Commitment Cost Allocation of Fast-Start Units for Approximate Extended Locational Marginal Prices

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***Abstract*—In the U.S. electricity markets, locational marginal prices (LMPs) are obtained in economic dispatch with fixed commitment decisions. The costs of committing or dispatching fast-start units at their minimum limits may not be covered by LMPs and significant uplift payments are thus needed. Extended LMPs (ELMPs) were established by MISO to appropriately reflect these costs, but are computationally expensive for market implementation. The approximate ELMP (aELMP) model is then developed with reduced complexity. An important design issue that highly affects aELMPs is to allocate commitment costs of fast-start units over time. However, it is difficult to obtain an allocation for the simplified model to effectively approximate the complex ELMPs. In this paper, allocation guidelines are derived in the day-ahead energy market without considering transmission capacity constraints for simplicity. The idea is to separate the ELMP problem into individual-hour problems resembling the aELMP model by using Lagrangain relaxation. Karush–Kuhn–Tucker (KKT) conditions are then innovatively used to derive guidelines for an easily implementable allocation that utilizes the commitment and dispatch results. To examine effectiveness of the guidelines, aELMPs are compared with ELMPs and LMPs. Numerical results show that the resulting aELMPs effectively approximate ELMPs and reduce uplift payments.**

***Index Terms*—Commitment cost allocation, electricity prices, extended LMPs, Lagrangian relaxation.**

I. Introduction

IN the U.S. wholesale electricity markets operated by Independent System Operators (ISOs), generators and their generation levels are selected to minimize the total offer cost while satisfying demand. In the day-ahead market, this unit commitment and economic dispatch (UCED) problem is solved for the next day with one hour as the time interval. With commitment decisions fixed at their optimal values, Locational Marginal Prices (LMPs) are then obtained in the economic dispatch process. Since commitment decisions are fixed, commitment costs including start-up and no-load costs cannot be included in the prices. Also, units that are scheduled at their minimum limits may not be the marginal units to set prices. As a result, the costs of committing or dispatching fast-start units[1] at their minimum limits may not be covered by LMPs, and significant uplift payments may be needed [1], [2].

To improve the prices, methods have been developed for pricing purpose only. For example, some ISOs relax the minimum limits of fast-start units to zero, so that they can be marginal units to set prices [3], [4]. Extended LMPs (ELMPs) were established by MISO as the optimal multipliers of the dual of the same UCED problem [5]. The optimal dual value as a function demand forms the convex hull of the total cost function [6], and its slope, i.e., the marginal cost of the convex hull is the ELMP. This marginal cost includes a share of the fixed costs incurred when a unit is committed or dispatched at its minimum limit. The costs of committing or dispatching fast-start units are thus incorporated in ELMPs and uplift payments are minimized. Commitment costs of slow-start units are not likely to participate in setting ELMPs, since these units are usually online for a long time and their commitment costs are covered by prices [7]. ELMPs, however, require the optimal multipliers for the mixed-integer UCED problem and are computationally expensive for market implementation [8].

The single-hour approximate ELMP (aELMP) model has been developed by MISO to reduce the complexity while capturing ELMPs' feature of incorporating the costs of committing or dispatching fast-start units at their minimum limits [8]. With the UCED problem solved first, commitment decisions of slow-start units are fixed at their optimal values. To approximate the convex hull of the multi-hour cost function in the ELMP model, commitment costs of fast-start units are allocated over a set of online hours to design a set of single-hour cost functions. These cost functions are then convexified by relaxing integrality requirements on commitment decisions for fast-start units, and the convexified functions serve as an approximation of the convex hull. The aELMPs are then obtained by minimizing the total energy cost and commitment costs of fast-start units [9]. Constraints at the hour are also satisfied including ramp rate constraints where the generation levels at hour $t-1$ are fixed as parameters and the multi-hour dispatch related costs are not captured in the aELMPs.

[1]Fast-start units, e.g., combustion turbine generators, can be started within a short time (10 minutes in MISO's current definition) and usually have high offer costs.

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The allocation of commitment costs highly impacts the aELMPs, and it is important for these prices to appropriately reflect the costs of fast-start units and reduce uplift payments. However, the complexity of the ELMP problem increases exponentially as the number of hours increases [10], [11]. It is difficult to obtain an allocation that can result in prices effectively approximating ELMPs and that can also be easily implemented. This paper derives guidelines for the allocation and examines their effectiveness in the day-ahead energy market for simplicity without considering transmission capacity constraints. In Section II, the two-settlement market and methods to incorporate the commitment costs of fast-start units into prices are reviewed. With the UCED problem, the ELMP model and the aELMP model formulated in Section III, allocation guidelines are derived in Section IV. The idea is to separate the ELMP problem into individual-hour problems resembling the aELMP model. Since the ELMP problem is coupled over time by state-transition constraints and ramp rate constraints, Lagrangian relaxation is used to relax the state-transition constraints on commitment decisions, while in ramp rate constraints the generation at hour $t-1$ is fixed. The relaxed problem at hour t resembles the aELMP problem with multipliers, i.e., shadow prices, coordinating the allocation of the associated commitment costs. KKT conditions are then analyzed and innovatively used to derive guidelines for an allocation that can be easily implemented by using the unit commitment and dispatch results.

Guidelines are to allocate start-up costs to the hours when a unit is most needed since the start-up is mostly driven by the demand at these hours. The objective of the paper is to provide theoretical guidelines for MISO's allocation. These guidelines are derived for the day-ahead market, but can also be applied to the real-time market which has a more complicated market structure[2]. As discussed at the end of Section IV, MISO considers the guidelines in both markets and allocates start-up costs of fast-start units to their minimum-run times after they are started up when most needed [9]. In Section V, to examine effectiveness of the guidelines, aELMPs are compared with ELMPs and LMPs. The examination is performed for problems of different sizes, including one with one-hundred simulation runs and one based on the MISO system. Numerical results show that the resulting aELMPs effectively approximate ELMPs and reduce uplift payments.

