, 2011.

[18] X. Zhao, P. B. Luh, and J. Wang, "The surrogate gradient algorithm for Lagrangian relaxation method," *J. Optimiz. Theory Appl.*, vol. 100, no. 3, pp. 328–339, March 1999.

[19] C. A. Kaskavelis and M. C. Caramanis, "Efficient Lagrangian relaxation algorithms for industry size job-shop scheduling problems," *IIE Trans.*, vol. 30, no. 11, pp. 1085–1097, 1998.

[20] M. A. Bragin, X. Han, P. B. Luh, and J. H. Yan, "Payment cost minimization using Lagrangian relaxation and a modified surrogate optimization approach," in *Proc. IEEE Power and Energy Society General Meeting*, Detroit, MI, USA, Jul. 2011.

[21] J. E. Kelley, "The cutting-plane method for solving convex programs," *J. Soc. Ind. Appl. Math.*, vol. 8, no. 4, Dec. 1960.

[22] A. S. Nemirovsky and D. B. Yudin, *Problem Complexity and Method Efficiency in Optimization.* New York, NY, USA: Wiley, 1983.

[23] O. Du Merle, J. L. Goffin, and J. P. Vial, "On improvements to the analytic center cutting plane methods," *Computat. Optimiz. Appl.*, vol. 11, pp. 37–52, 1998.

[24] M. Madrigal and V. H. Quintana, "An interior-point/cutting-plane method to solve unit commitment problems," *IEEE Trans. Power Systems*, vol. 15, no. 3, pp. 1022–1027, Aug. 2000.

[25] X. Guan, P. B. Luh, H. Yan, and J. A. Amalfi, "An optimization-based method for unit commitment," *Int. J. Elect. Power Energy Syst.*, vol. 14, no. 1, pp. 9–17, Feb. 1992.

[26] W. Fan, X. Guan, and Q. Zhai, "A new method for unit commitment with ramping constraints," *Elect. Power Syst. Res.*, vol. 62, pp. 215–224, 2002.

[27] G. C. Calafiore and F. Dabbene, "Aprobabilistic analytic center cutting plane method for feasibility of uncertain LMIs," *Automatica*, vol. 43, pp. 2022–2033, Aug. 2007.

[28] D. P. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization.* Belmont, MA, USA: Athena Scientific, 1997.

[29] G. B. Dantzig and M. N. Thapa, *Linear Programming.* New York, NY, USA: Springer-Verlag, Jan. 1997.

[30] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge, U.K.: Cambridge Univ. Press, 2004.

[31] J. Elzinga and T. G. Moore, "A central cutting plane algorithm for the convex programming problem," *Math. Program.*, vol. 8, pp. 134–145, Dec. 1975.

[32] "*ISO New England Manual for Market Operations M-11,*" Feb. 6, 2012. [Online]. Available: http://www.iso-ne.com/committees/comm_wkgrps/mrkts_comm/mrkts/mtrls/2012/may9102012/index.html.

[33] M. Murai and M. Kato, "Cutting plane methods for Lagrangian relaxation-based unit commitment algorithm," *Elect. Eng. Jpn.*, vol. 141, no. 3, 2002.

[34] P. B. Luh, D. J. Hoitomt, E. Max, and K. R. Pattipati, "Schedule generation and reconfiguration for parallel machines," *IEEE Trans. Robot. Autom.*, vol. 6, no. 6, pp. 687–696, Dec. 1990.

[35] Reliability Test System Task Force, "The IEEE reliability test system-1996," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999.

[36] F. Zhao, P. B. Luh, J. H. Yan, G. A. Stern, and S. Chang, "Payment cost minimization auction for deregulated electricity markets with transmission capacity constraints," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 532–544, May 2008.

[37] MISO presentation, in *20100824 ELMPTT Review*, Aug. 2010. [Online]. Available: https://www.midwestiso.org/Library/Meeting-Materials/Pages/ELMPTT.aspx.

Congcong Wang (S'09) received the B.S. degree in electrical engineering from Xi'an Jiaotong University, Xi'an, China, in 2008. She is currently pursuing the Ph.D. degree at the University of Connecticut, Storrs, CT, USA.

Her research interests include optimization and economics of electricity markets.

Tengshun Peng received the Ph.D. degree in electrical engineering from the Washington State University, Pullman, WA, USA, in 2006.

She is currently with Midwest ISO Inc. as a principal engineer in the Market Development and Analysis Department, mainly working on market initiatives and operational analysis. Her research interests include power systems operation, economics in power markets, and optimization.

Peter B. Luh (S'77–M'80–SM'91–F'95) received the B.S. degree from National Taiwan University, Taipei, the M.S. degree from the Massachusetts Institute of Technology, Cambridge, MA, USA, and the Ph.D. degree from Harvard, Cambridge, MA, USA.

He has been with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT, USA, since 1980, and currently is the SNET Professor of Communications & Information Technologies. He is also a member of the Chair Professors Group in the Department of Automation, Tsinghua University, Beijing, China. His interests include smart power systems; intelligent manufacturing; smart and green buildings; mathematical optimization of mixed-integer problems.

Prof. Luh is a member of the IEEE Technical Activities Board Periodicals Committee, and the Senior Advisor on Automation for the Robotics and Automation Society. He was VP Publication Activities for the IEEE Robotics and Automation Society (2008–2011), the founding Editor-in-Chief of the IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING (2003–2007), and the Editor-in-Chief of the IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION (1999–2003).

Paul Gribik received the B.S. degree in electrical engineering and the M.S. and Ph.D. degrees in industrial administration from Carnegie Mellon University, Pittsburgh, PA, USA.

He is Consulting Advisor, Market Development at the Midwest ISO, Carmel, IN. Before joining MISO, he was a consultant with PA Consulting Group, Perot Systems Corporation, and Mykytyn Consulting Group. He also held positions with Pacific Gas & Electric Company. He has worked on electricity market restructuring issues in the Midwest, New England, and California.

Li Zhang (S'99–M'02) received the B.S. and M.S. degrees in control systems from Xi'an Jiaotong University, Xi'an, China, and Shanghai Jiaotong University, Shanghai, China; the M.Eng. degree from the National University of Singapore, Singapore; and the Ph.D. degree from the University of Connecticut, Storrs, CT, USA, all in electrical engineering.

He is the Manager, Information Delivery and Market Analysis at the Midwest ISO Inc., Carmel, IN. He has been with Market Operation in ISO New England, Inc. His research interests include power systems operation, economics in power markets, control systems, and optimization.