II. Literature Review

For electricity markets operated by ISOs, e.g., MISO, ISO-NE, and CAISO, a two-settlement market structure is used: a day-ahead market which schedules generation to satisfy the demand for the next day with one hour as the time interval (the day-ahead UCED problem), and a real-time market which adjusts the outputs from online units to meet the load on a five-minute basis (the real-time dispatch problem). In addition, there are unit commitment processes (the reliability assessment commitment, look-ahead commitment) carried out throughout the day to modify the commitment decisions based on the latest forecasted load. Prices are needed to settle the markets. Since it is difficult to obtain prices in the existence of non-convexities arising from integer commitment decisions [12], [13], LMPs are obtained for both markets through economic dispatch on the hourly or five-minute basis with fixed commitment decisions. LMPs, however, may not reflect the costs of committing or dispatching fast-start units at minimum limits and significant uplift payments may be needed [1].

To improve the prices, ISO-NE allows fast-start units that are dispatched at their minimum limits to set prices by relaxing the limits to zero, but no commitment costs are included [3], [14]. NYISO allows fast-start units whose minimum limits equal the maximum limits to set prices, and includes no-load costs by increasing the incremental energy costs to cover the no-load costs [4], [14]. Start-up costs have been incorporated in studies based on German wholesale electricity markets. By using the unit commitment and dispatch results, the start-up costs are divided by the total outputs [15], [16] or the total capacities [17] over all the online hours as increased variable costs. However, the allocation of start-up costs to some low-demand hours may not be appropriate, and such methods have to use negative start-up costs to decrease the variable costs.

MISO established ELMPs based on the convex hull of the total cost function and those costs of fast-start units are reflected in ELMPs, i.e., the marginal costs of the convex hull [5]. ELMPs, however, require the optimal multipliers for the mixed-integer UCED problem which can be difficult to obtain [10], [11]. Methods have been developed to obtain ELMPs [18], [19], and the study of the prices obtained shows that ELMPs are mostly equal to LMPs. However, when an online fast-start unit sets ELMPs including its commitment costs, the ELMPs are higher than the LMPs. An offline fast-start unit can also set ELMPs under transient shortage conditions, and the ELMPs are then lower than the LMPs.

To implement ELMPs into markets, MISO developed an approximate model with reduced complexity while capturing their features above [8]. The idea of the aELMP model is that the convex hull of a separable function can be obtained through convex hulls of its constituents [20]. The multi-hour ELMP problem is thus separated into single-hour aELMP problems by allocating commitment costs over a set of online hours [8]. The aELMP is then obtained from a convex approximation of the single-hour total cost function, and can be easily implemented within the existing dispatch software at MISO [9]. The allocation of commitment costs is thus an important design issue that determines the quality of the approximation. While look-ahead dispatch is investigated at MISO, exploring the allocation of commitment costs may still be useful in view of the real-time moving window dispatch.

III. Problem Formulation

In the day-ahead energy market, the UCED problem is formulated in Section A following [5] without considering transmission capacity constraints for simplicity. The ELMP model is obtained as the dual of the UCED problem by relaxing the system demand constraints in Section B. The aELMP model as outlined in [9] is presented in Section C.

[2]Our studies have shown that ELMPs have a greater impact on the prices in the Real-Time market than in the Day-Ahead market. Consequently, we actually applied the guidelines to produce good aELMPs in the Real-Time market and then use the same allocation in the Day-Ahead market.

A. The UCED Problem

Consider an energy market with I units and a time horizon T. For unit i at time t, the state x_{it} is 1 if the unit is online and 0 if it is offline. The start-up decision u_{it} is 1 if the unit is turned from off to on and is 0 otherwise. The state-transition between x_{it} and u_{it} is given by:

$$x_{it} - x_{i,t-1} \leq u_{it}, \tag{1}$$
$$u_{it}, x_{it} \in \{0, 1\}, \quad \forall i, t. \tag{2}$$

Commitment decisions $\{u_{it}\}$, $\{x_{it}\}$ are coupled over time by state-transition constraints (1). Each unit satisfies generation capacity constraints, i.e., generation p_{it} is in-between the minimal and maximal levels p_i^{min} and p_i^{max} if the unit is online:

$$x_{it} p_i^{\min} \leq p_{it} \leq x_{it} p_i^{\max}, \quad \forall\, i, t. \tag{3}$$

Ramp rate constraints ensure that unit i cannot ramp up/down between two successive online hours beyond ramp limit Δ_{it}:

$$-\Delta_{it} \leq p_{it} - p_{i,t-1} \leq \Delta_{it}, \quad \forall\, i, t. \tag{4}$$

For unit i at time t, its offer costs include the energy cost $C_i(p_{it})$, the time-invariant start-up cost S_i^{up} if the unit is turned on, and the no-load cost S_{it}^{NL} if the unit is online. For a step offer curve with N blocks, the energy cost $C_i(p_{it})$ is convex and piecewise linear. It can be linearly formulated by splitting p_{it} into p_{int} for each block n with size s_{in} and price c_{in}:

$$C_i(p_{it}) = \min_{\{p_{\rm int}\}} \sum_{n=1}^{N} c_{in} p_{\rm int}, \text{ with} \tag{5}$$
$$\sum_{n=1}^{N} p_{\rm int} = p_{it}, \quad \forall\, i, t, \tag{6}$$
$$0 \leq p_{\rm int} \leq s_{in}, \quad \forall\, i, n, t. \tag{7}$$

In addition to individual unit constraints (1)–(4), the total generation should satisfy the demand p_t^D at all the time:

$$\sum_{i=1}^{I} p_{it} = p_t^D, \quad \forall\, t. \tag{8}$$

The objective is to minimize the total offer cost:

$$\min_{\{p_{it}\},\{u_{it}\},\{x_{it}\}} \sum_{t=1}^{T} \sum_{i=1}^{I} \left\{ C_i(p_{it}) + u_{it} S_i^{up} + x_{it} S_{it}^{NL} \right\}. \tag{9}$$

The UCED problem is thus formulated as a mixed-integer programming problem with integer variables $\{x_{it}\}$ and $\{u_{it}\}$.

The total cost function arising from the UCED problem is:

$$v(\{p_t^D\}) = \min_{\{p_{it}\} \in X(\{p_t^D\})} f(\{p_{it}\}), \tag{10}$$

where

$$X(\{p_t^D\}) = \left\{ \sum_{i=1}^{I} p_{it} = p_t^D, \forall t \right\}, \tag{11}$$

$$f(\{p_{it}\}) = \min_{\{u_{it}\},\{x_{it}\}} \sum_{t=1}^{T} \sum_{i=1}^{I} \left\{ C_i(p_{it}) + u_{it} S_i^{up} + x_{it} S_{it}^{NL} \right\}, \tag{12}$$

subject to constraints (1)–(4). The total cost function (10) is non-convex as a result of the integer commitment decisions with start-up cost S_i^{up} immediately incurred when a unit is turned on $u_{i,ton} = 1$ and no-load cost S_{it}^{NL} incurred at each online hour $x_{it} = 1, t \in [t_{on}, t_{off})$.

After the UCED problem is solved, e.g., by using branch and cut methods, commitment decisions in (9) are fixed at their optimal values $\{u_{it}^*\}$ and $\{x_{it}^*\}$, and LMPs are calculated by solving the resulting LP economic dispatch problem. Single-hour dispatch with $T = 1$ in (9) is used by ISOs such as MISO and ISO-NE for each time t, and the generation levels at $t-1$ in ramp rate constraints (4) are fixed as $\overline{p}_{i,t-1}$ [3], [9]:

$$-\Delta_{it} \leq p_{it} - \overline{p}_{i,t-1} \leq \Delta_{it}, \quad \forall i,\ t. \tag{13}$$

In this pricing problem with fixed commitment decisions $\{u_{it}^*\}$ and $\{x_{it}^*\}$, the commitment costs in (9) are constants and a unit generating at p_i^{min} in (3) may not be marginal. As a result, LMPs may not reflect the associated costs.

B. The ELMP Model

The Lagrangian of the UCED problem is obtained by relaxing the demand constraints (8) using multipliers $\{\lambda_t\}$:

$$L = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{I} \left(C_i(p_{it}) + u_{it} S_i^{up} + x_{it} S_{it}^{NL} \right) + \lambda_t \left(p_t^D - \sum_{i=1}^{I} p_{it} \right) \right\}. \tag{14}$$

The relaxed problem to minimize (14) given $\{\lambda_t\}$ can be decomposed into unit-level subproblems, one for each unit:

$$\min_{\{p_{it}\},\{u_{it}\},\{x_{it}\}} L_i, \text{ with}$$
$$L_i \equiv \sum_{t=1}^{T} \left\{ C_i(p_{it}) - \lambda_t p_{it} + u_{it} S_i^{up} + x_{it} S_{it}^{NL} \right\}. \tag{15}$$

subject to individual unit constraints (1)–(4). The subproblem (15) can be solved by using dynamic programming (DP). Denoting $L_i^*(\lambda)$ as the minimized subproblem cost, the dual problem is to select multipliers that maximize the concave and piecewise linear dual function [10]:

$$\max_{\lambda} \quad q(\lambda), \text{ with}$$
$$q \equiv \sum_{i=1}^{I} L_i^*(\lambda) + \sum_{t=1}^{T} \lambda_t p_t^D. \tag{16}$$

The optimal dual value $q^*(\{p_t^D\})$ as a function of demand forms the convex hull of the total cost function (10) [6], [20]:

$$v^c\left(\{p_t^D\}\right) = q^*\left(\{p_t^D\}\right). \tag{17}$$

The optimal multipliers $\{\lambda_t^*\}$ are the slopes, i.e., the marginal costs of the convex hull, and are established as ELMPs. The marginal costs of the convex hull (17) reflect the fixed costs in the non-convex cost function (10). The costs of committing or

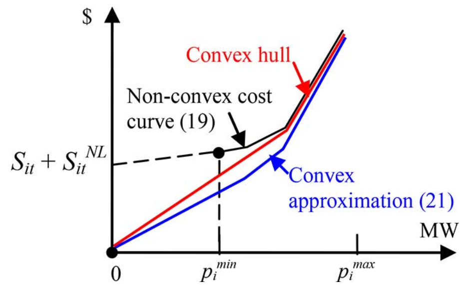

Fig. 1. Cost function of unit $i \in F$ at time t and its convex approximations.

dispatching a unit at p_i^{min} are thus reflected in ELMPs. However, ELMPs require the optimal multipliers for the mixed-integer UCED problem over $T = 24$ hours in (16), and are computationally expensive [10], [11].

C. The Approximate ELMP Model

The single-hour aELMP model is developed by MISO with reduced complexity for the market implementation of ELMPs. The aELMP is calculated after the results of unit commitment $\{x_{it}^*\}, \{u_{it}^*\}$ and the associated dispatch $\{p_{it}^*\}$ are obtained.

To capture ELMPs' feature of incorporating the costs of committing or dispatching fast-start units at their minimum limits, commitment costs of fast-start unit $i \in F$ are allocated to online hour t as $(S_{it} + S_{it}^{NL})$, where

$$S_i^{up} = \sum\nolimits_t S_{it},\ t \in [t_{on}, t_{off}). \tag{18}$$

The cost curve of fast-start unit i at hour t is thus:

$$f_{it}(p_{it}) = \begin{cases} C_i(p_{it}) + S_{it} + S_{it}^{NL};\ p_i^{\min} \le p_{it} \le p_i^{\max} \\ 0;\ p_{it} = 0. \end{cases} \tag{19}$$

This cost curve is convexified by using a commitment fraction satisfying (3) and:

$$0 \le x_{it} \le 1. \tag{20}$$

As shown in Fig. 1, the convexified cost curve is obtained as:

$$f_{it}(p_{it}, x_{it}) = C_i(p_{it}) + (S_{it} + S_{it}^{NL})x_{it}. \tag{21}$$

Commitment costs of slow-start units are not included in aELMPs, since these costs are usually covered by prices set by more expensive units and are not likely to participate in setting ELMPs [7]. The commitment decisions of these units are fixed at their optimal values $\{x_{i\notin F,t}^*\}$ of the UCED problem.

The aELMP at time t is thus obtained by minimizing the total energy cost and the commitment costs of fast-start units[3]:

$$\min_{\{p_{it}\},\{x_{i\in F,t}\}} \sum_{i=1}^{I} C_i(p_{it}) + \sum_{i\in F} x_{it}\left(S_{it} + S_{it}^{NL}\right). \tag{22}$$

subject to constraints (3), (8), (13) and (20). In this problem, parameters include the allocated start-up cost S_{it} in (22), commitment of slow-start units $x_{i\notin F,t}^*$ and the generation level $\overline{p}_{i,t-1}$ in ramp rate constraints (13). The aELMP is the optimal multiplier associated with the demand constraint (8).

[3]Usually, only online fast-start units are considered. Under transient shortage, offline fast-start units are also included to set aELMPs [9].

Different allocations of commitment costs $\{S_{it} + S_{it}^{NL}\}$ in (22) result in different values of the aELMP. Therefore, the allocation of commitment costs is a key design issue. With no-load cost S_{it}^{NL} included at each online hour, the start-up cost should be appropriately allocated over a set of online hours, i.e., the allocated start-up cost S_{it} associated with commitment fraction x_{it}, $t \in [t_{on}, t_{off})$ should be carefully designed.

IV. Solution Methodology

To derive guidelines for the allocation, the ELMP problem is separated into individual hour problems resembling the aELMP model in Section IV-A. KKT conditions are used in Section IV-B to derive the guidelines that can be easily implemented by using the commitment and dispatch results. In Section IV-C, a method to examine effectiveness of the guidelines is developed. MISO's implementation considering these guidelines is discussed in Section IV-D

A. Time-Decoupling by Lagrangian Relaxation

Established as the dual of the UCED problem, the ELMP problem is coupled over time by state-transition constraints (1) and ramp rate constraints (4). To study the allocation of the commitment costs of fast-start units, it is separated into individual-hour problems to resemble the aELMP model. With commitment decisions of slow-start units fixed and their integrality for fast-start units relaxed in the UCED problem, state-transition constraints that couple commitment decisions of fast start units over time are relaxed by using Lagrange multipliers $\{\beta_{i\in F,t}\}$. The relaxed problem is:

$$\min_{\{p_{it}\},\{u_{i\in F,t}\},\{x_{i\in F,t}\}} \sum_{t=1}^{T}\left\{\sum_{i=1}^{I} C_i(p_{it}) + \sum_{i\in F}\left(u_{it}S_i^{up} + x_{it}S_{it}^{NL} + \beta_{it}\left(x_{it} - x_{i,t-1} - u_{it}\right)\right)\right\} \tag{23}$$

subject to constraints (3), (4), (8) and

$$0 \le u_{it} \le 1, \forall\, i \in F, t \tag{24}$$

$$0 \le x_{it} \le 1, \forall\, i \in F, t. \tag{25}$$

By fixing the generation at hour $t-1$ in ramp rate constraints (4), the relaxed problem (23) at each hour t is obtained as:

$$\min_{\{p_{it}\},\{u_{i\in F,t}\},\{x_{i\in F,t}\}} \sum_{i=1}^{I} C_i(p_{it}) + \sum_{i\in F}\left(\left(S_{it}^{NL} + \beta_{it} - \beta_{i,t+1}\right)x_{it} + \left(S_i^{up} - \beta_{it}\right)u_{it}\right), \tag{26}$$

subject to constraint (3), (8), (13), (24) and (25). This problem resembles the aELMP problem (22) with the multipliers, i.e., shadow prices, coordinating the allocation of the associated commitment costs of fast-start units.

B. Optimality Analysis by KKT Conditions

With problem (26) obtained resembling the aELMP model, an allocation can be obtained from the optimal multipliers. However, the allocation is needed to be easily implemented without optimizing (26) numerically. KKT conditions are thus innovatively used to derive allocation guidelines that utilize the unit commitment and dispatch results.

The problem (26) involves many variables and constraints, while the allocation is coordinated by multipliers $\{\beta^*_{i\in F,t}\}$ of the state-transition constraints only. KKT conditions are thus applied in a flexible manner only to this subset of constraints, while other constraints are dealt with explicitly to maintain feasibility and optimality [10]. The KKT conditions applied to problem (26) are obtained as:

$$\beta^*_{it} \geq 0, \tag{27}$$
$$\beta^*_{it}\left(x^*_{it} - x^*_{i,t-1} - u^*_{it}\right) = 0, \tag{28}$$

and that $\{x^*_{it}\}$, $\{u^*_{it}\}$, and $\{p^*_{it}\}$ solves problem (26) given β^*_{it}.

These conditions are used to characterize the allocated start-up cost S_{it} in (22) by multiplier $\beta^*_{i\in F,t}$ in (26). For $u_{it} \in [0,1]$ to minimize the linear objective of problem (26), u^*_{it} can take any value in [0, 1] if its derivative is zero, $u^*_{it} = 0$ if the derivative is positive and $u^*_{it} = 1$ if the derivative is negative:

$$u^*_{it} \in [0,1] \quad for\ \beta^*_{it} = S^{up}_i, \tag{29a}$$
$$u^*_{it} = 0 \quad for\ \beta^*_{it} < S^{up}_i, \tag{29b}$$
$$u^*_{it} = 1 \quad for\ \beta^*_{it} > S^{up}_i. \tag{29c}$$

Nevertheless, (29c) violates the complementary slackness (28) at online hour $t \in (t_{on}, t_{off})$ since $x^*_{i,t-1} > 0$ and $\beta^*_{it}(x^*_{it} - x^*_{i,t-1} - u^*_{it}) < 0$, unless $t = t_{on}(x^*_{i,ton-1} = 0)$ and $x^*_{i,ton} = 1$. The cost in (26) is thus obtained for (29a) and (29b) as:

$$\min_{\{p_{it}\},\{x_{i\in F,t}\}} \sum_{i=1}^{I} C_i(p_{it}) + \sum_{i\in F}\left(S^{NL}_{it} + \beta^*_{it} - \beta^*_{i,t+1}\right)x_{it}, \tag{30}$$

and for (29c) only when $t = t_{on}$ and $x^*_{i,ton} = 1$ as:

$$\min_{\{p_{it}\},\{x_{i\in F,t}\}} \sum_{i=1}^{I} C_i(p_{it}) + \sum_{i\in F}\left(S^{NL}_{it} + S^{up}_i - \beta^*_{i,t+1}\right)x_{it}. \tag{31}$$

Comparing (30) and (31) with the cost in the aELMP problem (22), the allocated start-up cost is:

$$S_{it} = \beta^*_{it} - \beta^*_{i,t+1}, \tag{32}$$

or in the special case of $t = t_{on}$ and $x^*_{i,ton} = 1$ is:

$$S_{it} = S^{up}_i - \beta^*_{i,t+1}. \tag{33}$$

The allocated start-up cost S_{it} associated with x_{it} is obtained as above, because commitment decision x_{it} is involved in two state transitions $x_{it} - x_{i,t-1} \leq u_{it}$ with the shadow price β^*_{it} and $x_{i,t+1} - x_{it} \leq u_{i,t+1}$ with the shadow price $\beta^*_{i,t+1}$.

With allocation $\{S_{it}\}$ characterized by $\{\beta^*_{i\in F,t}\}$, the optimal multipliers are obtained by using the unit commitment and dispatch results. If the state-transition constraint is active, i.e., $x_{it} - x_{i,t-1} = u_{it}$, then the shadow price $\beta^*_{it} = S^{up}_i$ in (29a), since the state-transition costs associated with the increase u_{it} of commitment fraction x_{it} is $u_{it}S^{up}_i$ in (9). If the constraint is not active, i.e., $x_{it} - x_{i,t-1} < u_{it}$, then $\beta^*_{it} = 0$, and $u^*_{it} = 0$ as in (29b). It is a marginal case when x_{it} remains the same and $\beta^*_{it} \in [0, S^{up}_i]$ as in (29a) and (29b). The following are thus obtained:

$$x^*_{it} - x^*_{i,t-1} = u^*_{it} > 0 \Rightarrow \beta^*_{it} = S^{up}_i, \tag{34a}$$
$$x^*_{it} - x^*_{i,t-1} < u^*_{it} = 0 \Rightarrow \beta^*_{it} = 0, \tag{34b}$$
$$x^*_{it} - x^*_{i,t-1} = u^*_{it} = 0 \Rightarrow \beta^*_{it} \leq S^{up}_i. \tag{34c}$$

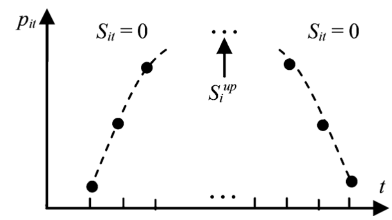

Fig. 2. Start-up cost allocation over a set of online hours.

Category (34a) describes the hours when a fast-start unit is turned on and its usage is then increased. When the usage is decreased until the unit is turned off, these hours are of category (34b). At the hours in-between, the usage is usually at the peak but may further vary and these hours may be represented by all the three categories.

Proposition:

i) If $p_{i,\tau-1} < p_{i\tau}$, $\forall\tau \in [t_{on}, t]$, then $\beta^*_{it} = S^{up}_i$;
ii) If $p_{i\tau} < p_{i,\tau-1}$, $\forall\tau \in [t, t_{off})$, then $\beta^*_{it} = 0$.

Proof: At $t = t_{on}$, $x_{it} > 0$, $x_{i,t-1} = 0$. With $x_{it} - x_{i,t-1} > 0$, $\beta^*_{i,ton} = S^{up}_i$ in (34a). For $t > t_{on}$, considering that $\beta^*_{i,t+1} \leq S^{up}_i$, the gradient of x_{it} in (26) $g(x_{it}) = S^{NL}_{it} + \beta^*_{it} - \beta^*_{i,t+1} > 0$. To minimize the linear objective, $x_{it} = p_{it}/p^{max}_i$ in (3). In view that $x_{i,t+1} \geq p_{i,t+1}/p^{max}_i$, it is thus shown that $x_{i,t+1} - x_{it} > 0$. The decreasing usage hours when the state-transition constraint in (34b) is not active can be similarly shown. □

Combining this proposition with the allocation in (32) or (33), allocation guidelines are obtained as shown in Fig. 2.

Allocation Guidelines:

i) If $p_{i,\tau-1} < p_{i\tau}$, $\forall\tau \in [t_{on}, t]$, then $S_{it} = 0$;
ii) If $p_{i\tau} < p_{i,\tau-1}$, $\forall\tau \in [t, t_{off})$, then $S_{it} = 0$.

Start-up costs are allocated to the hours in-between when a fast-start unit is most needed, because the demand at these hours is the main factor driving the start-up of the unit. The allocated costs add up to the total start-up cost satisfying (18):

$$\sum_t S_{it} = \beta^*_{it} - \beta^*_{i,t+1} + \beta^*_{i,t+1} - \ldots = \beta^*_{i,ton} - \beta^*_{i,toff} = S^{up}_i. \tag{35}$$

C. Effectiveness Examination via Approximate ELMPs

To examine effectiveness of the guidelines, aELMPs are compared with ELMPs and LMPs. The comparison is based on the hourly average price, uplift payments and the total load payment following [7]. The uplift payments include the costs not covered by prices and opportunity costs as defined in [5], and the total load payment is the payment by prices plus the uplift payments. By using the guidelines, allocation method (c) is selected from a set of alternative allocation methods:

a) Allocate to the first online hour;
b) Allocate evenly to all the online hours;

TABLE I
SUPPLY OFFERS OF EXAMPLE 1

Unit	Start-up $	No-load $	Energy cost $/MW	p^{min} MW	p^{max} MW
G1	1000	45	25	300	400
G2	1000	45	30	100	130
G3*	100	45	35	10	130
G4*	100	45	36	10	100
G5*	100	45	37	10	10

c) Allocate to the peak usage hours;
d) Allocate based on the energy usage [21];
e) Allocate based on the capacity usage [21].

The aELMPs are obtained for each of these five methods, and are compared with ELMPs and LMPs. The guidelines are effective if aELMPs of method (c) are closest to ELMPs and have the most uplift payment reduction compared to LMPs.

D. Discussion of MISO's Implementation

The guidelines are considered in MISO's allocation design for both the day-ahead and the real-time markets. However, to obtain aELMPs in real-time, the commitment and dispatch of a fast-start unit planned for the forecasted load in the unit commitment processes as reviewed in Section II may not be accurate. Currently, MISO only incorporates into prices the commitment costs for fast-start units that can be started within 10 minutes and have min-run times within 1 hour. These units are usually started up when they are most needed and may be turned off after the min-run time. Therefore, start-up costs are allocated to the min-run time after start-up. In the future, when commitment costs for more units are considered and a multi-interval aELMP model is used, the guidelines will be valuable.

V. NUMERICAL RESULTS

The aELMP model was implemented in IBM ILOG CPLEX 12.2 on a Dell M4600 laptop with Intel Core i7-2820QM CPU 2.30 GHz RAM 8 GB. Three examples are presented. Example 1 is a simple five-unit four-hour problem to illustrate the process of obtaining aELMPs with allocations by directly using the guidelines and by using MISO's allocation. Example 2 is a 32-unit 24-hour problem and its one hundred simulation runs based on the IEEE Reliability Test System to show that the allocation method selected by the guidelines obtains the best approximation of ELMPs. Example 3 is a MISO-sized problem to show the effectiveness of the guidelines in a practical-sized problem.

Example 1: Consider a five-unit four-hour problem. Supply offers of the five units are specified in Table I with each identical over the four hours. For simplicity, these units are assumed initially off. Units G3, G4 and G5 are fast-start units, among which G4 and G5 have minimum run times of one hour.

The UCED problem (9) is solved by using the branch-and-cut method in CPLEX as shown in Table II. Based on these UCED results, commitment costs are allocated by using the guidelines for fast-start units G3 and G4 over their online hours. Both units G3 and G4 are most needed at hour 3 since they generate at the peak among all the online hours. The start-up cost is thus allocated to hour 3 and the no-load cost is included at each online hour as shown in Table III. Given the commitment cost allocation $\{S_{it} + S_{it}^{NL}\}$, aELMPs$^{(1)}$ are obtained by solving (22) using the LP solver of CPLEX.

TABLE II
DEMAND AND THE UCED SOLUTIONS OF EXAMPLE 1

Hour	1	2	3	4
Demand	600	625	663	647
G1	400	400	400	400
G2	130	130	130	130
G3*	70	95	123	117
G4*	Off	Off	10	Off
G5*	Off	Off	Off	Off

TABLE III
COMMITMENT COST ALLOCATION BY USING THE GUIDELINES FOR EXAMPLE 1

Hour		1	2	3	4
$S_{it} + S_{it}^{NL}$	G3*	45	45	145	45
	G4*	0	0	145	0

TABLE IV
PRICES AND THEIR HOURLY AVERAGE, UPLIFT PAYMENTS AND THE TOTAL LOAD PAYMENT OF EXAMPLE 1

Hour	1	2	3	4	Aver.	Uplift	Total
LMP	35	35	35	35	35	435	89160
ELMP	35.35	35.35	37.45	35.35	35.88	185.45	91190
aELMP$^{(1)}$	35.35	35.35	37.45	35.35	35.88	185.45	91190
aELMP$^{(2)}$	35	35	37.45	35	35.61	282.65	90632

The aELMPs$^{(2)}$ are obtained by using MISO's allocation as discussed in Section IV-D, and only G4 with the minimum-run time of one hour is qualified for the allocation. Its start-up cost is allocated to the hour 3 after start-up as $S_{43} + S_{43}^{NL} = 145$. This allocation is consistent with the allocation for G4 obtained by using the guidelines in Table III. LMPs are also obtained by using CPLEX and ELMPs are obtained by solving problem (16) using the subgradient simplex cutting plane method in [18] for comparison as presented in Table IV.

As can be seen, LMPs cannot cover the commitment costs of G3, and cannot be set by G4 that is dispatched at its minimum limit. As a result, uplift payments of $435 are used. ELMPs incorporate the costs of committing G3 and dispatching G4 at its minimum limits, and uplift payments are reduced to $185.45. Moreover, commitment costs of slow-start units G1 and G2 are covered by the prices and do not participate in setting ELMPs as discussed in Section I.

The aELMPs$^{(1)}$ obtained following the guidelines effectively captured these features of ELMPs' and uplift payments are reduced by 57% compared with those of LMPs. For this small example, aELMPs$^{(1)}$ are the same as ELMPs, but this may not always be true. The aELMPs$^{(2)}$ obtained following MISO's allocation have a difference of 0.6% in total load payments compared with ELMPs and a reduction of 35% in uplift payments compared with LMPs. Therefore, MISO's allocation for G4 is consistent with that by using the guidelines and obtains aELMPs$^{(2)}$ as good approximations of ELMPs.

Example 2: Consider a 32-unit 24-hour problem based on the IEEE Reliability Test System of 1996 [22]. The system includes generating units, hourly system load of a year and a transmission network which is not used here. Generation offers as presented

TABLE V
GENERATION OFFERS OF EXAMPLE 2

Unit type	No. of Units	P^{min} MW	P^{max} MW	Incremental cost $/MWh	Start-up $
U12	5	3	12	67.15	247
U20*	4	10	20	65.9	32.5
U50	6	0	50	15	0
U76	4	10	76	12.16	727.1
U100	3	15	100	56.60	1625
U155	4	20	155	10.42	317.2
U197	3	20	197	57.42	2880
U350	1	35	350	10.85	2336
U400	2	40	400	43.03	6000

TABLE VI
DEMAND AND UCED SOLUTIONS OF FAST-START UNITS FOR EXAMPLE 2

Hour	11	12	13	14	15
p^D	3271.5	3300	3271.5	3300	3300
U20-1	10	20	10	20	20
U20-2	0	15	0	15	15

TABLE VII
COMMITMENT COST ALLOCATION OF EXAMPLE 2

t	$S_{it}^{(a)}$ $		$S_{it}^{(b)}$ $		$S_{it}^{(c)}$ $		$S_{it}^{(d)}$ $		$S_{it}^{(e)}$ $	
	G1	G2	G1	G2	G1	G2	G1	G2	G1	G2
11	32.5	0	6.5	0	0	0	4.06	0	3.25	0
12	0	32.5	6.5	32.5	10.83	32.5	8.13	32.5	8.67	32.5
13	0	0	6.5	0	0	0	4.06	0	3.25	0
14	0	32.5	6.5	16.25	10.83	16.25	8.13	16.25	8.67	16.25
15	0	0	6.5	16.25	10.83	16.25	8.13	16.25	8.67	16.25

in Table V are obtained from production costs. All units are assumed initially off for simplicity. Units of type U20 are fast-start units. The peak-load day within the year is selected, and is appropriately increased as shown in Table VI, since the original values are satisfied with no fast-start units committed.

By solving the UCED problem (9), two fast-start units are committed as shown in Table VI at hours 11–15. Based on the UCED results, allocation $\{S_{it} + S_{it}^{NL}\}$ in problem (22) is obtained in Table VII by using each of the five allocation methods in Section IV-C. For method (c), since there are multiple peak usage hours, start-up costs of are allocated evenly to the hours that also have peak load.

The aELMPs are obtained by solving (22) and compared with ELMPs and LMPs in Table VIII. For simplicity of presentation, prices are shown only for hours 11–15 since all sets of prices are the same at other hours. As can be seen, among the five allocation methods, aELMPs (c) is closest to ELMPs in hourly average price and total load payments. Compared with LMPs, uplift payments of aELMPs (c) are reduced by 35.2%. This reduction is less than the 76.0% reduction of ELMPs, but is the most among all the five sets of aELMPs. Therefore, the allocation method selected by the guidelines obtains the best aELMPs.

To examine effectiveness of the guidelines for different problems, 100 Monte Carlo simulation runs are performed by perturbing the load of the above case. For the convenience to preserve the load pattern, a uniform distribution is used at each hour with interval length selected as 2% of the base case value. The examination is conducted in each simulation run similarly as in

TABLE VIII
PRICES AND THEIR AVERAGE, UPLIFT AND TOTAL PAYMENTS FOR EXAMPLE 2

t	11	12	13	14	15	Aver.	Uplift	Total
(a)	67.53	67.53	65.9	67.53	65.9	55.12	292.5	3812349
(b)	66.23	67.53	66.23	66.71	66.71	55.08	220.9	3809087
(c)	65.9	67.53	65.9	66.71	66.71	55.05	173.1	3806913
(d)	66.10	67.53	66.10	66.71	66.71	55.07	203.0	3808272
(e)	66.06	67.53	66.06	66.71	66.71	55.07	197.0	3808000
ELMP	65.9	67.53	62.65	67.53	65.9	54.92	64.2	3796172
LMP	57.42	65.9	57.42	65.9	65.9	54.21	267.1	3740797

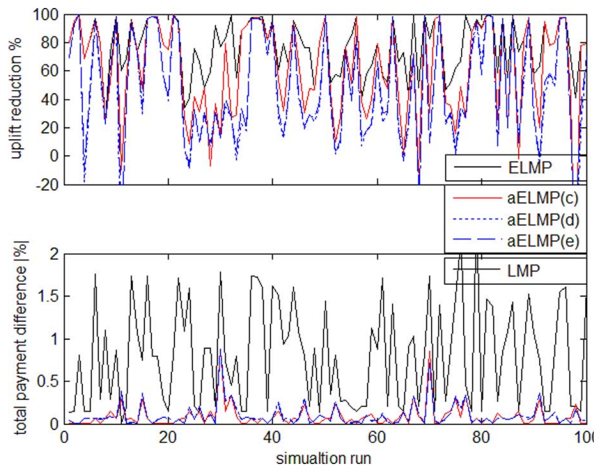

Fig. 3. Reduction in uplift payments compared with LMPs and the difference in total payment compared with ELMPs for Example 2.

the base case. In addition, allocation methods (a) and (b) that resulted in worse approximations of ELMPs than other methods are not repeated anymore.

The aELMPs are compared with ELMPs and LMPs below. Over the 100 simulation runs, reduction (in percentage) of the uplift payments compared with those of LMPs is shown in Fig. 3 for ELMP, aELMPs (c), (d) and (e). The difference (in absolute percentage) in total load payment compared with that of ELMPs is also shown for each set of aELMPs and LMPs, and the difference in hourly average price has a similar pattern. As is shown, ELMPs obtain the most uplift reduction, since they minimize uplift payments as discussed in Section I. By effectively approximating ELMPs, aELMPs (c) generally reduce more uplift payments than aELMPs (d) and (e) except for a few simulation runs (4/100). Also, aELMPs (c) have a smaller difference in total load payment than aELMPs (d) and (e) in most of the simulation runs (71/100). The guidelines are thus effective for different problems.

Example 3: Consider a MISO-sized problem in the day-ahead market with 1207 units over 24 hours. Allocation is performed for fast-start units that can be started-up within 10 minutes. The guidelines are examined following a similar process as that in Example 2. Moreover, there are different options of method (c) when there are multiple peak usage hours. To investigate the sensitivity, two options are studied: (c_1) Allocate to the peak usage hour which has the peak load; (c_2) Allocate evenly to the peak usage hours.

For this practical-sized example, the aELMPs are obtained by using similar amounts of CPU time as those of LMPs, since the computational complexity of aELMPs is much reduced than

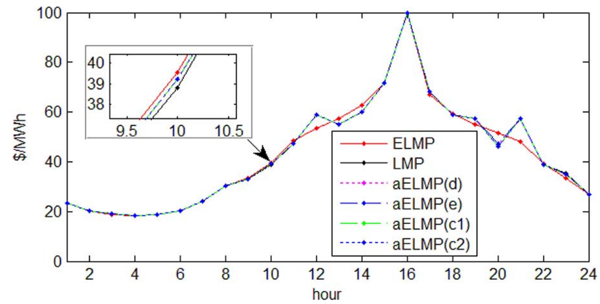

Fig. 4. Approximate ELMPs, ELMPs and LMPs for Example 3.

TABLE IX
COMPARISON OF AVERAGE, UPLIFT AND TOTAL PAYMENTS FOR EXAMPLE 3

	aELMP$^{(c1)}$	aELMP$^{(c2)}$	aELMP$^{(d)}$	aELMP$^{(e)}$	ELMP	LMP
Aver.	42.84	42.84	42.88	42.88	42.50	42.84
Uplift	109347	109347	109853	109836	74920	110249
Total	81196254	81196254	81273621	81270984	80528537	81180013

that of ELMPs. The aELMPs of allocation methods (c_1), (c_2), (d) and (e) are depicted in Fig. 4 comparing with ELMPs and LMPs. As can be seen, the four sets of aELMPs, ELMPs and LMPs are very close in this day-ahead example. Their hourly average value, uplift payments, and the total load payment are then further compared in Table IX.

Compared with ELMPs, the aELMPs (c_1) and (c_2) have less difference in the average price or the total load payment than aELMPs (d) and (e). Uplift payments of aELMPs (c_1) and (c_2) are reduced by 0.82% compared with those of LMPs, while the reductions of aELMPs (d) and (e) are only 0.36% and 0.37%, respectively. Moreover, aELMPs (c_1) and (c_2) are not sensitive to the two options of allocating over multiple peak usage hours given the limited number of 10-minute fast-start units [7]. The allocation guidelines are thus shown effective for this practical-sized problem.

VI. CONCLUSION

The aELMP model is developed by MISO for the market implementation of ELMPs which can appropriately reflect the costs of committing or dispatching fast-start units at their minimum limits and thus minimize uplift payments. This paper derived guidelines for an important design issue of allocating the commitment costs of fast-start units over time. By separating the ELMP problem to individual-hour problems resembling the aELMP model, allocation guidelines are to allocate start-up costs of fast-start units to the hours when they are most needed, since the start-up is mostly driven by the demand at these hours. These guidelines are demonstrated effective for different problems and are considered in MISO's design which allocates for fast-start units to their minimum run time after start-up when they are most needed.

REFERENCES

[1] S. Stoft, *Power System Economics*. New York, NY, USA: Wiley-Interscience, 2002, pp. 277–285.

[2] C. Ruiz, A. Conejo, and S. A. Gabriel, "Pricing non-convexities in an electricity pool," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1334–1342, Aug. 2012.

[3] *"ISO New England, Manual for Market Operations Manual M-11, Revision 39,"* Feb. 6, 2012.

[4] *"Transmission and Dispatching Operations Manual, Version 2.3,"* NYISO, Oct. 2012.

[5] P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-Clearing Electricity Prices and Energy Uplift," Dec. 31, 2007 [Online]. Available: http://ksghome.harvard.edu/~whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf

[6] A. M. Geoffrion, "Lagrangian relaxation for integer programming," *Mathematical Programming Study*, no. 2, pp. 82–114, 1974.

[7] in *Proc. ELMP Task Team Meeting Presentation*, Mar. 04, 2011 [Online]. Available: https://www.midwestiso.org/Library/Meeting-Materials/Pages/ELMPTT.aspx

[8] in *Proc. ELMP Task Team Meeting Presentation*, Oct. 05, 2010 [Online]. Available: https://www.midwestiso.org/Library/Meeting-Materials/Pages/ELMPTT.aspx

[9] in *Proc. ELMPTT Procedure Detailed Outline*, Sept. 23, 2011 [Online]. Available: https://www.midwestiso.org/Library/Meeting-Materials/Pages/ELMPTT.aspx

[10] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA, USA: Athena Scientific, 2003.

[11] M. L. Fisher, "The Lagrangian relaxation method for solving integer programming problems," *Management Sci.*, vol. 50, no. 12, pp. 1861–1871, Dec. 2004.

[12] H. E. Scarf, "The allocation of resources in the presence of indivisibilities," *J. Economic Perspectives*, vol. 8, no. 4, pp. 111–128, 1994.

[13] H. E. Scarf, "Mathematical programming and economic theory," *Oper. Res.*, vol. 38, no. 3, pp. 377–385, Jun. 1990.

[14] in *Proc. ELMPTT Comparison Between MISO Recommendation and Other RTOs Practice*, Apr. 29, 2011 [Online]. Available: https://www.midwestiso.org/Library/MeetingMaterials/Pages/ELMPTT.aspx

[15] D. Martinez, "A methodology for the consideration of start-up costs into the marginal cost estimated with production cost models," in *Proc. 5th Int.Conf. Eur. Electricity Markets*, 2008, pp. 1–10.

[16] H. Schwarz and C. Lang, "The rise in German wholesale electricity prices: Fundamental factors, exercise of market power, or both?," *Working Paper*, Feb. 2006.

[17] F. Sensfuß and M. Genoese, "Agent-based simulation of the German electricity markets—An analysis of the German spot market prices in the year 2001," in *Proc. 9th Symp. Energy Innovation*, Graz, Austria, 2006.

[18] C. Wang, T. Peng, and P. B. Luh, "The subgradient simplex cutting plane method for extended locational marginal prices," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2758–2767, Aug. 2013.

[19] G. Wang and U. V. Shanbhag, "An extreme-point subdifferential method for convex hull pricing in energy and reserve markets—Part I: Algorithm structure," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2121–2127, Aug. 2013.

[20] J. E. Falk, "Lagrangian multipliers and nonconvex programs," *SIAM J. Control*, vol. 7, no. 4, pp. 534–545, 1969.

[21] "ELMPTT Allocating Start-up Cost to periods," Jan. 07, 2011 [Online]. Available: https://www.midwestiso.org/Library/Meeting-Materials/Pages/ELMPTT.aspx

[22] "The IEEE reliability test system—1996," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999, Reliability Test System Task Force.

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